FLOATING MULTIPLE PROBE SYSTEMS FOR PLASMA MEASUREMENTS

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Summary

A new analysis of the current voltage characteristic of a floating double probe system is given, showing that the electron temperature must be estimated from measurements at the point of inflection of the characteristic if positive ion movement contributes to the probe circuit current. Temperature measurements will still be in error if the rates of variation of positive ion current to the two probes are not closely similar as the potential difference between them is varied. An alternative experimental technique is described which will yield correct temperature estimates in such circumstances, and is simpler and less perturbing to the plasma than the floating triple probe method.

I. Introduction

In the application of floating probe systems to dynamic measurements of electron temperatures and carrier concentrations in hydrogen plasma, some errors in earlier accounts of these techniques have been noted. These become significant when variations of positive ion currents to the individual probes cannot be neglected, and the probe characteristic is asymmetrical.

This communication presents a new analysis of the floating double probe characteristic, and indicates the circumstances in which it should yield correct electron temperature estimates. The floating triple probe method of Okuda and Yamamoto (1960) is discussed, and it is concluded that electron energy spectra and temperature estimates can be obtained using floating double probes of widely different areas, without the introduction of a third probe.

II. Floating Double Probes with Mobile Positive Ions

Consider two probes P_1, P_2, of effective areas A_1 and A_2, immersed in a plasma at points where the space potential has the values V_{P1}, V_{P2}, and where the random positive ion space current densities are j_{p1} and j_{p2} and the random electron space current densities j_{e1} and j_{e2}. So long as the probes are electrically isolated, they will assume potentials which are negative to the local plasma and which are determined, in the manner of wall potentials, by ambipolar diffusion. Let V_1 and V_2 be the drops in potential from V_{P1} and V_{P2} to the respective probes. Then, if the electron energy distribution is Maxwellian, the electron currents flowing from the plasma to the probes are

$$i_{e1}=A_1 j_{e1}\mathrm{e}^{-\varphi V_1}\,;\quad i_{e2}=A_2 j_{e2}\mathrm{e}^{-\varphi V_2}, \tag{1}$$

where $\varphi=e/kT_e$. The positive ion currents will be space charge limited, and the conventional diode expression, showing dependence on $V^{3/2}/d$, is applicable; however d, the thickness of the sheath adjacent to the probe over which the

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majority of V appears, itself varies with V, and experiments indicate a linear dependence of probe positive ion current on plasma-probe potential fall. This variation in positive ion current can be observed directly as the variation of total current to the probe when it is so negative to the plasma that it draws a negligible electron current. If A_1 and A_2 are sufficiently different, the proportionality constants (S_1, S_2) connecting i_{p1} with V_1 and i_{p2} with V_2 may be noticeably

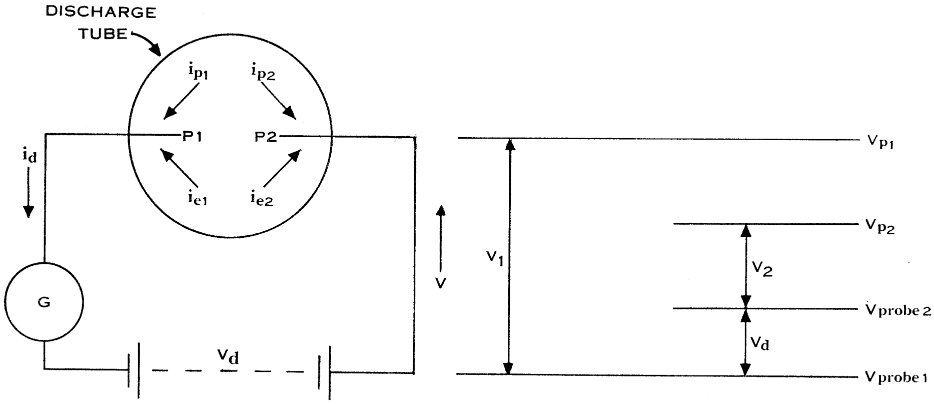

Fig. 1 Fig. 2

Fig. 1.—Schematic circuit of floating double probes.
Fig. 2.—Potential diagram: floating double probes.

