COUNTS OF EXTRAGALACTIC RADIO SOURCES

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Summary

In this paper a formula is derived giving the cumulative totals of class II radio sources, to successive limits of flux density, when changes in the space-density and in the radiative properties of these sources are taken into account.

The formula is then compared with the observations and it is shown that it embodies the various attempts which have been made so far to explain the $-3/2$ power law.

I. INTRODUCTION

The number of radio sources in the space-time

$$ds^2=dt^2-\frac{R^2(t)}{c^2}\left\{\frac{dr^2+r^2d\theta^2+r^2\sin^2\theta d\varphi^2}{(1+\frac{1}{4}\alpha r^2)^2}\right\}, \tag{1}$$

lying within $0\rightarrow r$, is given by the formula (McVittie 1956)

$$N=\frac{4\pi}{R_0^3}\int_0^{\zeta}\frac{R^3(t)\nu(t)\zeta^2d\zeta}{(1+\zeta^2/a^2)^3}, \tag{2}$$

where

$$\zeta=R_0r, \quad a^2=4R_0^2/\alpha, \tag{3}$$

and $\nu(t)$ is the number of radio sources per unit volume at time t.

In relativistic cosmology, it is usually assumed that the space density parameter

$$\varepsilon=R^3(t)\nu(t) \tag{4}$$

is constant.

In order to account for the observed distribution of extragalactic radio sources (Mills, Slee, and Hill 1958) one has been led to one of the following alternatives :

1. All class II radio sources are assumed to be of equal power and to be at rest and scattered at random with constant space density in the Euclidian space. But, as pointed out by McVittie (1959), there are serious objections to this interpretation.

2. The space density parameter ε in formula (4) is assumed to be constant, but it is assumed that secular changes in the radiative properties of the sources are taking place (McVittie 1957).

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3. One assumes that all radio sources have equal power but that these sources are colliding galaxies, i.e. one abandons the assumption of constant space density parameter (Priester 1958 ; Davidson 1959).

It is the purpose of the present paper to derive a formula connecting the cumulative number N of radio sources to successive limits of flux density S, allowing for secular changes both in the radiative properties and in the space density of these sources.

II. Number of Radio Sources and Luminosity Distance

Using the expansion

$$(1+\zeta^2/a^2)^{-3}=1-3\zeta^2/a^2,$$

formula (2) may be written

$$N=\frac{4\pi}{R_0^3}\int_0^{\zeta} R^3(t)\nu(t)\left[\zeta^2-3\frac{\zeta^4}{a^2}\right]\mathrm{d}\zeta. \tag{5}$$

But (McVittie 1956)

$$R=R_0(1-x+gx^2), \tag{6}$$

where

$$x=h_1D/c, \tag{7}$$

and

$$g=(3h_1^2+h_2)/2h_1^2, \tag{8}$$

D being the luminosity distance, h_1 the Hubble constant $\dot{R}_0/R_0$, and $h_2=\ddot{R}_0/R_0$. Also

$$\zeta=(cx/h_1)(1-x+bx^2), \tag{9}$$

where

$$b=3/2+\tfrac{1}{2}h_2/h_1^2+c^2/h_1^2a^2. \tag{10}$$

Substituting (6) and (9) in (5) we have

$$N=4\pi\int_0^x \nu(x)(1-x+gx^2)^3\left\{\frac{c^2x^2}{h_1^2}(1-x+bx^2)^2 - \frac{3}{a^2}\left(\frac{cx}{h_1}\right)^4(1-x+bx^2)^4\right\}\frac{c}{h_1}(1-2x+3bx^2)\mathrm{d}x. \tag{11}$$

We now consider the space density as a function of the luminosity distance, given by the following expansion to the second order in x,

$$\nu=\nu_0+\nu_1x+\nu_2x^2. \tag{12}$$

Substituting (12) in (11), working out the various expansions and integrating we have

$$N=\frac{4\pi c^3x^3}{h_1^3}\left[\tfrac{1}{3}\nu_0+\tfrac{1}{4}(\nu_1-7\nu_0)x+\left\{\nu_2-7\nu_1+\nu_0\left(3g+19+5b+1-\frac{3c^2}{a^2h_1^2}\right)\right\}\frac{x^2}{5}\right]. \tag{13}$$

It is easily seen that

$$\varepsilon=\nu R^3=\nu_0+(\nu_1-3\nu_0)x+[\nu_2-3\nu_1+3\nu_0(g+1)]x^2, \tag{14}$$

and that for constancy of the space density parameter, that is,

$$\varepsilon=\nu_0,$$

we should have

$$\nu_1=3\nu_0, \tag{15}$$

and

$$\nu_2=3\nu_0(2-g). \tag{16}$$

Substituting these values of ν_1 and ν_2 in (13) we have, using (10),

$$N=(4\pi/3)\nu_0 D[1-3x+3\{2+(h_1^2+h_2)/2h_1^2+2c^2/5a^2h_1^2\}x^2], \tag{17}$$

which is the formula commonly used in relativistic cosmology (McVittie 1956).

