THE ANGULAR DISTRIBUTION OF 1·79 MeV NEUTRONS SCATTERED FROM HELIUM

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Summary

The angular distribution of $1\cdot79 \pm 0\cdot05$ MeV neutrons scattered from ^{4}He has been measured in an effort to settle a discrepancy in the phase shifts. The laboratory angular range 25° to 135° was covered. The results are compared with differential cross sections computed from the phase shifts of Dodder and Gammel (1952) and Demanins *et al.* (1962), and the data are seen to agree best with the former.

I. Introduction

The neutron time-of-flight method has been used to measure the angular distribution of $1\cdot79 \pm 0\cdot05$ MeV neutrons scattered from helium. The neutrons were obtained by bombarding a tritium gas target assembly with $2\cdot75$ MeV protons. Previous experiments in this general energy range have all used a recoil alpha-particle detection technique, which is intrinsically limited to large scattering angles. On the other hand, when the scattered neutron is detected, as in this experiment, considerably smaller scattering angles can be reached. Forward angle data are of interest in that such data may resolve the present disagreement among various experimenters as to the value of the p_- phase shift in the region around 2 MeV (Dodder and Gammel 1952; Seagrave 1953; Clementel and Villi 1955; Levintov, Miller, and Shamshev 1957; Striebel and Huber 1957; Demanins *et al.* 1962).

II. Experimental Details

A 10 μA beam from the University of Texas 3 MeV electrostatic accelerator was pulsed after acceleration at a frequency of $3\cdot6$ Mc/s. The amplitude of the deflecting plate voltage and the width of the "chopping" slits were such as to produce bursts of approximate length 5 ns. After collimation and focusing, about $0\cdot3$ to $0\cdot4$ μA of pulsed beam was available on target.

The neutron-producing target consisted of a cylinder containing tritium gas at one atmosphere. The cell was $1\cdot5$ cm long and was separated from the accelerator by a $0\cdot0001$ in. nickel foil. The tritium was stored when not in use in the standard way as UT_3 (Johnson and Banta 1956).

The helium scatterer was contained in a steel high-pressure cell located $7\cdot5$ cm from the tritium target. The high-pressure cell was cylindrical in shape and was oriented with its axis vertical. The cell was filled and emptied by means of an external gas-handling system. All measurements reported here were made using a pressure of 5000 lb/in^2.

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The high-pressure cell is of a sufficiently simple design to allow easy fabrication and is shown in Figure 1. The main section of the cell was constructed from "Vega" tool steel, supplied by the Carpenter Steel Co. This was chosen rather than a stainless steel since it can be machined before it is hardened. The thin portion of wall was machined to a thickness of $0 \cdot 032 \pm 0 \cdot 0003$ in. It was then heat treated by a commercial organization to a Brinell hardness of C 45. The remaining pieces of the cell assembly were made from mild steel.

Initial pressure tests (using strain gauges) indicated linear deformation up to 6500 lb/in^2. In a later test, pressures as high as 11 000 lb/in^2 were used in an attempt to rupture the vessel, but no failure occurred. However, after cycling the pressure from 0 to 7500 lb/in^2 about 400 times, the cell failed.

The cell was pressurized with a free piston pump which was driven by a Sprague* air-operated oil pump of maximum pressure capacity 33 000 lb/in^2. This system required about 3 minutes to raise the helium pressure from 0 to 5000 lb/in^2.

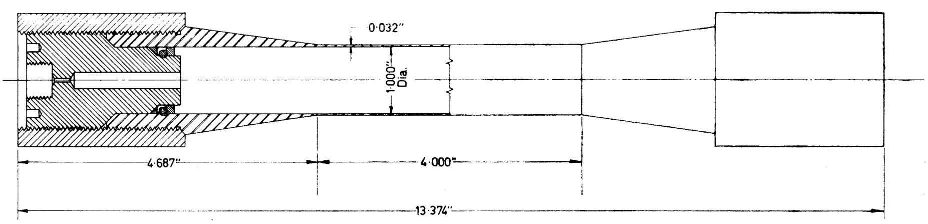

Fig. 1.—Helium high-pressure cell.

