QUIET DAY SOLAR DAILY MAGNETIC VARIATIONS AT TOOLANGI, VICTORIA

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Summary

The results of analysis of the quiet day solar daily magnetic variations (S_q) are given in the form $\Sigma\, c_n \sin(nt+\phi_n)$, for $n = 1, 2, 3, 4$, of all magnetic elements as recorded at Toolangi Observatory, Victoria, during the years 1949–1958, as well as the times of occurrence of extrema and the percentage contributions of each harmonic to each extrema.

Comparison of times of occurrence of the extrema seems to indicate that the extrema in ΔX and ΔY occur about 10 min earlier in years of high sunspot activity than in years of low sunspot activity, and vice versa for ΔZ.

Graphs of S_q in ΔX, ΔY, and ΔZ are given.

I. Introduction

Published hourly values of horizontal intensity, vertical intensity, and declination as recorded at Toolangi Observatory, Victoria, are available (van der Waal, Sorenson, and Brooks 1960) and the years 1949 to 1958 inclusive were selected for analysis.

Table 1

Annual mean relative sunspot numbers for selected years

Year	Sunspot No.	Year	Sunspot No.
1949	135·1	1954	4·4
1950	83·9	1955	38·0
1951	69·4	1956	141·7
1952	31·4	1957	189·9
1953	13·9	1958	182·4

II. Results

The annual mean sunspot numbers for these years (Chernosky and Hagan 1958) are given in Table 1.

To best determine the effect of sunspot activity on the magnetic variations two groups of years were selected, namely, 1949, 1956, 1957, and 1958, as a group of years with high sunspot activity, the annual relative mean sunspot number for the group being 162·3; and 1952, 1953, 1954, and 1955, as a group of years with low sunspot activity, the annual relative mean sunspot number for the group being 21·9.

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TABLE 2

AMPLITUDES AND PHASES OF MAGNETIC ELEMENTS*

Element	Season	Sunspot Activity	C_1	ϕ_1	C_2	ϕ_2	C_3	ϕ_3	C_4	ϕ_4
ΔX	S	High	13·3	284·4	11·8	286·9	6·5	309·3	2·1	332·7
ΔX	E	High	16·4	259·0	10·3	260·4	3·8	277·3	1·1	292·9
ΔX	W	High	11·0	233·4	9·9	228·6	4·8	226·7	1·1	223·6
ΔX	S	Low	9·0	275·1	7·4	283·7	4·7	300·1	2·1	322·5
ΔX	E	Low	9·0	242·4	6·7	252·8	3·3	271·1	1·3	310·1
ΔX	W	Low	5·6	194·0	6·6	219·2	3·7	213·6	1·0	213·5
ΔY	S	High	27·6	3·0	25·6	11·4	11·8	26·8	1·5	52·7
ΔY	E	High	20·3	358·9	19·0	357·0	10·2	9·8	2·8	16·1
ΔY	W	High	8·9	3·1	10·8	334·6	6·1	330·2	3·1	314·8
ΔY	S	Low	20·2	5·7	16·6	4·7	7·8	20·7	1·2	20·3
ΔY	E	Low	11·2	16·2	12·1	354·6	6·0	3·4	2·2	5·4
ΔY	W	Low	3·0	35·3	6·7	329·8	3·6	321·3	2·3	309·4
ΔZ	S	High	4·7	70·7	3·4	64·6	2·1	63·1	0·5	46·7
ΔZ	E	High	2·7	76·4	2·2	65·6	2·5	58·6	1·0	45·1
ΔZ	W	High	2·6	127·9	1·4	141·7	0·7	80·6	0·8	17·7
ΔZ	S	Low	1·9	82·2	1·5	58·8	1·0	49·1	0·4	12·2
ΔZ	E	Low	1·9	109·8	1·0	79·0	1·2	52·3	0·7	26·2
ΔZ	W	Low	2·9	133·5	1·2	154·0	0·4	96·8	0·7	18·2
ΔH	S	High	14·8	302·8	12·8	307·0	7·2	325·5	2·1	339·6
ΔH	E	High	15·9	271·6	10·3	278·8	4·0	303·0	1·2	316·5
ΔH	W	High	10·0	240·2	9·4	239·6	4·6	239·5	1·2	251·2
ΔH	S	Low	9·5	296·4	8·3	303·6	5·0	315·4	2·1	327·3
ΔH	E	Low	7·6	252·9	6·5	270·9	3·4	288·8	1·5	321·7
ΔH	W	Low	5·1	191·9	6·2	229·1	3·5	223·2	1·1	235·5
ΔF	S	High	8·9	280·1	6·8	283·1	3·5	292·6	0·7	300·9
ΔF	E	High	8·4	267·0	5·7	267·5	3·2	263·5	1·0	251·4
ΔF	W	High	5·2	266·0	3·9	259·0	2·3	245·1	1·1	216·4
ΔF	S	Low	5·2	285·1	3·9	284·5	2·1	290·1	0·6	298·6
ΔF	E	Low	4·4	266·6	3·3	267·6	2·1	262·8	0·7	257·3
ΔF	W	Low	2·3	269·4	2·3	257·7	1·6	235·2	1·0	212·6

