THE DAMPING OF WATER WAVES BY SURFACE FILMS

By R. G. Vines*

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Summary

Measurements have been made of the stilling of small water ripples by surface films of cetyl alcohol. The damping is considerable and is somewhat in excess of that predicted by existing theories. Imperfections in the ripples induce a directed surface drift (surface mass transport), and it is possible that this is indirectly responsible for the extra damping. Under natural conditions surface films not only impede the formation of small waves but they are also very effective in damping them out.

I. Introduction

The calming action of oil on a water surface has been known from ancient times, but the phenomenon was not studied scientifically until the nineteenth century (see, for example, Aitken 1883). Lamb (1932) examined the theoretical aspects of the damping of water waves and a section of his work was devoted to the influence of surface films on capillary waves. More recently similar treatments were put forward independently by Levich (1941) and Dorrestein (1951).

Experimental measurements by Brown (1936) showed a marked increase in the damping of small ripples on water when surface films were present. However, no quantitative results were reported except for clean water; since the wave frequency was high ($\sim$300 c/s) damping must have been pronounced, and with surface films the experimental errors were probably large. Similar measurements have now been carried out at lower frequencies in the presence and absence of cetyl alcohol monolayers. The results are in only fair agreement with existing theoretical treatments, and the observed differences are well beyond the likely experimental error.

II. Experimental

The apparatus used is shown in Figure 1; it was similar to the arrangement described by Brown (1936) in that stroboscopic illumination of the waves was provided by a small vibrating mirror.

Light from a ribbon projection lamp P passed through a cylindrical lens L_1 and a narrow horizontal slit S situated close to the lamp. The concave mirror M_1 (radius 26 cm), rigidly fixed to one arm of a 50 cycle electric tuning fork, was used to produce an image of the slit at the point X, and the light was then rendered parallel by a large lens L_2 separated from X by a distance equal to its focal length ($\sim$15 cm). A plane mirror M_2, attached to a travelling telescope T, now deflected the light so that it fell normally upon the water surface in the

* Division of Physical Chemistry, C.S.I.R.O., Chemical Research Laboratories, Melbourne.

enamelled tank B; from here it was reflected back vertically into the telescope. The tank B was $\sim$3 cm deep, 30 cm wide, and 35 cm long and it was mounted on sponge rubber to reduce the effect of stray vibrations.

Small waves were generated in the tank by the dipper D which just touched the water surface. This dipper was a microscope slide $\sim 7\frac{1}{2}$ cm across, and it was held by a thin wooden rod directly connected to the tuning fork F with a wire (Watson 1901). As a result of this arrangement the frequency of the tuning fork was reduced to 49·4 c/s. When the fork was in operation ripples were obtained on the water which were exactly synchronized with the movement of the mirror M_1.

Through the telescope a series of stationary, narrow lines of light could be seen. These were images of the slit produced by the curvature of the ripples at their troughs and crests. The images from the troughs were above and those from the crests below the water surface, at heights which changed progressively

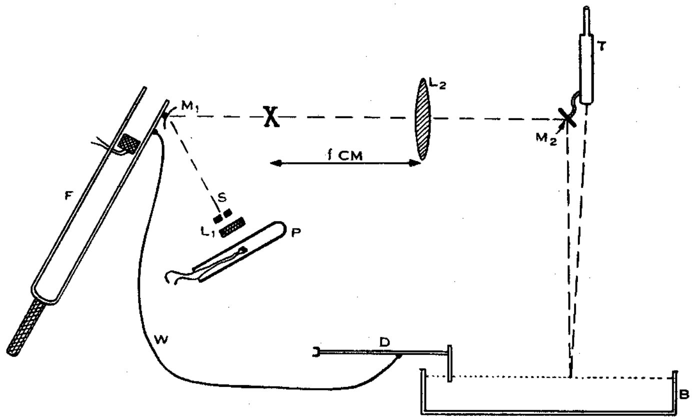

Fig. 1.—Diagram of equipment used. B, tank; D, dipper; F, 50 c/s tuning fork; L_1, cylindrical lens; L_2, lens of focal length f; M_1, M_2, mirrors; P, lamp (18A, 6V); S, slit; T, telescope; W, wire; X, image of slit.