The neutron detector consisted of a 4 by 3 by 2 in. plastic scintillator viewed by two RCA 6810 photomultiplier tubes through Lucite light pipes. Coincidences between the two phototubes provided the "start" signal for a time to pulse-height converter of the Oak Ridge type (Marion and Fowler 1960). The stop signal was obtained from the deflection plate oscillator. The converter output was amplified and pulse-height analysed. The pulse-height analyser was gated "on" only if the stretched pulses from each phototube satisfied an amplitude criterion.

The experimental assembly is illustrated in Figure 2. The detector was placed in a large shield made of lithium carbonate and paraffin. An adjustable wedge of iron and tungsten was used as an additional shield against neutrons coming directly from the tritium target. The entire shield assembly was free to rotate around the helium cell.

The neutrons were monitored by a 1 by 2 by $0 \cdot 040$ in. plastic scintillator placed a few inches above the tritium cell. The rather short distance was used to make the monitor insensitive to geometry changes caused by rotation of the large shield assembly.

The relative efficiency of neutron detection as a function of energy was determined by scattering from a Lucite target. The time resolution of the system was sufficiently good to distinguish neutrons which scattered from protons from

* Sprague Engineering Corporation, 19300 South Vermont Avenue, Gardena, California, U.S.A.

those which scattered from carbon or oxygen. Thus, since the n-p angular distribution is well known, it was possible to obtain the relative efficiency from the counting rate at various angles. One difficulty resulted from the energy limitation of the accelerator, which made it impossible to cover the full energy range of helium-scattered neutrons. The efficiency at energies greater than that corresponding to helium-scattered neutrons at 60° was found by extrapolation of the lower energy efficiency. To minimize the error in the extrapolation, the observed efficiency data were fitted to a theoretical curve.

The data taken at each angle consisted of a run with the helium cell filled to 5000 lb/in² and a run with the cell filled to 1 atm. As can be seen from Figure 3,

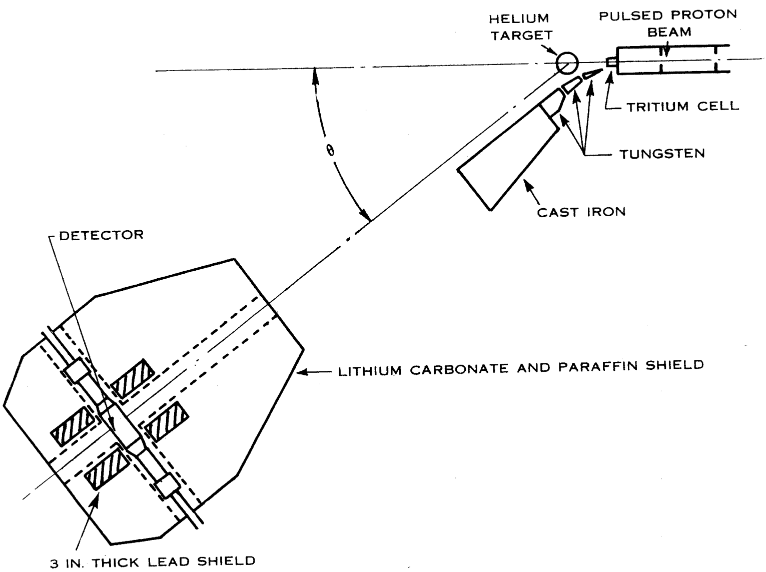

Fig. 2.—Experimental arrangement.

the background was quite large. Scattering by the walls of the pressure cell was relatively unimportant.

III. Multiple-scattering Correction

A multiple-scattering correction was applied to the data. Since the thickness of the scatterer used in the experiment was only about 1/20 of a mean free path, a relatively crude treatment was performed. The following discussion holds true only if the scattering law has axial symmetry (as is the case in the present experiment).