different. For the time being, we will assume that these constants are the same, recognizing this as an approximation unless the probe areas are similar. Thus we have $S_1=S_2=S$,

$$i_{p1}=A_1j_{p1}+SV_1; \qquad i_{p2}=A_2j_{p2}+SV_2. \tag{2}$$

If now an external electrical circuit is provided (Fig. 1) connecting the probes, containing a galvanometer of negligible resistance measuring current i_d flowing from P_1 to P_2, and a source of e.m.f. V_d of variable polarity and magnitude, continuity of circuit current requires

$$i_d=i_{p1}-i_{e1}=i_{e2}-i_{p2}. \tag{3}$$

Also, if V_d is assumed positive on the P_2 side, the earlier definitions yield (see Fig. 2)

$$V_1-V_2=V_d+(V_{P1}-V_{P2}). \tag{4}$$

Substituting for i_{e1}, i_{p1}, etc. in (3) from (1) and (2), and differentiating with respect to V_d we obtain

$$\frac{\mathrm{d}i_d}{\mathrm{d}V_d}=(S+\varphi i_{e1})\frac{\mathrm{d}V_1}{\mathrm{d}V_d}=-(S+\varphi i_{e2})\frac{\mathrm{d}V_2}{\mathrm{d}V_d}. \tag{5}$$

Now a second differentiation gives

$$\left.\begin{aligned}\frac{\mathrm{d}^2i_d}{\mathrm{d}V_d^2}&=-\varphi^2 i_{e1}\left(\frac{\mathrm{d}V_1}{\mathrm{d}V_d}\right)^2+(S+\varphi i_{e1})\frac{\mathrm{d}^2V_1}{\mathrm{d}V_d^2},\\ &=\varphi^2 i_{e2}\left(\frac{\mathrm{d}V_2}{\mathrm{d}V_d}\right)^2-(S+\varphi i_{e2})\frac{\mathrm{d}^2V_2}{\mathrm{d}V_d^2}.\end{aligned}\right\} \tag{6}$$

In principle, the electron temperature can be determined from (5), as the only unknown in φ, once we know the slope of the probe characteristic (see Fig. 3). However, the electron current can only be estimated after subtracting the positive ion current, and the two can only be resolved when the probe potential V_1 or V_2 is known; neither potential is available from observations on the conventional double probe system described, and this is also true of their derivatives with respect to V_d. In the special case where the variation in positive ion current can be neglected ($S=0$), the electron current to either probe is directly measurable, and $\mathrm{d}V_1/\mathrm{d}V_d$ and $\mathrm{d}V_2/\mathrm{d}V_d$ can be found, for any point on the characteristic, by computations deriving from equations (1) and (3).

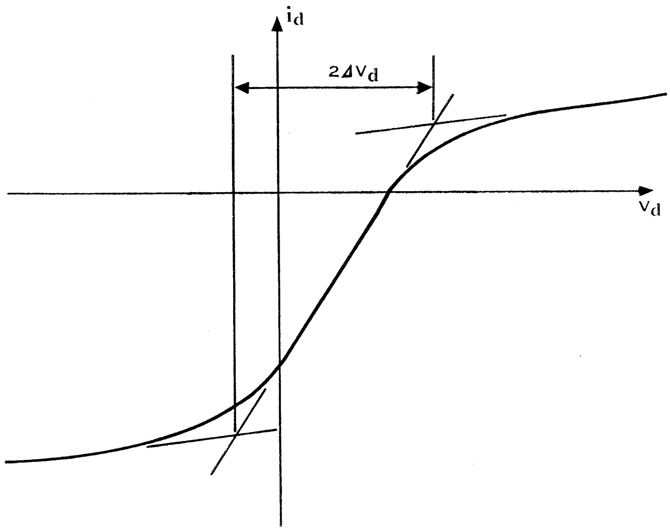

Fig. 3.—Typical current-voltage characteristic: floating double probes.