with distance as the amplitude of the ripples decreased (cf. Watson 1901). In the present experiments only the images above the water surface—reflections from the troughs—were followed. The telescope was focused and one of the slit images centred on the cross-wires: a small pointer mounted on a travelling stage was then raised or lowered until it, too, was in focus, and its height above the water surface was measured. The telescope was now moved along an integral number of wave lengths, refocused, and the height of the images above the surface determined again with the pointer. In this way it was possible to observe, indirectly, the change in amplitude of the ripples at varying distances from the dipper, or wave source.

In some experiments the dipper was dispensed with and ripples were generated with a jet of air from a modified fish-tail burner. With care it was possible to stabilize the air stream so that waves of frequency exactly twice that of the tuning fork were obtained: under these circumstances stationary images were again observed through the telescope. It was more difficult to produce air-blown ripples at 49·4 c/s ; however, a few rough measurements were made at this frequency.

The wave lengths of the ripples were determined at the same time as the other measurements were made ; over the entire distance of travel of the telescope (see Fig. 2) there was no detectable change in wave length.

III. Theoretical

The amplitude of damped straight ripples originating from a long dipper is given by

$$a = a_0 e^{-\mu x}, \qquad (1)$$

where μ is the damping factor and x is the distance from the wave source at which the ripples possess an initial amplitude a_0. When parallel light falls vertically on water ripples of this type, the troughs act as cylindrical mirrors and series of focal lines are produced above the surface. The focal length f of any one of the mirrors is related to the wavelength λ and amplitude a of the ripple, by the equation (Brown 1936)

$$a = \lambda^2/8\pi^2 f. \qquad (2)$$

Thus, from (1),

$$1/f = (1/f_0) e^{-\mu x}, \qquad (3)$$

where f_0 is the focal length of a ripple of amplitude a_0. It follows that the damping factor μ is given by the slope of the linear plot*

$$\ln f = \mu x + \text{constant}. \qquad (4)$$

Theoretical treatments of damping have been concerned with extended wave trains, where *all* ripples have the same initial amplitude a_0, which everywhere decreases with time t according to the equation

$$a = a_0 e^{-t/\tau}, \qquad (5)$$

τ being the modulus of decay.

Although equations (1) and (5) refer to different physical situations, the modulus of decay and the damping factor μ are simply related to each other in terms of the " group velocity " V of the wave train (Dorrestein 1951, p. 272), that is,

$$\tau = 1/\mu V. \qquad (6)$$

According to theory (Lamb 1932, p. 624) the modulus of decay τ_1 when viscosity is the controlling factor in damping, is given by

$$\tau_1 = 1/2\nu k^2 : \qquad (7)$$

* To obtain a value for μ it is not necessary to know the actual position of the wave source, and this was never recorded in the experiments. All measurements were made at distances greater than $\sim$5 cm from the dipper, because of excessive deformation of the surface in its immediate vicinity.

here ν is the kinematic viscosity and $k=2\pi/\lambda$, λ again being the wavelength. For water at 20 °C ($\nu=0\cdot01$ cm²/sec) this gives

$$\tau_1 \simeq 1\cdot25\lambda^2. \qquad (8)$$

When surface films are present, the extensions and contractions of the surface associated with the passage of waves are opposed by the variation in surface tension so produced. It is this which is mainly responsible for the damping of the ripples. Under these conditions (Dorrestein 1951)

$$\tau_2=(8/\nu k^2\sigma)^{\frac{1}{2}},$$

where

$$\sigma^2=gk+T'k^3: \qquad (9)$$

here g is the acceleration due to gravity, T' the "specific" surface tension (surface tension/density), and k, as before, is $2\pi/\lambda$. For water the density ρ is unity and T' is numerically equal to the surface tension T, which is derived from the well-known equation for the velocity of propagation v of surface waves under the combined influence of gravity and surface tension (Thomson 1871),

$$v=(g\lambda/2\pi+2\pi T/\lambda\rho)^{\frac{1}{2}}. \qquad (10)$$

This wave velocity v is equal to the product $n\lambda$ where n is the wave frequency.

