EQUILIBRIUM MODELS OF STARS COMPOSED OF PURE HELIUM

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Summary

Nine equilibrium models of pure helium stars have been calculated for a range of 1 to 17 solar masses. It is shown that for stars larger than two solar masses radiation pressure cannot be neglected. The main effects are a marked reduction in luminosity and a larger core. The change in luminosity does not alter the "main sequence" line in the Hertzsprung-Russell diagram, but shifts the masses down along this line. There is a small decrease in radius accompanied by a corresponding increase in central density.

I. Introduction

Models of stars composed of pure helium were first constructed by Crawford (1953), and more recently by Cox and Giuli (1961). These authors showed that all the energy generation occurred in a convective core, which was surrounded by a radiative envelope. The opacity is mainly due to electron scattering, and electron degeneracy can be neglected for masses larger than one solar unit.

In their paper, Cox and Giuli assumed the effects of radiation pressure to be negligible. Using the formula

$$\beta = [1+\mu a T^3/3\mathscr{R}\rho]^{-1}$$

and substituting the values of T_c and ρ_c obtained in their models they found the values of β_c given in Table 1. The last column of this table gives the values of β_c obtained when radiation pressure is fully taken into account. It can be seen from this table that $1-\beta$ is certainly small for masses which do not exceed two solar units.

In order to show that the effects of radiation pressure can be neglected for higher masses, Cox and Giuli compared their results with those of Taylor (1954). This comparison shows in fact that, for a star composed of pure hydrogen ($\mu = \frac{1}{2}$) with mass $9\cdot 9M_{\odot}$, the results obtained by neglecting radiation pressure are still substantially correct. This is to be expected since Taylor finds for these stars a value of β_c equal to $0\cdot 97$.

For higher masses a rough estimate of the amount of radiation pressure to be expected can be obtained from the standard model. In the standard model β (which is constant throughout) is determined as a function of mass and composition by

$$\mu^2 M = 8\cdot 073 \left[\frac{3\ \mathscr{R}^4}{a\pi G^3}\cdot\frac{1-\beta}{\beta^4}\right]^{\frac{1}{2}}.$$

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For a mass of $9 \cdot 9M$ and a composition of pure helium ($\mu = 4/3$), $\mu^2 M/M_\odot = 17 \cdot 6$. Interpolating in the table given by Chandrasekhar (1939) we find that

$$1-\beta = 0 \cdot 271.$$

The central value β_c derived in the present paper for a star of mass $9 \cdot 027 M_\odot$ is in fact $0 \cdot 7$.

TABLE 1

VALUES OF β_c

$M/M_\odot$	β_c Cox-Giuli	β_c Present Paper
0·500	1·00	
1·000	0·98	
1·071		0·98
1·828		0·95
2·000	0·93	
2·928		0·90

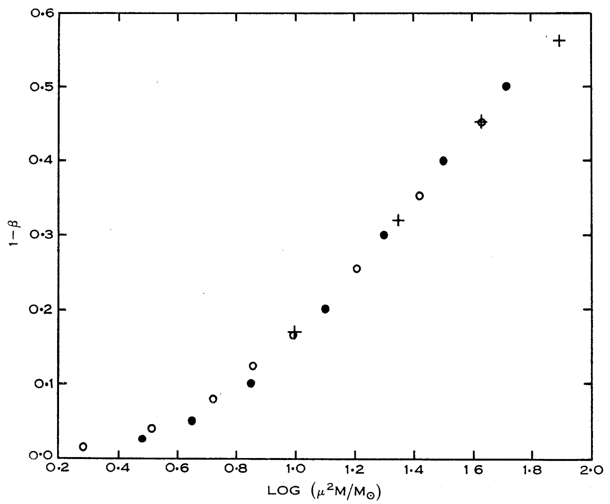

Fig. 1.—Variation of β with mass and composition. ● Standard model; + Schwarzschild and Härm; ○ pure helium.

It is interesting to note that there exists a relationship of the same type as the one found between $1-\beta$ and $\mu^2(M/M_\odot)$ for the standard model, if we substitute for β the average $\bar{\beta}$ defined by $\bar{\beta} = \frac{1}{2}(\beta_c+\beta_f)$ where β_f is the value of β at the boundary of the convective core and radiative envelope.

The values of $1-\bar{\beta}$ and $\mu^2(M/M_\odot)$ have been plotted in Figure 1 for the following massive stars.

(*a*) Stars with mixed composition, $\mu = 0 \cdot 5952$ (Schwarzschild and Härm 1958).
(*b*) Stars composed of pure helium, $\mu = 4/3$ (present models).

The values of $1-\beta$ for the standard model are also indicated in this diagram.

It is seen from this figure that the values of β derived from the standard model give a good approximation of the average value of β in the core.

