EXPERIMENTAL RELATIONS BETWEEN IONOSPHERIC TRUE HEIGHT, GROUP HEIGHT, AND PHASE HEIGHT

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Summary

Computations of ionospheric phase height at a particular frequency have been made using true height analyses of $h'f$ curves. Temporal changes in these computed values agree well with experimentally measured changes of phase path both in the case of regular diurnal variations and during an eclipse. The true height analyses are thus shown to be experimentally reliable, and at the same time we can allot an " absolute " value to the normal phase-path records.

I. Introduction

On the basis of ray optics the phase path of a radio wave vertically reflected from the ionosphere at a true height " h " is given by

$$P=2\int_0^h \mu \mathrm{d}h,$$

where μ is the refractive index (at the frequency concerned) corresponding to the element of height $\mathrm{d}h$. The group path is given by the companion expression

$$P'=2h'=2\int_0^h \mu'\mathrm{d}h,$$

where μ' is the group refractive index defined by $\mu'=\mu+f\partial\mu/\partial f$.

Both the group path and the changes in phase path at a fixed frequency may be experimentally determined. Estimates of true height of reflection may be made by various means based on measured (P', f) curves, and we are therefore in a position to compare the measured values of P' and ΔP with those derived from estimates of true height.

II. Experimental Measurements

Phase-path measurements have been made at Brisbane at a frequency of $5\cdot8$ Mc/s by the method described by McNicol and Thomas (1960). Enlargement of the resulting films enables the phase fringes to be counted starting from some arbitrary origin of time, a decrease of phase path of one wavelength being registered as -1, and an increase as $+1$. By this means we obtain plots of ΔP values throughout the daylight hours as indicated in Figure 1. Because of interference and the necessity for changing film it is not possible to follow the phase throughout a complete day.

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Care is taken to count fringes corresponding to *o*-ray reflections only. The estimated accuracy of such a count at the end of a day is ± 10 fringes; this is much greater than any shift due to oscillator instabilities, but still represents a negligible error in a count of some 6000 fringes. There is considerable variation from day to day in such curves; these day-to-day variations are usually associated with similar day-to-day variations in group path.

Simultaneous h', f records have been taken throughout the day as part of the routine Ionospheric Prediction Service recording and these are available for reduction to (N, h) profiles. Values of h' at $5 \cdot 8$ Mc/s may be taken from either the phase-path films or the $h'f$ curves. Piggott (1959) has recently demonstrated

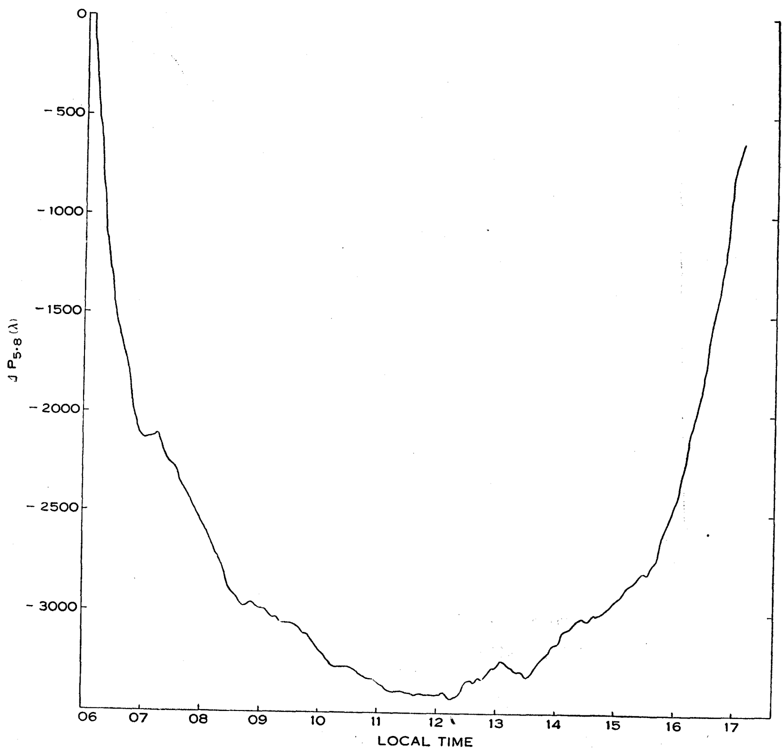

Fig. 1.—Plot of phase-path changes throughout the day (Brisbane, April 9, 1959). The small irregularities shown here are typical.