From equations (**7**) and (**9**) values of μ corresponding to τ_1 and τ_2 may be obtained in terms of equation (**6**). These are:

$$\mu_1=2\nu k^2/V, \qquad (11)$$

$$\mu_2=(\tfrac{1}{8}\nu k^2\sigma)^{\frac{1}{2}}/V. \qquad (12)$$

The value of the group velocity V is calculated from the relation (Lamb 1932)

$$V=v-\lambda\mathrm{d}v/\mathrm{d}\lambda. \qquad (13)$$

IV. Results

(*a*) *Clean Water*

The tank B was thoroughly cleaned and filled with water at ~20 °C and it was allowed to overflow gently to reduce the concentration of impurities at the surface. Typical results are shown in Figure 2 (*a*), where the logarithm of f, the distance of the focal lines above the water surface, is plotted as a function of distance x along the tank (the zero is quite arbitrary and x does *not* represent the exact distance from the dipper). The slope of the line drawn is $0\cdot029$/cm and this must be multiplied by $2\cdot303$ to obtain the value of μ_{obs} ($0\cdot06_5$ cm). The observed wavelength was $0\cdot595$ cm (wave frequency $49\cdot4$ c/s) and, as may be seen in Table 1, the corresponding theoretical value of μ_1 (cf. equation (**11**), with $V=40\cdot9$ cm/sec) is $0\cdot055$/cm. From the present measurements the surface tension of water, equation (**10**), is $72\cdot3$ dyne/cm.

Similar measurements by Brown (1936) give $T=72\cdot1$ dyne/cm for the water used in his experiments. Corresponding to a wave frequency of 300 c/s ($\lambda=0\cdot172$ cm) his value for μ_{obs} is $0\cdot45$/cm: the theoretical value (equation (**11**) with $V=77\cdot0$ cm/sec) is $0\cdot35$/cm. Theory requires that τ_1, the modulus of decay, be proportional to λ^2, equation (8), and calculation of the proportionality constant gives 1·03 from the present results, and 0·97 from Brown's results. Although these figures are reasonably constant in themselves, they are both about 20 per cent. lower than the theoretical value of $\sim$1·25 ; thus the damping is somewhat greater than predicted.