We will now show that, provided the slope of the probe characteristic is measured at the point of inflection, the electron current to each probe can be determined even though $S\neq 0$.

Differentiation of (4), twice, yields

$$\frac{\mathrm{d}V_1}{\mathrm{d}V_d}=1+\frac{\mathrm{d}V_2}{\mathrm{d}V_d}, \tag{7}$$

and

$$\frac{\mathrm{d}^2V_2}{\mathrm{d}V_d^2}=\frac{\mathrm{d}^2V_1}{\mathrm{d}V_d^2}, \tag{8}$$

and, if we add the two equations in (6) and substitute for derivatives of V_2 from (7) and (8), some rearrangement leads to

$$2\frac{\mathrm{d}^2i_d}{\mathrm{d}V_d^2}=\varphi(i_{e1}-i_{e2})\left[\frac{\mathrm{d}^2V_1}{\mathrm{d}V_d^2}-\varphi\left(\frac{\mathrm{d}V_1}{\mathrm{d}V_d}\right)^2\right]+\varphi^2 i_{e2}\left(1-2\frac{\mathrm{d}V_1}{\mathrm{d}V_d}\right). \tag{9}$$

When $i_{e1}=i_{e2}$, the first term in (9) vanishes; further, (5) then requires

$$\frac{\mathrm{d}V_1}{\mathrm{d}V_d}=-\frac{\mathrm{d}V_2}{\mathrm{d}V_d}, \tag{10}$$

and (10) and (7) yield

$$\frac{\mathrm{d}V_1}{\mathrm{d}V_d}=\tfrac{1}{2}; \quad \frac{\mathrm{d}V_2}{\mathrm{d}V_d}=-\tfrac{1}{2}. \tag{11}$$

Substitution of (11) into (9) makes it evident that the second term also vanishes when the electron currents to the probes from the plasma are equal.

From the form of the double probe characteristic, there can only ever be one zero of the second derivative (9); so we can infer that the electron currents to the probes are always equal at the point of inflection of the characteristic, and that (11) gives the dependence of probe potentials on applied probe circuit e.m.f. at that point. It is advisable to recall at this point the assumption that S is the slope of the positive ion current to each probe; if the areas are sufficiently different that the end portions of the characteristic show this assumption to be invalid, then equal electron currents will not cause (9) to vanish and neither will (11) be true under these conditions. The slow variation of i_p compared with i_e allows use of the above results for analysis with little error, however, even when there is an apparent difference in S_1 and S_2.

If the probes are of identical areas and situated at points in the plasma where the random current densities are also identical, then the electron currents to them will be equal when $V_1=V_2$, and the circuit current i_d will be zero at the point of inflection. However, because of the greatly different sensitivities of dependence on plasma-probe fall of the electron and ion currents, small differences in effective areas which do not invalidate the assumption

$$S_1=S_2=S,$$

or small differences in random current densities which do not upset the assumption of equal temperatures near both probes, may still result in substantial differences between electron and positive ion currents at the inflection point. In such circumstances, the characteristic is termed asymmetrical. Apart from this asymmetry about the axis $i_d=0$, the characteristic will not pass through the origin whenever $V_{P1}\neq V_{P2}$—which is mostly the case, but of little importance, since it does not affect the analysis at the point of inflection. We will proceed on the assumption that both forms of asymmetry exist.