the necessity for correcting the values of h' for the large receiver delays commonly found in $h'f$ recorders. For the equipment used the delay is substantially independent of frequency and is given by $D=9-0 \cdot 4W$ where W is the width (in kilometres) of the recorded trace and D is the correction in kilometres. These corrections have been applied to the data analysed here.

III. True Height Analysis

Using the method developed by Schmerling (1958) and the tabulated values of sampling frequencies given by either Ventrice and Schmerling (1958) or Wright and Norton (1959), a manual reduction of a number of $h'f$ curves has been carried out to derive the corresponding (h,f) curves. A typical example of the results of such a reduction is shown in Figure 2. The oscillation of the height values about the general trend is due to the limited sampling method employed, and has been discussed earlier by Piggott (1954). The best one can do in such circumstances is to fit a " mean " curve such that the sum of the areas between the mean curve and the oscillatory curve is balanced. It is obvious that great care must be exercised in using the Schmerling analysis for day-time

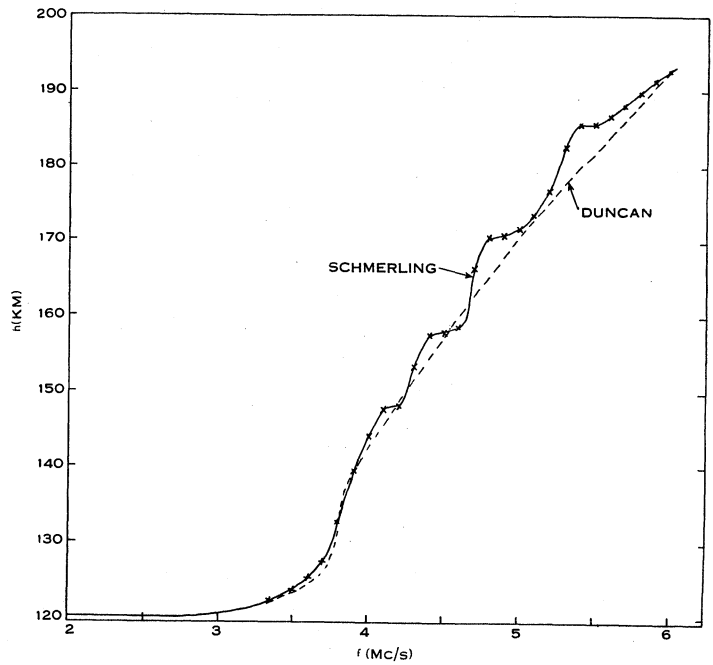

Fig. 2.—A comparison of (h,f) data derived from machine analysis (Duncan), and sampling analysis (Schmerling) of $(h'f)$ data for 1440 hr on April 10, 1959.

records. The height calculations must be carried out at very close frequency intervals if false data are not to be obtained. This difficulty does not exist at night except in the neighbourhood of the F_2 penetration frequency.

For 15 cases the corresponding (h, f) data derived using the machine analysis of Duncan (1958) have also been plotted in Figure 2. It will be noticed that the two sets of data are in substantial agreement up to about 4 Mc/s, but beyond this frequency the Schmerling values of true height are consistently higher than the Duncan values. This is so for all 15 cases, the difference at 6 Mc/s being usually about 3 km.* Such differences may often be neglected, but become important when one computes the phase and group refractive indices as functions of height.

* Since this work was carried out Duncan (personal communication) has inserted a slight correction into his programme ; this correction brings the two methods into very close agreement.

IV. Calculation of Refractive Indices

Using the methods of Whale and Stanley (1950) and Shinn and Whale (1952), we may compute the values of μ and μ' for a frequency of 5·8 Mc/s for vertical o-ray propagation at Brisbane, neglecting collisions. Using the previously calculated (h, f) profiles we obtain μ and μ' as a function of true height at a number of times throughout the day.