For an infinite medium one may speak of a set of functions $S_K(\theta)$ which characterize the angular distribution of particles which have undergone K collisions. Aside from normalization (we choose $2\pi\int_0^\pi S_K(\theta)\sin\theta\, d\theta = 1$), $S_1(\theta)$ is the same as the differential cross section $\sigma(\theta)$.

The various $S_K(\theta)$ can be related to $S_1(\theta)$ by a simple formula (Goudsmit and

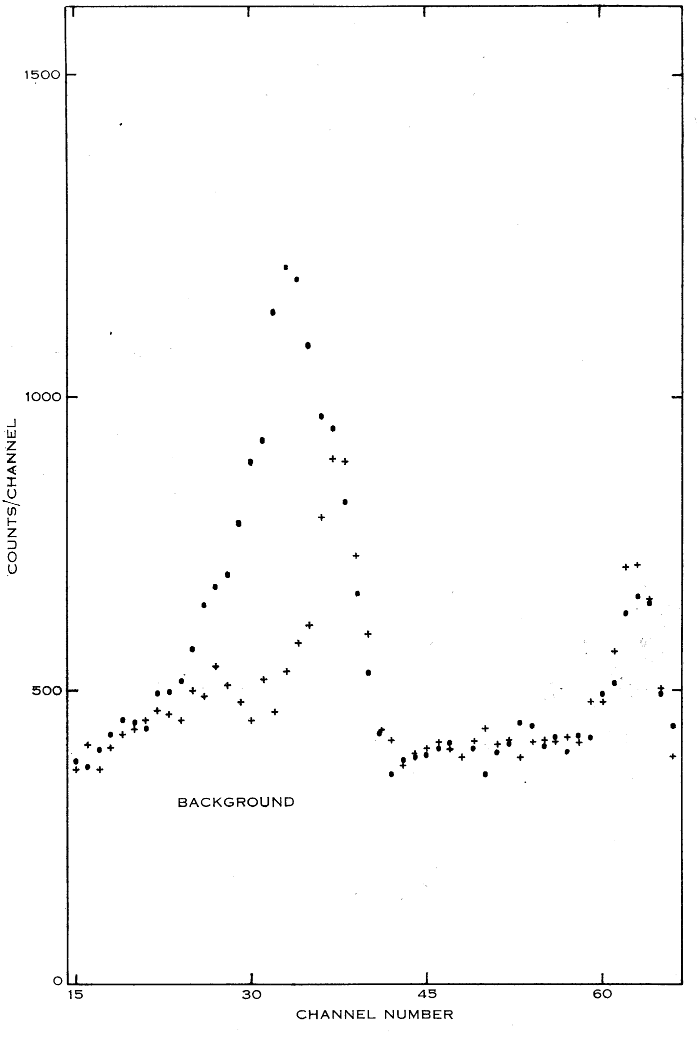

Fig. 3.—Neutron spectra at 70° lab. with the high-pressure target full of helium (solid circles) and with the target empty (crosses). The time scale is approximately 2 ns per channel. From left to right the large peaks are due to helium-scattered neutrons, iron-scattered neutrons, and gamma-rays resulting from inelastic scattering at the iron target.

Saunderson 1940)

$$S_K(\theta) = \frac{1}{4\pi}\sum_{l=0}^{\infty}(2l+1)\left[\overline{P_l(\cos\theta)}\right]^K P_l(\cos\theta),$$

where $\overline{P_l(\cos\theta)}$ is the mean value of the Legendre polynomial $P_l(\cos\theta)$ after a single scattering:

$$\overline{P_l(\cos\theta)} = \int_0^{\pi}\int_0^{2\pi} S_1(\theta)P_l(\cos\theta)\sin\theta\, d\phi d\theta.$$

It follows that the effective angular distribution is given by an appropriate weighting of the $S_K(\theta)$,

$$S(\theta) = \sum_{K=1}^{\infty} a_K\, S_K(\theta),$$

where a_K is the fraction of particles which are scattered K times. The expression given above for $S_K(\theta)$ assumes that the relative angular distribution does not change with energy.