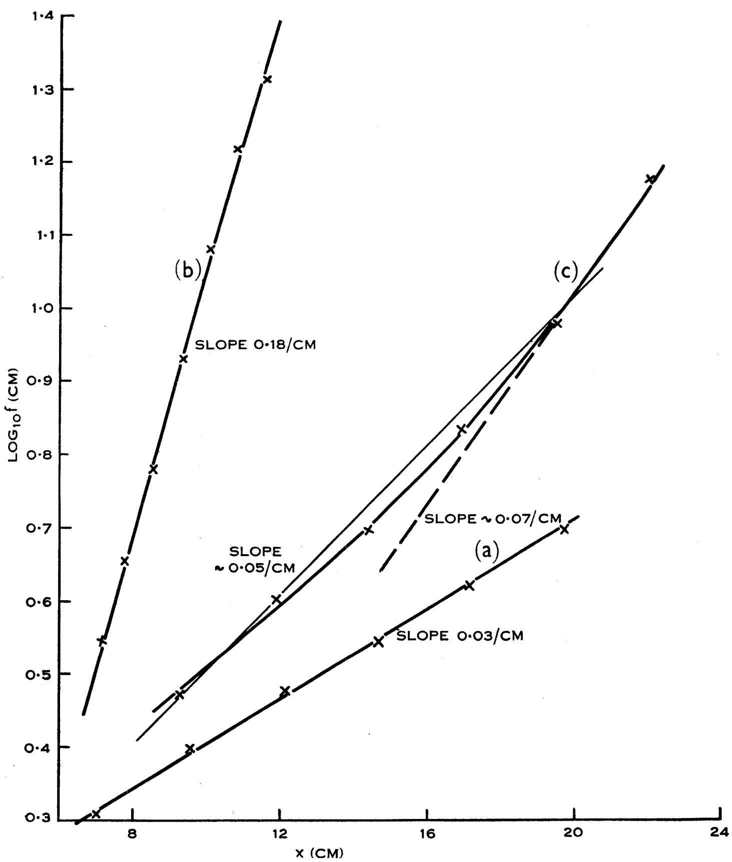

Fig. 2.—The damping of small water ripples as a function of distance x. (*a*) Clean water surface, (*b*) with high pressure cetyl alcohol film, (*c*) with trace of ethyl stearate.

Better agreement with theory was occasionally obtained during the present experiments, but in general the value of μ_{obs} was rather too high. This may have been due in part to the presence of slight traces of surface impurities ; however, in view of the results of subsequent measurements with cetyl alcohol films, it is probably a genuine effect.

(*b*) *Surface Films*

A surface film was introduced by spreading one drop of a solution of cetyl alcohol in a low boiling petroleum ether. The monolayer obtained was of very high pressure ($\sim$47 dyne/cm) and, as is often the case when spreading takes place from solution, this exceeds the equilibrium pressure of a cetyl alcohol film in contact with the solid. A typical result is shown in Figure 2 (*b*); it may be seen that, in comparison with Figure 2 (*a*), the damping is greatly increased, for the slope of the line has now risen to 0·18/cm ($\mu_{obs}=0\cdot41$/cm when $T=25\cdot5$ dyne/cm). Other measurements were made at different surface pressures, and the results of these and the earlier experiments are summarized in Table 1. In this table μ_1 is the theoretical damping factor as derived from equation (**11**) (viscosity damping) and μ_2 is the corresponding value obtained from equation (**12**) (damping due to variations in surface tension).

TABLE 1

MECHANICALLY GENERATED RIPPLES (WAVE FREQUENCY 49·4 C/S)

	Surface Tension T (dyne/cm)	Wave-length λ (cm)	Slope of $(\log_{10} f)/x$ Graph	V/v (cf. eqn. (**13**))	μ (per cm)			
					μ_1	μ_2	$\mu_1+\mu_2$	μ_{obs}
Clean water	72·3	0·595	0·029	1·39	0·055	—	—	0·065
	72·1	0·172*	$0\cdot19_5$	1·49	0·35	—	—	0·45
Surface films	65·5	0·575	0·11†	1·39	0·06	$0\cdot17_5$	$0\cdot23_5$	$0\cdot25_5$
	58·5	0·555	0·12	$1\cdot38_5$	0·07	$0\cdot18_5$	$0\cdot25_5$	$0\cdot27_5$
	42·5	0·500	$0\cdot13_5$	1·37	$0\cdot09_5$	0·23	$0\cdot32_5$	0·31
	25·5	0·425	0·18	1·35	$0\cdot15_5$	$0\cdot32_5$	0·48	0·41

* Measurements by Brown (1936), at wave frequency 300 c/s.

† This result was obtained at a considerable distance from the dipper: nearer to it the damping was less (cf. Section IV (*d*)).

It may be seen from Table 1 that, when surface films are present, μ_{obs} is distinctly greater than μ_1 or μ_2 in every case: in fact μ_{obs} is roughly equal to the sum of μ_1 and μ_2 over the experimental range. This suggests that damping effects due to bulk viscosity and to surface films are largely independent and that the total damping is a combination of them both. However, such a view receives little support from Dorrestein's treatment, in which damping by surface films is shown to be associated with changes in the nature and extent of hydrodynamic oscillations beneath the surface. Thus the relation $\mu_1+\mu_2 \simeq \mu_{obs}$ is probably coincidental.