V. Computed Group Path

By plotting $\mu'_{5\cdot 8}$ as a function of height we obtain curves such as Figure 3. The area beneath this curve is $\int_0^h \mu' \mathrm{d}h$ and should thus agree with $h' = \frac{1}{2}P'$ if the previous calculations have been correctly carried out. In practice the area is computed up to the height at which the plasma frequency (f_p) is 5·6 Mc/s ($\mu' = 3\cdot 98$). Between this height and the reflection height the electron density

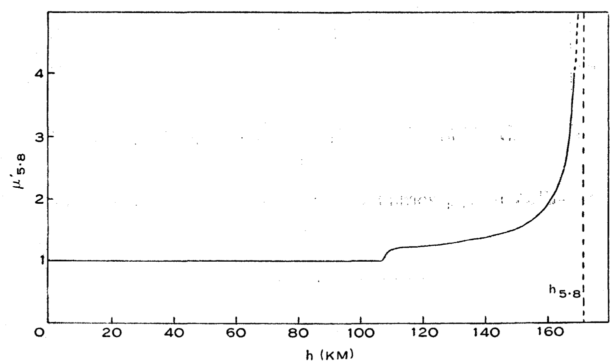

Fig. 3.—Plot of $\mu'_{5\cdot 8}$ versus true height for Brisbane at 1024 hr on April 8, 1959.

is assumed to increase linearly, and the transformation is made to the variable $t = \sqrt{(1 - f_p^2/5\cdot 8^2)}$. As Shinn and Whale have indicated, we can now obtain a finite answer to the value of

$$\int_{h_{5\cdot 6}}^{h_{5\cdot 8}} \mu' \mathrm{d}h = C(h_{5\cdot 8} - h_{5\cdot 6}) \int_{t_{5\cdot 6}}^{t_{5\cdot 8}\ (=0)} \mu' t \mathrm{d}t,$$

since $\mu' t$ tends to a limiting value as $t \to 0$. For Brisbane conditions this reduces to 12·1 $(h_{5\cdot 8} - h_{5\cdot 6})$ km, and this value must be added to the previously calculated value of the area up to $h_{5\cdot 6}$.

Comparisons between the computed group-path values and those actually observed can now be made. In all cases the two agree within a few kilometres. For those curves analysed using the Duncan method the computed values tend to be systematically low (3·5 km on the average). For those analysed using the Schmerling method, however, the computed values show rather more scatter but on the average agree with the observed values.

VI. Computed Phase Path

By plotting $\mu_{5\cdot 8}$ as a function of height we obtain curves such as Figure 4. The area beneath this curve is $\int_0^h \mu \mathrm{d}h$ and should thus agree with $\frac{1}{2}P$. We have no records of P as such, but only changes in P over a period of time. The computed values of P should vary throughout the day in the same way as does the change in P derived from the fringe count. A comparison of the two sets of data for the eclipse day of April 8, 1959, is given in Figure 5.

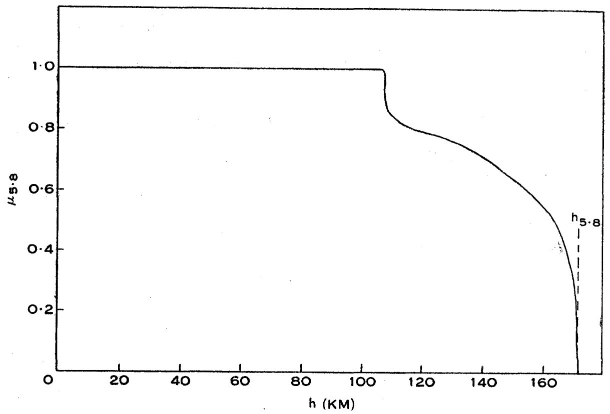

Fig. 4.—Plot of $\mu_{5\cdot 8}$ versus true height for Brisbane at 1024 hr on April 8, 1959.