* All phase angles ϕ_n are given in degrees at 1200 hr L.T. Amplitudes of ΔH, ΔX, ΔY, ΔZ, and ΔF are given in gammas (1 gauss = 10^5 gammas). Amplitudes of ΔD and ΔI are given in tenths of a minute of arc, where for ΔD, 1 min of arc corresponds to 6·6 gammas, for ΔI, 1 min of arc corresponds to 17·7 gammas. S indicates summer months, January, February, November, December; E indicates equinoctial months, March, April, September, October; W indicates winter months, May, June, July, August.

TABLE 2 (*Continued*)

Element	Season	Sunspot Activity	C_1	ϕ_1	C_2	ϕ_2	C_3	ϕ_3	C_4	ϕ_4
ΔD	S	High	40·6	7·9	37·8	16·0	17·3	32·3	2·2	66·9
ΔD	E	High	31·2	6·6	28·7	2·3	15·2	13·5	4·2	19·9
ΔD	W	High	15·2	11·5	16·9	342·9	9·4	337·7	4·6	318·4
ΔD	S	Low	30·1	10·2	24·5	9·2	11·4	26·8	1·6	36·7
ΔD	E	Low	18·3	21·4	18·4	359·9	9·0	8·8	3·0	10·7
ΔD	W	Low	5·8	30·1	10·6	338·4	5·7	330·6	3·5	313·7
ΔI	S	High	7·2	309·1	6·4	312·6	3·7	332·1	1·1	344·9
ΔI	E	High	7·8	272·7	5·0	281·7	2·0	316·8	0·7	334·7
ΔI	W	High	5·0	234·3	4·9	236·2	2·3	238·2	0·5	266·9
ΔI	S	Low	4·7	299·2	4·2	307·6	2·6	319·9	1·2	330·4
ΔI	E	Low	3·7	249·2	3·2	271·7	1·6	296·1	0·9	330·7
ΔI	W	Low	3·0	182·1	3·3	224·8	1·8	220·7	0·4	246·5

The months of each year were divided into three seasons each of four months, for the purpose of estimating the seasonal changes in the magnetic variations:

Summer months: January, February, November, December
Equinoctial months: March, April, September, October
Winter months: May, June, July, August.

The non-cyclic variation was calculated for each quiet day using Chapman's (1960) formula

$$\tfrac{1}{2}\{(24\tfrac{1}{2})+(23\tfrac{1}{2})-(0\tfrac{1}{2})-(-0\tfrac{1}{2})\},$$

where the figures in brackets refer to the hourly mean values centred at the epochs indicated. Using the average non-cyclic variation over the five international quiet days in each month the monthly quiet day means were thus corrected for non-cyclic variation.

Seasonal averages of S_q were computed for each of the two groups of years for the magnetic elements, declination D, vertical intensity Z, and horizontal intensity H. Corresponding mean values of D, H and I for the main field were found and S_q in the elements, X the geographical north component, Y the geographical east component, F the total intensity, and I the inclination, were computed using the relations:

$$\begin{aligned}\Delta X &= \Delta H \cos D - 2{\cdot}909\times10^{-4}H \sin D\,(\Delta D)',\\ \Delta Y &= \Delta H \sin D + 2{\cdot}909\times10^{-4}H \cos D\,(\Delta D)',\\ \Delta F &= \Delta H \cos I + \Delta Z \sin I,\\ (\Delta I)^{\gamma} &= \Delta Z \cos I - \Delta H \sin I,\end{aligned}$$

and at Toolangi

$$(\Delta I)^{\gamma} = (\Delta I)'\times F\times 2{\cdot}909\times10^{-4},$$

$(\Delta I)^{\gamma}$ being the number of gammas required to produce a variation of 1 min of arc in inclination.