According to Dorrestein the most general situation is that in which the film, besides showing variation of surface tension with area, also possesses an appreciable surface viscosity; under these conditions the total damping factor is sometimes considerably greater than μ_2. Nevertheless, calculation shows that in the present experiments any such effect is very small indeed,* and it is quite

* Dorrestein indicates that μ may be greater than μ_2 if $N>0\cdot06$, where N is a dimensionless number related to the surface compressibility. The compressibility of cetyl alcohol films has been studied by Nutting and Harkins (1939), and in the present measurements the values of N range from about 3 to 40; however, these values are so large that μ exceeds μ_2 by only a small amount. If the surface viscosity is also taken into account μ is correspondingly reduced, and its final value is almost indistinguishable from μ_2.

clear that the observed damping is always in excess of the theoretical value, although the discrepancies are never very large. In Table 1 the difference between μ_{obs} and μ_2 is consistently equal to $\sim 0 \cdot 08$; this suggests the presence of an extra damping effect which is not included in the theoretical treatment.

(c) *Wind-blown Ripples*

Results for ripples produced by the small air jet are shown in Table 2. The surface tension as derived from equation (**10**) ($\lambda = 0 \cdot 535/2$ cm when $n = 98 \cdot 8$ c/s) is $\sim$28 dyne/cm, corresponding to a film pressure of $\sim$45 dyne/cm. This is somewhat lower than the maximum pressure obtained previously, and the air blast is possibly responsible; nevertheless, the film pressure is still high and equal to the equilibrium pressure of solid cetyl alcohol. It will be observed that μ_{obs} is again greater than μ_1 or μ_2. However, as in the previous experiments

TABLE 2

WIND-GENERATED RIPPLES: CETYL ALCOHOL FILM AT FULL PRESSURE

Wave Frequency n (c/s)	Surface Tension T (dyne/cm)	Wave-length λ (cm)	Slope of $(\log_{10} f)/x$ Graph	V/v (cf. eqn. (**13**))	μ (per cm)			
					μ_1	μ_2	$\mu_1+\mu_2$	μ_{obs}
98·8	$\sim$28	0·535/2	0·32	1·44	0·29	0·53	0·82	0·74
49·4	assumed 28	0·44*	0·16	1·36	0·13	0·29	0·42	0·37

* This value may be inaccurate since it was difficult to stabilize the waves in these low frequency measurements. The wavelength was not determined directly, but calculated on the assumption that, as before, $T = 28$ dyne/cm.

at high surface pressures, μ_{obs} is rather less than the sum $\mu_1 + \mu_2$, the discrepancies being roughly the same in both cases. Again, as before, $\mu_{obs} - \mu_2 = 0 \cdot 08$ for the experiment at $49 \cdot 4$ c/s, but the difference is much more pronounced at $98 \cdot 8$ c/s: if some extraneous damping effect is responsible it increases markedly with wave frequency.

No measurements were carried out on clean water since there were traces of oil in the compressed air line, which quickly contaminated the surface.

(d) *Films of Very Low Pressure*

According to Adam (1941), ethyl stearate gives rise to a gaseous film if the surface pressure is less than $\sim 0 \cdot 03$ dyne/cm. It was of interest to determine whether films of this kind could exert any damping effect, and a minute trace of ethyl stearate was therefore introduced to a clean water surface on which ripples were generated with the dipper, as before. A small but distinct change was immediately observed and the damping was increased.

The results are shown in Figure 2 (*c*), which, this time, is in the form of a curve, i.e. the damping is more pronounced as the distance from the dipper becomes greater. Similar behaviour, though much less marked, is observed with a low pressure cetyl alcohol film (see Table 1). The effect is probably due to movement of the film by surface mass transport, whereby the concentration of the monolayer is decreased in the vicinity of the dipper and increased towards the far end of the tank. Such movement induced by the waves undoubtedly takes place in any film, but its influence is proportionately less with films of high surface pressure.