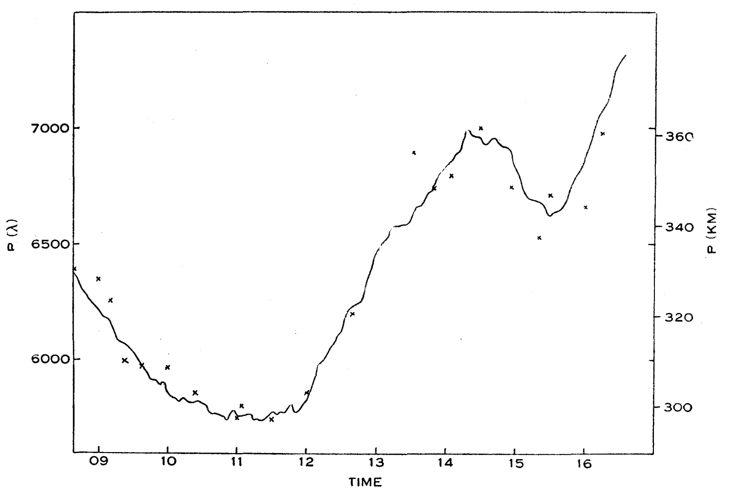

Fig. 5.—Comparison of measured phase-path variations with computed values (×) of phase path for April 8, 1959.

We thus have, from the good agreement shown here and throughout other days, a direct independent experimental check on the validity of the true height estimations using the Schmerling and Duncan methods. It should be noted that, whereas in the computation of $\int \mu' \mathrm{d}h$ the (h, f) curve near the reflection point (where μ' is high) must be very accurately known, for the computation of $\int \mu dh$ the base height of the E region (where μ is high) is rather more important. It is just this region which is sometimes in doubt because of strong day-time absorption, and recourse has to be made to extrapolation of the $h'f$ curve. This undoubtedly explains some of the larger deviations shown in Figure 5.

By adjusting the vertical position of the ΔP curve to get the best least squares fit to the $\int \mu dh$ points, we can allot an " absolute " value to the phase-path results, i.e. we have a measure of the *total* phase path. The curve of Figure 5

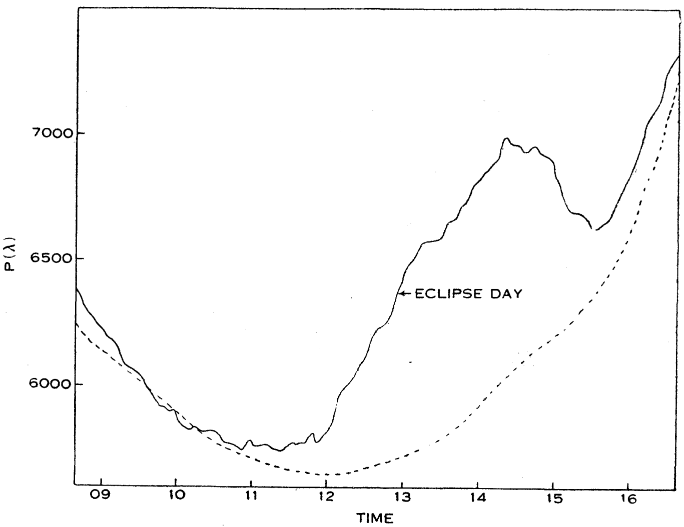

Fig. 6.—Comparison of " absolute " phase path for eclipse and control days.

has been so fitted and we can thus allot an absolute phase of 5810 wavelengths at 1200 hr. The same can, of course, be done for other days with an error of less than 20 wavelengths ($\sim$1 km)—we have thus regained some of the information lost through inability to record right through the night.

We can now compute, using " absolute " phases, the average absolute phase path throughout a series of days and use this for comparison with unusual effects produced on certain days, e.g. by an eclipse. Such a comparison is shown in Figure 6.

Such comparisons are useful mainly in giving an idea of the integrated effect of any ionization changes taking place. The eclipse day effect will be further discussed elsewhere.

VII. Conclusion

We have shown that computed values of ionospheric electron density as a function of height are experimentally reliable, and that they enable us to allot an absolute value to the normal phase path records. True height computations appear to be valid during an eclipse period.

VIII. Acknowledgments

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