TABLE 3

MAXIMA AND MINIMA OF MAGNETIC ELEMENTS*

Element	Season	Sunspot Activity	Local Time (hr)	Value	Type	Contributions of Harmonics (%)			
						1st	2nd	3rd	4th
ΔX	S	High	11·2	−33·5	Min.	40	35	19	6
ΔX	E	High	12·1	−31·0	Min.	52	33	12	3
ΔX	W	High	13·2	−25·9	Min.	40	38	18	4
ΔX	S	Low	11·4	−23·1	Min.	39	32	20	9
ΔX	E	Low	12·2	−18·8	Min.	44	35	17	4
ΔX	W	Low	13·6	−14·4	Min.	24	46	25	6
ΔY	S	High	8·7	−56·5	Min.	35	45	18	2
			14·3	50·8	Max.	33	50	17	−1
ΔY	E	High	9·3	−43·3	Min.	31	44	22	4
			14·5	39·4	Max.	31	46	22	2
ΔY	W	High	10·4	−21·4	Min.	15	49	28	9
			15·0	24·4	Max.	27	40	24	9
ΔY	S	Low	8·9	−36·9	Min.	36	45	18	1
			14·5	36·1	Max.	38	45	16	1
ΔY	E	Low	9·5	−22·9	Min.	18	52	24	5
			14·5	26·6	Max.	34	42	20	4
ΔY	W	Low	10·7	−10·8	Min.	−7	58	33	16
			15·0	14·0	Max.	21	41	25	13
ΔZ	S	High	12·8	10·6	Max.	44	32	19	5
ΔZ	E	High	12·7	8·4	Max.	33	26	29	12
ΔZ	W	High	11·9	3·9	Max.	55	24	17	5
ΔZ	S	Low	13·0	4·9	Max.	40	32	20	9
ΔZ	E	Low	12·7	4·4	Max.	36	22	27	15
ΔZ	W	Low	10·4	3·4	Max.	81	36	3	−19
ΔH	S	High	10·7	−36·4	Min.	40	35	20	6
ΔH	E	High	11·5	−31·2	Min.	51	33	13	4
ΔH	W	High	12·9	−24·4	Min.	39	38	19	4
ΔH	S	Low	10·9	−24·7	Min.	38	33	20	9
ΔH	E	Low	11·9	−18·0	Min.	40	36	18	6
ΔH	W	Low	13·3	−13·1	Min.	20	47	27	6

* Amplitudes of ΔH, ΔX, ΔY, ΔZ, and ΔF are given in gammas (1 gauss = 10^5 gammas). Amplitudes of ΔD and ΔI are given in tenths of a minute of arc, where for ΔD, 1 min of arc corresponds to 6·6 gammas, for ΔI, 1 min of arc corresponds to 17·7 gammas. S indicates summer months, January, February, November, December; E indicates equinoctial months, March, April, September, October; W indicates winter months, May, June, July, August. The contribution of each harmonic to the extrema is expressed as a percentage of the actual value of the extrema.

TABLE 3 (*Continued*)

Element	Season	Sunspot Activity	Local Time (hr)	Value	Type	Contributions of Harmonics (%)			
						1st	2nd	3rd	4th
ΔD	S	High	8·6	−81·8	Min.	34	46	18	2
			14·2	76·3	Max.	35	49	17	−1
ΔD	E	High	9·2	−62·6	Min.	29	45	22	3
			14·4	62·8	Max.	33	44	21	2
ΔD	W	High	10·1	−31·6	Min.	14	51	29	6
			14·9	40·7	Max.	31	39	22	9
ΔD	S	Low	8·7	−53·8	Min.	35	46	18	1
			14·4	54·0	Max.	40	45	15	0
ΔD	E	Low	9·3	−33·9	Min.	18	54	24	4
			14·4	42·3	Max.	36	41	19	3
ΔD	W	Low	10·4	−17·1	Min.	−4	58	33	13
			14·9	23·5	Max.	23	41	24	12
ΔI	S	High	10·5	−18·1	Min.	38	35	21	6
ΔI	E	High	11·3	−15·2	Min.	51	33	12	4
ΔI	W	High	12·9	−12·1	Min.	39	40	19	3
ΔI	S	Low	10·8	−12·5	Min.	36	34	20	9
ΔI	E	Low	11·6	−8·8	Min.	38	36	19	8
ΔI	W	Low	13·4	−6·5	Min.	19	51	27	3
ΔF	S	High	11·5	−20·0	Min.	45	34	18	4
ΔF	E	High	12·2	−18·3	Min.	46	31	18	5
ΔF	W	High	12·6	−12·4	Min.	42	31	19	8
ΔF	S	Low	11·5	−11·7	Min.	44	33	18	5
ΔF	E	Low	12·2	−10·4	Min.	42	32	20	6
ΔF	W	Low	12·7	−7·1	Min.	32	32	23	13

The three sets of values in each of ΔD, ΔH, ΔZ, ΔX, ΔY, ΔF, and ΔI were then subjected to classical harmonic analysis and each series expressed in the form

$$\sum_{n=1}^{4} c_n \sin(15nt+\phi_n),$$

with the phase angle ϕ_n being given at 1200 hr L.T.; the results are given in Table 2.