II. Numerical Calculations

The four basic equilibrium equations (Schwarzschild 1958)

$$\frac{\mathrm{d}M_r}{\mathrm{d}r} = 4\pi r^2\rho,$$

$$\frac{\mathrm{d}L_r}{\mathrm{d}r} = 4\pi r^2\rho\epsilon,$$

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{GM_r\rho}{r^2},$$

$$\frac{\mathrm{d}T}{\mathrm{d}r} = \frac{2(4-3\beta)}{32-24\beta-3\beta^2}\cdot\frac{T}{P}\frac{\mathrm{d}P}{\mathrm{d}r} \quad \text{(in convective core)},$$

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3\kappa}{16\pi ac}\cdot\frac{\rho}{T^3}\cdot\frac{L_r}{r^2} \quad \text{(in radiative envelope)},$$

together with the equation of state

$$P = \frac{\mathscr{R}}{\mu}\rho T + \tfrac{1}{3}\mathrm{a}T^4,$$

were integrated numerically, using the I.B.M. 1620 computer of the Research School of Physical Sciences at the Australian National University.

A constant opacity

$$\kappa = 0 \cdot 2004(1+X)\ \mathrm{cm}^2/\mathrm{g}$$

due to electron scattering was adopted, and the energy production per unit mass due to the Salpeter triple alpha reaction was taken as

$$\epsilon = 1 \cdot 38\times 10^{17}\times\rho^2\ Y^3\times(10^6/T)^3\times \mathrm{e}^{-4320\times 10^6/T}\ \mathrm{erg\ g^{-1}\ s^{-1}}.$$

The results of these calculations for the nine models are set out in Table 2, in which the subscripts c, f, and R refer to values at the centre, fitting point, and surface respectively.

The pressure and temperature have been assumed to be zero at the outer boundary.

III. Discussion of Results

The mass luminosity relation in the present models takes the form of

$$\kappa L/(1-\beta_{\mathrm{R}}) = 4\pi cGM.$$

This has been plotted in Figure 2, together with the mass luminosity relation for the models of Cox and Giuli, which is given by

$$L/L_\odot = 320 \cdot 4(M/M_\odot).$$

It can be seen that the two curves differ markedly from one another in the higher mass range. This is mainly due to the fact that in the triple alpha process the energy

TABLE 2

RESULTS OF CALCULATIONS FOR NINE MODELS OF HELIUM STARS

β_c	0·98	0·95	0·9	0·85	0·8	0·75	0·7	0·6	0·5
β_f	0·989	0·972	0·941	0·907	0·870	0·832	0·790	0·696	0·591
β_R	0·994	0·984	0·964	0·938	0·907	0·872	0·831	0·736	0·625
$T_c \times 10^{-8}$	1·399	1·521	1·633	1·711	1·775	1·831	1·883	1·978	2·071
$T_e \times 10^{-5}$	0·613	0·735	0·863	0·961	1·044	1·120	1·188	1·308	1·408
ρ_c	546·9	272·5	159·8	115·7	91·2	75·1	63·5	47·4	36·2
$P_c \times 10^{-19}$	4·830	2·700	1·794	1·442	1·252	1·134	1·056	0·966	0·928
$R \times 10^{-10}$	1·226	1·850	2·592	3·229	3·846	4·482	5·166	6·790	9·004
$\log (L/L_\odot)$	2·602	3·275	3·846	4·224	4·522	4·775	5·001	5·406	5·779
x_f	0·294	0·314	0·337	0·363	0·389	0·421	0·447	0·499	0·550
q_f	0·338	0·387	0·443	0·506	0·567	0·637	0·692	0·786	0·864
$M \times 10^{-34}$	0·213	0·363	0·584	0·817	1·083	1·402	1·792	2·908	4·834

generation depends very critically on the central temperature. The slight reduction in temperature brought about by taking account of radiation pressure leads to a decrease in luminosity by a factor of two at the upper end of the mass range.

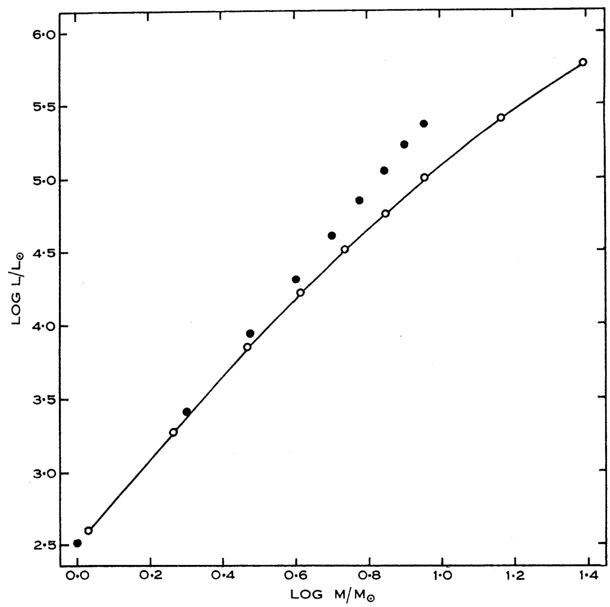

Fig. 2.—Mass-luminosity relation. ● Cox and Giuli; ○ present models.