For the ethyl stearate film the wavelength observed, ($\lambda=0\cdot595$ cm at a wave frequency of $49\cdot4$ c/s), is the same as that for clean water (cf. Table 1). This again gives $T=72\cdot3$ dyne/cm so that the surface pressure must be very small. Corresponding figures for the damping factor are $\mu_1=0\cdot055$/cm (from equation (**11**)) and $\mu_2=0\cdot16$/cm (from equation (**12**)). The mean slope of the curve c in Figure 2 is $\sim0\cdot05$ ($\mu_{obs}\simeq0\cdot12$/cm), and that of the asymptotic dotted line is $\sim0\cdot07$ ($\mu_{obs}\simeq0\cdot16$ cm); in either case $\mu_1+\mu_2>\mu_{obs}>\mu_1$. It follows that the damping effect is quite pronounced, but rather less than is found with denser films. This is a reasonable result.

One interesting experiment was carried out after a film of unknown impurities had collected overnight on a clean water surface. The film had a definite structure and was quite rigid, and even though its pressure was fairly low ($T\simeq66$ dyne/cm) the damping was pronounced. After some time damping decreased and the surface layer could be seen breaking up under the continued action of the ripples. This shows that films of high rigidity exert a strong damping effect.

V. Conclusions

In this work the amplitude of the individual ripples is small, being of the order of 10^{-3} to 10^{-4} cm;* no results are available for larger waves. Theoretical descriptions of the damping of these capillary ripples are confirmed to only a limited degree, for the observed damping is always somewhat in excess of the predicted values. This suggests that the theories are not completely adequate and that some additional damping mechanism is involved. Circular waves are damped more rapidly than straight waves; however, any extra damping due to curvature of the wave front must be small in the present experiments. The tank used was not large, but the depth of water was always greater than five wavelengths so that, for ripples of such small amplitude, any effects of depth are scarcely significant. It is possible that damping could be augmented by the general circulation in the bulk of the fluid resulting from surface mass transport (cf. Section IV (d)). This arises because the waves are imperfect, and the results of surface movement of this kind are not considered in any theory. All the factors mentioned above lead to an increase in damping, and depend markedly on wave frequency in accord with what is observed in Table 2; they may all be involved in the measurements.

The experimental facts are at variance with a suggestion of Dorrestein that the stilling of small ripples by a surface film is primarily due to the action of the film in preventing ripple *formation* rather than to a direct damping effect. He based this view on his result that for capillary ripples of wavelength

* From equation (2) $a\simeq10^{-4}$ cm, when f is ~10 cm.

$0 \cdot 1$–$1 \cdot 7$ cm the ratio of the decay moduli τ_1/τ_2 was never very large, varying only from about $1 \cdot 6$ to 4; this implied, he believed, that the influence of surface films on the rate of decay of capillary waves was relatively small. However, if the damping is considered as a function of distance rather than time (i.e. if the factor μ in equation (**1**) is thought of, and not the factor τ in equation (**5**)) the effect of a film is more obvious: the ratio μ_2/μ_1 is, in fact, greater than the ratio τ_1/τ_2 for capillary waves, since the decrease in surface tension which follows the introduction of a surface film also produces a corresponding decrease in the wave (and group) velocity. The factor $(\mu_1+\mu_2)/\mu_1$ is greater still, and from the present results this would appear to be a more relevant measure of damping efficiencies.

One additional point should be mentioned. Wavelengths are shortened in the presence of monolayers, and since the shorter waves are damped more easily a further damping is effected indirectly.* It follows that surface films not only help to prevent the formation of small ripples as Dorrestein maintains; they are also very effective in damping them out.

VI. Acknowledgments

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* This was strikingly demonstrated when a fragment of solid cetyl alcohol was placed on a clean water surface: the film pressure rose very slowly and the damping of the ripples increased progressively as their wavelength decreased.