To evaluate T_e from (5), we need the electron current to one probe and the derivative of the appropriate plasma-probe fall with respect to V_d. At the point of inflection, the derivatives are both given by (11); the electron currents are equal and writing (3) in the form

$$i_{p1}+i_{p2}=i_{e1}+i_{e2}, \tag{3a}$$

each is clearly equal to one-half the total positive ion current to the system. This last may be found from

$$i_{p1i}+i_{p2i}=i_{p1s}+i_{p2s}-2S\Delta V, \tag{12}$$

a result derived from (2), in which ΔV is the shift in potential of either probe as its electron current increases from zero to the inflection value. The subscript " i " indicates values at the inflection, while " $1s$ ", " $2s$ " indicate values when P_1, P_2 respectively draw zero electron current. While the probes may, in fact, shift their potentials by different amounts in moving from positive ion saturation to the inflection, any such difference will be compensated by a corresponding difference in the true values of S_1 and S_2. The relation between ΔV and the corresponding observed change ΔV_d has been worked out by Johnson and Malter (1950), and the analysis will not be reproduced here ; the result is

$$\Delta V = 0\cdot 85 \Delta V_d. \tag{13}$$

Combining (12) and (13)

$$i_{e1i} = i_{e2i} = \tfrac{1}{2}(i_{p1s} + i_{p2s} - 1\cdot 70 S \Delta V_d), \tag{14}$$

and substituting from (11) and (14) into (5), rearranged to read

$$\frac{1}{\varphi} = i_{e1i} \Big/ \left(\frac{\mathrm{d}i_d}{\mathrm{d}V_d} \frac{\mathrm{d}V_d}{\mathrm{d}V_1} - S \right)_i$$

we get finally

$$\frac{1}{\varphi} = \frac{kT_e}{e} = \frac{(i_{d1s} + i_{d2s}) - 1\cdot 70 S \Delta V_d}{2(2\mathrm{d}i_d/\mathrm{d}V_d - S)_i}, \tag{15}$$

where we have used the relations

$$i_{d1s} = i_{p1s} \; ; \quad i_{d2s} = i_{p2s}.$$

Yamamoto and Okuda (1956) give a result similar to equation (15)

$$\frac{kT_e}{e} = \tfrac{1}{2} A \Big/ \left(\frac{1}{R_0} - \tfrac{1}{2} S \right)$$

(eqn. (29) in their paper. Note that there is a misprint in both (29) and that immediately preceding it, the oblique stroke of division has been omitted.) In this expression,

$$R_0 = \left(\frac{\mathrm{d}V_d}{\mathrm{d}i_d} \right)_{V_d = 0},$$

and $A = i_{p1} = i_{p2}$ when $V_d = 0$, it being assumed that the probe characteristic is symmetrical. The analysis here has shown this result to be generally true, even though the characteristic is strongly asymmetrical, provided the variables are evaluated at the point of inflection of the characteristic and provided that S_1 and S_2 do not differ too greatly. Evaluation at $V_d = 0$ leads to serious errors for even a small departure from symmetry, since not only are i_{p1} and i_{p2} wrongly estimated, but the temperature equation above is only true when the electron currents to both probes are equal to one-half the total positive ion current. Yamamoto and Okuda obtained their result by correcting the " equivalent resistance " analysis of Johnson and Malter to allow for non-zero S, but the method of correction used is valid only if the characteristic is symmetrical. Johnson and Malter, in developing the equivalent resistance method, implicitly assumed a symmetrical characteristic, but believed that the method was applicable so long as the ordinate $V_d = 0$ lay between V_{d1s} and V_{d2s} ; as the above analysis shows, the point corresponding to $V_d=0$ has no special significance. If $S=0$, the point chosen will not affect the accuracy of the temperature estimate, but the analysis is greatly simplified if measurements are made at the point of inflection; if $S\neq 0$, the point of inflection is the only point at which measurements will yield a correct temperature estimate—and then only so long as S_1 and S_2 are equal to a good approximation. Yamamoto and Okuda properly subtracted out the component of the characteristic slope due to positive ion current variation, but did not appreciate the significance of the point of inflection.