Substituting the values for g and b from (8) and (10) in formula (13) we have

$$N=\frac{4\pi c^3x^3}{h_1^3}\cdot\frac{\nu_0}{3}\left\{1+\tfrac{3}{4}\left(\frac{\nu_1}{\nu_0}-7\right)x+3\left(\frac{\nu_2}{\nu_0}-7\frac{\nu_1}{\nu_0}+32+\frac{4h_2}{h_1^2}+\frac{2c^2}{a^2h_1^2}\right)\frac{x^2}{5}\right\}. \tag{18}$$

III. Number of Radio Sources and Flux Density

The flux density in watts/m^2 measured by the observer in the range $\lambda_1\to\lambda_2$ of observed wavelength is

$$L=\frac{A}{4\pi D^2}\cdot\frac{1}{1+\delta}\int_{\lambda_1}^{\lambda_2}\sigma(\lambda)B\left(t,\frac{\lambda}{1+\delta}\right)\mathrm{d}\lambda, \tag{19}$$

where A is the total emitting area of the source,

$B(t,\lambda)\mathrm{d}\lambda$ is the energy emitted by the source at the time t in the range $\mathrm{d}\lambda$ in watts/m^2.

$\sigma(\lambda)$ is an empirical function depending on the atmospheric extinction and the response of the apparatus.

In this paper, in order to simplify the analysis, we assume that the function $B(t,\lambda)$ is the same for all sources, i.e. we assume that all sources have equal power. In fact this can only be a first approximation since recent observations (Mills 1960) seem to indicate a marked dispersion in the radiative power of the sources. The function $B(t,\lambda)$ should here be regarded as a mean value.

If L_0 is the flux density for a nearby standard source at distance d ($\delta=0$) we have

$$L_0=\frac{A}{4\pi d^2}\int_{\lambda_1}^{\lambda_2}\sigma(\lambda)B(t_0,\lambda)\mathrm{d}\lambda. \tag{20}$$

Then

$$\frac{L}{L_0}=\frac{d^2}{D^2}\cdot\frac{1}{1+\delta}\cdot\frac{I}{I_0}\cdot\frac{1}{I}\cdot\int_{\lambda_1}^{\lambda_2}\sigma(\lambda)B\left(t,\frac{\lambda}{1+\delta}\right)\mathrm{d}\lambda, \tag{21}$$

where

$$I=\int_{\lambda_1}^{\lambda_2}\sigma(\lambda)B(t,\lambda)\mathrm{d}\lambda, \tag{22}$$

$$I_0=\int_{\lambda_1}^{\lambda_2}\sigma(\lambda)B(t_0,\lambda)\mathrm{d}\lambda. \tag{23}$$

But, if S is the flux density in watts m^{-2} $(c/s)^{-1}$, we have

$$S=\frac{L}{|\nu_1-\nu_2|}=\frac{L\lambda_1\lambda_2}{c(\lambda_2-\lambda_1)},$$

and

$$S/S_0=L/L_0. \tag{24}$$

We take

1. for the correction due to secular changes in the radiative properties of the sources,

$$I/I_0=1+W_1x+W_2x^2, \tag{25}$$

2. for the K-correction,

$$\frac{1}{(1+\delta)I}\int_{\lambda_1}^{\lambda_2}\sigma(\lambda)B\left(t,\ \frac{\lambda}{1+\delta}\right)d\lambda=1+C_1x+C_2x^2. \tag{26}$$

Substituting (24), (25), (26) in (21) we get, to the second order in x,

$$S/S_0=(d^2h_1^2/c^2x^2)(1+\beta x+\alpha x^2), \tag{27}$$

where

$$\beta=C_1+W_1, \tag{28}$$

and

$$\alpha=W_1C_1+(W_2+C_2), \tag{29}$$

or

$$x[1+\beta x+\alpha x^2]^{-\frac{1}{2}}=y, \tag{30}$$

where

$$y=(dh_1/c)(S_0/S)^{\frac{1}{2}}. \tag{31}$$

It follows easily from (30) that

$$x=y+\tfrac{1}{2}\beta x^2+(\tfrac{1}{2}\alpha-\tfrac{3}{8}\beta^2)x^3. \tag{32}$$

It then follows from (32) that

$$\left.\begin{aligned} x&=y,\\ x&=y+\tfrac{1}{2}\beta y^2,\\ x&=y+\tfrac{1}{2}\beta y^2+(\tfrac{1}{2}\alpha+\tfrac{1}{8}\beta^2)y^3,\end{aligned}\right\} \tag{33}$$

are the first, second, and third approximations of x as a power series in y.