For a non-infinite medium the $S_K(\theta)$ are not as given above. For example, suppose a collimated neutron beam is incident along the cylindrical axis of an oblate spheroidal target. Then it follows that those neutrons which are scattered through an angle near 90° on their first scattering will have a higher probability of scattering a second time than those which are scattered near 0°. Thus, the actual angular distribution after second scatterings will be increased at large angles compared to the $S_2(\theta)$ calculated above. However, for our purposes we will assume that the $S_K(\theta)$ calculated for an infinite medium are sufficiently accurate. Blok and Jonker (1952) calculate that this approximation results in an error of less than $\frac{1}{2}$% for their particular geometry.

We make two further assumptions:

1. Only up to three scatterings need be considered, i.e. $a_1+a_2+a_3 = 1$.
2. The number of triply scattered neutrons is the same fraction of the doubly scattered neutrons as the number of doubly scattered neutrons is of the singly scattered neutrons, i.e. $a_3/a_2 = a_2/a_1$.

With these assumptions, it is only necessary to evaluate a_1.

We may write the following expression for a_1:

$$a_1 = \int P(x,y,z)e^{-n\sigma(E)r(x,y,z,\theta,\phi)}S_1(\theta)\sin\theta\, \mathrm{d}\theta\mathrm{d}\phi\mathrm{d}x\mathrm{d}y\mathrm{d}z,$$

where $P(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z$ is the relative probability that a first scattering occurs in $\mathrm{d}x\mathrm{d}y\mathrm{d}z$, $S_1(\theta)\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi$ is the probability that this scattering is within the solid angle $\sin\theta\,\mathrm{d}\theta\mathrm{d}\phi$, and $e^{-n\sigma(E)r(x,y,z,\theta,\phi)}$ is the probability that the particle escapes without further scattering. The normalization of $P(x,y,z)$ is as follows:

$$\int_{\text{volume}} P(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z = 1.$$

The quantity n is the number of helium nuclei per cubic centimetre, $\sigma(E)$ is the total cross section, which depends on the scattering angle through the energy E of the scattered particle, and r is the distance the particle must travel to escape from the scatterer.

The exact evaluation of this integral could only be done through computor techniques. A crude approximation will suffice for the accuracy required here.

We approximate as follows:

1. $P(x,y,z)$ will be regarded as a constant, i.e. the probability of first scattering throughout the volume will be assumed to be uniform.
2. $S(\theta)$ will be assumed isotropic.
3. $\sigma(E)$ will be replaced by an appropriate mean value. These approximations

yield

$$1-a_1 = \frac{1}{4\pi V}\int (1-\mathrm{e}^{-n\bar{\sigma}r}) \sin\theta \,\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}x\mathrm{d}y\mathrm{d}z,$$

where V is the volume of the scatterer. Expanding and retaining only the first term, this becomes

$$1-a_1 = \frac{\bar{\sigma}n}{4\pi V}\int r \sin\theta \,\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}x\mathrm{d}y\mathrm{d}z.$$

A crude numerical evaluation of this integral for cylindrical geometry having a height to radius ratio of four yields

$$1-a_1 = 0\cdot 87\bar{\sigma}nR,$$

where R is the radius of the cylinder.
Thus we obtain

$$a_1 = 0\cdot 945,$$

$$a_2 = 0\cdot 052,$$

$$a_3 = 0\cdot 003.$$

A further problem in this case is the effect of the container wall on the angular distribution. For this portion of the correction, we regard the scattering as occurring in a plane. The effect on the angular distribution due to neutrons which are scattered first by the helium into angle ϕ and then by the container wall may be represented approximately by

$$(1-\epsilon)S(\theta)+\epsilon\int_0^{2\pi} S(\phi)S_{\mathrm{Fe}}(\theta-\phi)\mathrm{d}\phi,$$