Vagner (1958), commenting on an analysis by Tverdokhlebov (1957, 1958) recognized that measurements at $V_d=0$ will yield false results if substituted into a formula formally similar to (15), unless the probe characteristic is symmetrical about the origin. They concluded that the point on the characteristic at which measurements should be made was that corresponding to $i_d=0$. As we have seen, although this is a better choice than $V_d=0$, it can still be well removed from the point of inflection if the probe effective areas differ slightly, and the results will then be in error.

It should be noted that the point of inflection may be difficult to locate with certainty from the observed characteristic, which is frequently nearly linear over a range of values of voltage and current between the break-points. If the areas of the probes are not greatly different, the inflection may be assumed located at the midpoint of the voltage range between the break-points. This approximate location corresponds to the approximation already noted in equation (12)—that V is the same for both probes. The best way to resolve any doubt as to the true location of the inflection is to make replicated observations of the probe current i_d at four values of V_d lying between the break-points of the characteristic. If these voltage values are equally spaced, it is a comparatively simple matter to fit to the four results a cubic regression curve, the inflection of which may then be found readily.

III. Floating Probe Systems with Greatly Different Probe Areas: Mapping the Energy Distribution

The electron current to either probe can never exceed the positive ion current to the other; thus, if the probe areas are equal, and since $j_e \gg j_p$ in gas discharges to which probes are applicable, only a small fraction of the electron energy distribution will be sampled by one probe before the circuit current is limited by positive ion saturation at the other. Johnson and Malter suggested that this could be overcome by using probes of widely different areas, the smaller probe being used as anode so that all electrons approaching it may be accepted before all positive ions approaching the (larger) cathode are accepted; thus the characteristic terminates by the small probe reaching space potential, as with the single Langmuir probe. The theory of such a system has been developed by Okuda and Yamamoto (1960), who introduced a third probe P_3 into the plasma, which was held at the floating potential by drawing no current from it. The potential of the smaller of the double probe pair was then measured relative to P_3; these measurements can be related to the space potential either through the theoretical connection between wall and space potential, or from observation of the measuring probe potential above P_3 when the electron current to it reaches Aj_e—the break-point in the probe characteristic. The characteristic obtained in this way should be identical with that obtained by the single Langmuir probe technique, the only difference being that no net current is here being drawn from the plasma. Thus the electron energy distribution can be mapped by plotting the second derivative d^2i_e/dV^2 against V (Druyvesteyn 1930). The electron current i_e to the measuring probe can be obtained readily for any point on the characteristic from (3) and (2), once the potential of the probe has been related to the plasma potential from the break-point of the characteristic. If the distribution is Maxwellian, the electron temperature can then be determined directly from (5).

The introduction of a third probe into the plasma is the only unfortunate feature of this extension of floating probe technique; one of the double probe pair must already have a considerable area, in order to permit the other to reach saturation electron current before i_d is limited by positive ion saturation. Although the probes draw no net current from the plasma, they do modify the discharge geometry and alter the spatial distribution of charges. However, there is really no need to introduce a third probe into the discharge to act as a reference against which to measure the potential of the measuring probe. So long as no current is drawn from the measuring probe in measuring its potential, the probe system remains floating; thus any convenient point of constant potential (e.g. earth, or either electrode of the discharge if it is a steady d.c. discharge) can be used as reference level, and the potential of the measuring probe measured relative to it potentiometrically. The zero of the probe potential is, again, fixed by the position of the break-point of the characteristic, and the floating potential is readily determined, if required, as that potential at which the measuring probe draws zero current from the discharge.

This method of determining plasma-probe fall experimentally is less convenient and less readily automated than the analysis based on measurements at the point of inflection, if floating double probes are being used simply to estimate the electron temperature. However, if the probe areas are so different that S_1 and S_2 cannot be taken as equal, or if there is any doubt that the electron energy distribution is Maxwellian, one must either use the triple probe technique of Okuda and Yamamoto, or the simpler modification of the technique of asymmetrical-area double probes described here.

IV. Acknowledgment

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