The present models, together with those of Cox and Giuli, have also been plotted on the Herzsprung-Russell diagram, and the results are compared in Figure 3. The two helium "main sequences" do not differ appreciably from each other. There is, however, a general shift of masses down along this line, the magnitude of this shift increasing as the masses increase.

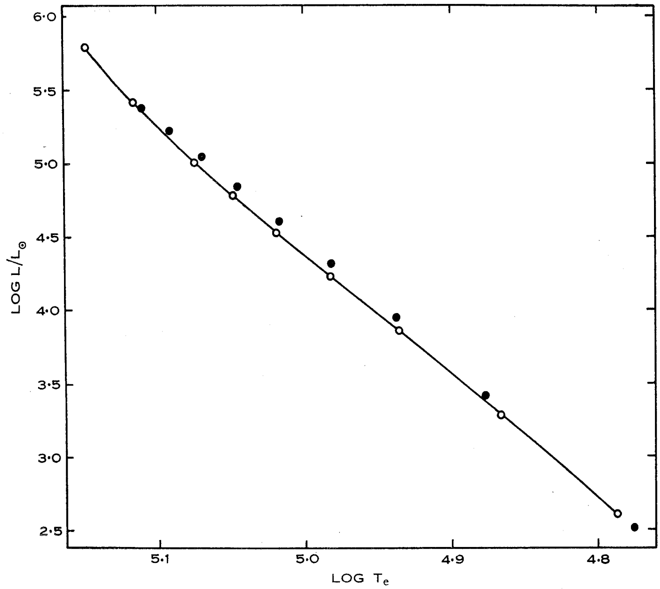

Fig. 3.—Hertzsprung-Russell diagram. ○ Present models; ● Cox and Giuli.

Figure 4 shows the variation of the four main quantities—mass, luminosity, pressure, temperature—throughout the interior of the model of mass $9 \cdot 027 M_{\odot}$.

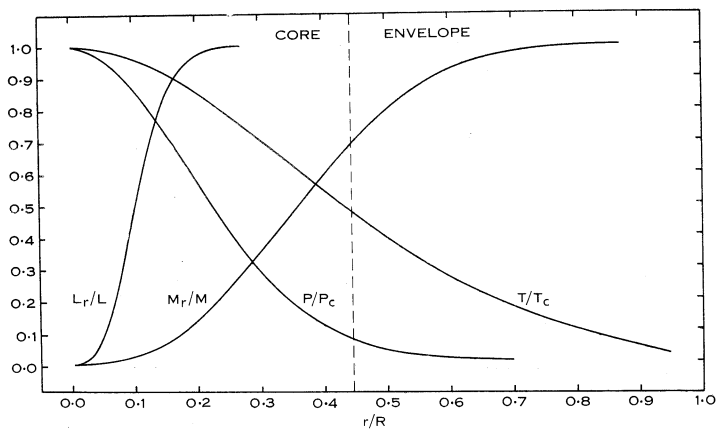

Fig. 4.—Variation of luminosity, mass, pressure, and temperature with distance from centre.

As is to be expected, all the energy generation takes place in a small part of the core, which in fact contains only about 17% of the total mass. The point of inflection of the temperature curve comes a little way inside the core at $r/R = 0 \cdot 38$.

A comparison was also made with results obtained by Cox and Giuli for a star of mass $9M_{\odot}$, which is at the upper limit of the masses considered in their paper.

The results are given in Table 3 and the ratios of the various quantities are given in the last column.

This table emphasizes again the following facts.

Radiation pressure cannot be neglected in the computation of models of stars composed of pure helium, for masses as high as nine solar masses. Even in the outer layers the gas pressure only accounts for 83% of the total pressure.

TABLE 3

COMPARISON WITH RESULTS OF COX AND GIULI

	Cox and Giuli	Present Model	Ratio
β_c	1·000	0·700	1·429
β_f	1·000	0·790	1·266
β_R	1·000	0·831	1·203
$T_c \times 10^{-8}$	2·176	1·883	1·156
$R/R_{\odot}$	0·971	0·744	1·305
$\rho_c \times 10^{-3}$	0·274	0·635	0·432
$L/L_{\odot} \times 10^{-5}$	2·336	1·003	2·329
$T_e \times 10^{-5}$	1·290	1·188	1·086
q_f	0·312	0·692	0·451
x_f	0·283	0·447	0·634

Although the change in radius is not very large, it has a marked effect on the value of the central density, which is more than double the value found by Cox and Giuli. On the other hand the luminosity is only half the value found by these authors.

Finally it should be mentioned that, when radiation pressure is taken into account, the size of the convective core is nearly doubled and that it contains a much larger fraction of the stellar mass.

IV. References

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