where $S(\theta)$ is the angular distribution from helium in the absence of the container walls, $S_{\mathrm{Fe}}(\theta)$ is the angular distribution of neutrons scattered from iron, and ϵ is the fraction of neutrons scattered in passing through the container wall. The effect of scattering first by the container wall and then by the helium can be similarly represented, and we have finally

$$S_{\mathrm{ex}}(\theta) = (1-2\epsilon)S(\theta)+\epsilon\int_0^{2\pi}\Big[S(\phi)S_{\mathrm{Fe}}(\theta-\phi)+S_{\mathrm{Fe}}(\phi)S(\theta-\phi)\Big]\mathrm{d}\phi,$$

where $S_{\mathrm{ex}}(\theta)$ is the observed angular distribution.

The above expression assumes:

1. The energy dependence of the iron cross section may be neglected.
2. All helium scatterings occur at the centre of the cylinder.
3. The geometry may be considered to be plane.

Numerical values of the corrections are presented in Table 1. The various approximations used result in a 30% uncertainty in these values.

TABLE 1

SCATTERING CORRECTIONS

θ	% Correction He Mult. Scat.	% Correction Wall	Total % Correction
0°	+2·8	+1·9	+4·7
30°	+2·5	+1·5	+4·0
60°	+1·0	+0·3	+1·3
90°	−4·1	−4·1	−8·2
120°	−5·9	−6·1	−12·0
150°	−0·6	−1·5	−2·1
180°	+0·7	−0·5	+0·2

IV. RESULTS OF THE EXPERIMENT

For the purpose of normalization, a least squares fit of the corrected angular distribution to a parabola was made. The parabola was normalized to a total cross

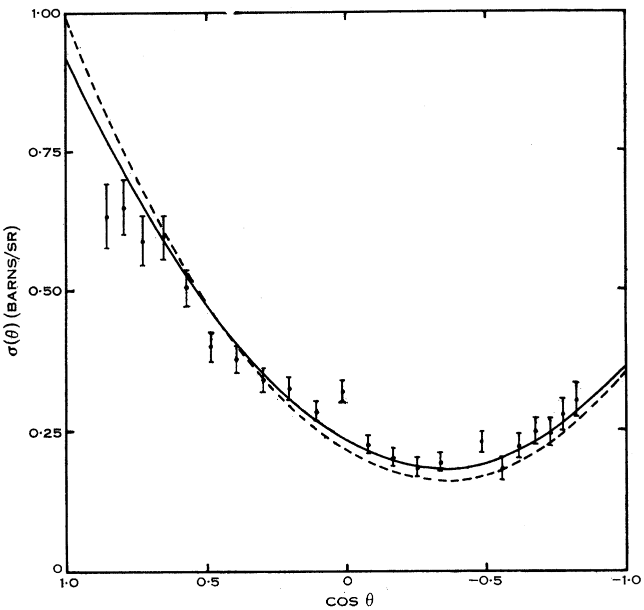

Fig. 4.—Comparison of the normalized data with curves calculated from the phase shifts of Dodder and Gammel (solid curve) and Demanins *et al.* (dashed curve).

section of 4·50 barns. A parabola was sufficient, since at this energy only s and p waves need be considered.

In Figure 4 the normalized data are shown. The designated errors are due to statistical uncertainty and uncertainty in the relative efficiency calibration. Also shown in the figure are differential cross sections computed from the phase shifts of Dodder and Gammel and from extrapolated phases from the data of Demanins *et al.* In the case of the former, the phases $\delta_0 = -33°$, $\delta_1{}^+ = 114°$, and $\delta_1{}^- = 12°$ were used, whereas for the latter $\delta_0 = -34°$, $\delta_1{}^+ = 113°$, and $\delta_1{}^- = 2°$ were used.

The experimental data are seen to fit Dodder and Gammel's curve somewhat better than that of Demanins *et al.* The measurements are certainly not of sufficient accuracy to be conclusive, but they do lend support to the higher value of the p_- phase shift as obtained by Dodder and Gammel.

V. Acknowledgments

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VI. References

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