Substituting (33) in (18) we have, after a few reductions,

$$N=\frac{4\pi c^3\nu_0}{3h_1^3}y^3[1+(3B_1+A_1)y+(3B_2+3B_1^2+4A_1B_1+A_2)y^2], \tag{34}$$

where

$$\begin{aligned} A_1&=\tfrac{3}{4}(\nu_1/\nu_0-7),\\ A_2&=\tfrac{3}{5}[\nu_2/\nu_0-7\nu_1/\nu_0+32+4h_2/h_1^2+2c^2/a^2h_1^2],\\ B_1&=\tfrac{1}{2}(C_1+W_1),\\ B_2&=\tfrac{1}{2}[W_1C_1+(W_2+C_2)+\tfrac{1}{4}(C_1+W_1)^2].\end{aligned}$$

IV. Comparison with Observations

Equation (34) to the first order in y could be written

$$N=\frac{4\pi\nu_0}{3}d^3\frac{S_0^{3/2}}{S^{3/2}}\left[1+(3B_1+A_1)\frac{dh_1}{c}\frac{S_0^{\frac{1}{2}}}{S^{\frac{1}{2}}}\right], \tag{35}$$

and in order to satisfy the experimental law, we should have

$$(3B_1+A_1)\frac{dh_1}{c}\cdot\frac{S_0^{\frac{1}{2}}}{S^{\frac{1}{2}}}=0. \tag{36}$$

This could be satisfied in a variety of ways.

We could take $h_1=0$, i.e. assume that the sources do not participate in the general expansion, and this would correspond to the hypothesis 1 of the introduction where the sources are supposed to be at rest and scattered at random with constant number density.

As shown by McVittie (1959) there are serious difficulties to this interpretation. Moreover, quite a few radio sources have now been identified with galaxies or pairs of galaxies (Mills 1960).

For small distances

$$d=(c/h_1)\delta_0, \tag{37}$$

and substituting this in (36) we would have the condition

$$(3B_1+A_1)\delta_0 S_0^{\frac{1}{2}}/S^{\frac{1}{2}}=0. \tag{38}$$

This could be satisfied if we assume that

$$3B_1+A_1=0,$$

that is,

$$2C_1+2W_1+\nu_1/\nu_0-7=0. \tag{39}$$

(a) *Space Density Parameter* ε *assumed Constant*

If we assume that the space density parameter ε is constant, i.e. that

$$\nu_1=3\nu_0,$$

then condition (39) reduces to

$$W_1+C_1=2, \tag{40}$$

and, since $C_1=-0\cdot2$ (see Appendix), we have

$$W_1=2\cdot2, \tag{41}$$

and from (25)

$$I/I_0=1+2\cdot2\delta. \tag{42}$$

A secular change of this order of magnitude was found by McVittie (1957) using the standard sources:

NGC 1275 (IAU 03N4A) with $S_0=240\times10^{-26}$ W m^{-2} (c/s)$^{-1}$,

$$\delta_0=0\cdot018,$$

and Cygnus A (IAU 19N4A) with $S_0=19{,}000\times10^{-26}$ W m^{-2} (c/s)$^{-1}$,

$$\delta_0=0\cdot056.$$

McVittie also found that in the case of NGC 1275 it was possible to get a good fit to the $-3/2$ power law, without the assumption of a secular change in the radiative power, but not if Cygnus A were taken as standard source.

This is due to the fact that (35) would reproduce the observed law fairly well if

$$|(3B_1+A_1)\delta_0 S_0^{\frac{1}{2}}/S^{\frac{1}{2}}|<1 \tag{43}$$

for all observed S. The best fit would be for the large values of the observed flux density.

If we do not assume secular changes in density and luminosity, we have

$$W_1=\nu_1=0,$$

and then (43) reduces to

$$\left|\left(3\frac{C_1}{2}-3\times\frac{7}{4}\right)\frac{\delta_0 S_0^{\frac{1}{2}}}{S_m^{\frac{1}{2}}}\right|<1, \tag{44}$$

where $C_1=-0\cdot2$ and

S_m=the smallest observed value of the flux density

$=7\times10^{-26}$ W m^{-2} (c/s)$^{-1}$.

Then (44) reduces to

$$\delta_0 S_0^{\frac{1}{2}}<0\cdot48\times10^{-13}. \tag{45}$$

For NGC 1275

$$\delta_0 S_0^{\frac{1}{2}}=0\cdot28\times10^{-13},$$

and therefore condition (45) is satisfied.

For Cygnus A

$$\delta_0 S_0^{\frac{1}{2}}=7\cdot7168\times10^{-13},$$

and then the agreement is very bad indeed.

It follows then from this analysis that the $-3/2$ power law can be approximately satisfied, without further assumptions, if the standard source is taken as being a nearby faint one.

(*b*) *Space Density Parameter not Constant*

We can also assume that the space density parameter is no longer constant. This is the case, if we assume that the majority of extragalactic radio sources are colliding galaxies.

Then, in the absence of changes in the radiative power, $W_1=0$ and formula (39) reduces to

$$\nu_1=\nu_0[7-2C_1],$$

or

$$\nu_1=7\cdot4\nu_0,$$

that is,

$$\nu=\nu_0[1+7\cdot4\delta]. \tag{46}$$

Priester (1958) and Davidson (1959) take for the number density of colliding galaxies

$$\nu(t)=\beta n^2(t), \tag{47}$$

where

$$n(t)=n_0R_0^3/R^3(t),$$

that is,

$$\nu(t)=k/R^3(t), \tag{48}$$

k being a constant.

But, in first approximation,

$$R=R_0(1-\delta),$$

therefore

$$\nu=(k/R_0^6)(1-\delta)^{-6},$$

or

$$\nu=(k/R_0^6)(1+6\delta). \tag{49}$$

A comparison of formulae (49) and (46) shows why the collision hypothesis cannot account exactly for the $-3/2$ power law, a fact already pointed out by McVittie (1959) in a different way.

Of course, one should keep in mind that the counts need correction at low flux densities because of effects of random noise and finite aerial resolution (Mills and Slee 1957). In addition, there is a recently discovered effect (Mills, Slee, and Hill 1960) which gives a systematic tendency to over-estimate the flux density of a source when the signal-to-noise ratio is low.

The $-3/2$ power law does not seem to be established with sufficient certainty yet, and it is to be expected that the various possibilities discussed in this paper might not be the only ones.

V. References

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Appendix

The K-correction is given by the formula

$$K=K_1\delta+K_2\delta^2,$$

where

$$K_1=(2\cdot5/E)(1+J_0/I_0),$$

with

$$E=\ln 10=2\cdot303,$$

$$J_0=\int_0^\infty \sigma(\lambda)\lambda\frac{\partial B(t_0,\lambda)}{\partial\lambda}\mathrm{d}\lambda,$$

$$I_0=\int_0^\infty \sigma(\lambda)B(t_0,\lambda)\mathrm{d}\lambda.$$

For a narrow wave-band $\mathrm{d}\lambda$ in the neighbourhood of wavelength λ, with $\sigma(\lambda)=0$ outside this wave band,

$$K_1=\frac{2\cdot5}{E}\left(1+\frac{\lambda\frac{\partial}{\partial\lambda}B(t_0,\lambda)}{B(t_0,\lambda)}\right), \tag{50}$$

t_0 being the present epoch of observation.

But (Whitfield 1957)

$$B(t_0,\lambda)=\lambda^{-0.8}.$$

It follows then that

$$\frac{\lambda\frac{\partial}{\partial\lambda}B(t_0,\lambda)}{B(t_0,\lambda)}=-0\cdot8.$$

and (50) can be written

$$K_1=\frac{2\cdot5}{2\cdot303}(1-0\cdot8),$$

or

$$K_1=0\cdot218. \tag{51}$$

But (McVittie 1956)

$$K=-2\cdot5\log_{10}\left\{\frac{1}{I(1+\delta)}\int_0^\infty \sigma(\lambda)B\left(t,\frac{\lambda}{1+\delta}\right)\mathrm{d}\lambda\right\}, \tag{52}$$

where

$$I=\int_0^\infty \sigma(\lambda)B(t,\lambda)\mathrm{d}\lambda.$$

Using (26)

$$\begin{aligned}K&=-2\cdot5\log_{10}\{1+c_1x+c_2x^2\}\\&=-(2\cdot5/2\cdot303)\ln\{1+c_1\delta+\ldots\},\end{aligned}$$

and using

$$\ln(1+x)=x-\tfrac{1}{2}x^2,$$

we have

$$K=-(2\cdot5/2\cdot303)C_1\delta.$$

Comparing this to

$$K=K_1\delta+K_2\delta^2,$$

and using the value of K_1 given by (51) we have

$$C_1=-0\cdot2. \tag{53}$$

The exponent $-0\cdot8$ in formula (51) does not seem to be generally accepted by radio astronomers. A different value of the exponent would result in a different value of K_1 with a corresponding change in the value of C_1.