The Fourier series expression

$$\sum_{n=1}^{4} c_n \sin(15nt+\phi_n),$$

was then evaluated at $t = 0, 1, 2, \ldots, 23$, and these values were taken to be the "smoothed" values of the data. The times of occurrence of maxima and minima were then found by first evaluating the derivative at each tabulated point using the formula

$$y_3' = \frac{1}{60}(-y_0+9y_1-45y_2+45y_4-9y_5+y_6),$$

then locating the zeros of the derivative by inverse interpolation using Bessel's modified interpolation formula

$$f_a = f_0 + a\delta f_{\frac{1}{2}} + \mathrm{B}^{\mathrm{ii}}(a)\{\delta^2 f_0 + \delta^2 f_1 - 0\cdot 184(\delta^3 f_{\frac{1}{2}} + \delta^3 f_{-\frac{1}{2}})\} + \mathrm{B}^{\mathrm{iii}}(a)\delta^4 f_0,$$

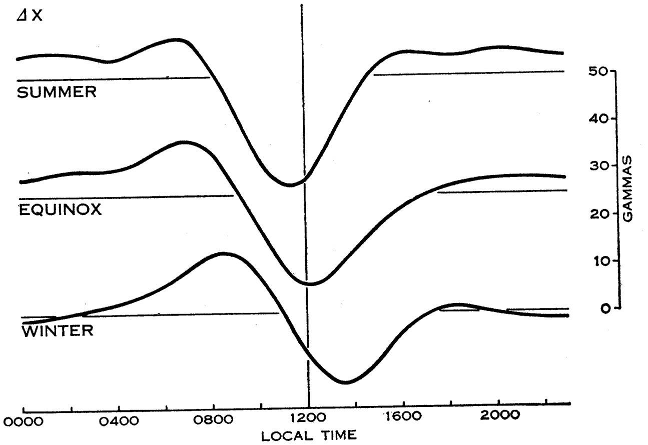

Fig. 1.—Solar daily variation on quiet days in X for years of low sunspot activity.

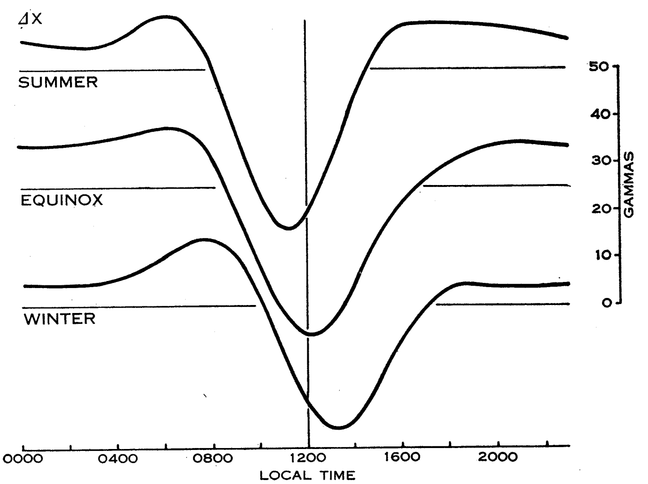

Fig. 2.—Solar daily variation on quiet days in X for years of high sunspot activity.

then the Fourier series was evaluated at this point, and each term in the Fourier series was expressed as a percentage of the value of the extrema; the results are given in Table 3.

Slaucitajs (1947) offers the following formula for the location of extrema. Given

$$\mathrm{f}(x) = a_1 \sin(A_1 + x) + a_2 \sin(A_2 + 2x) + a_3 \sin(A_3 + 3x),$$

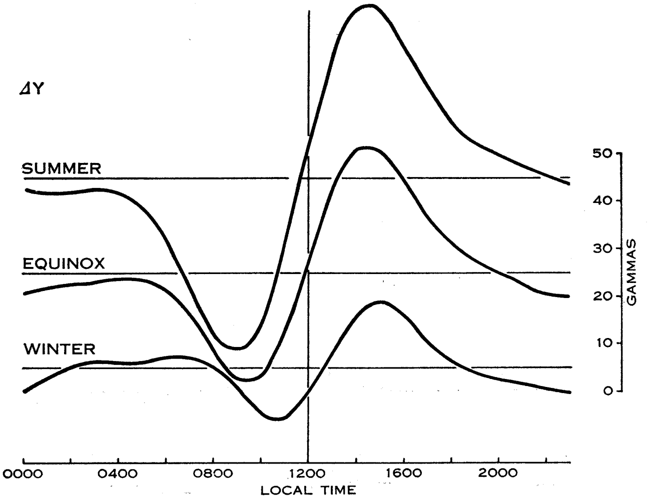

Fig. 3.—Solar daily variation on quiet days in Y for years of low sunspot activity.

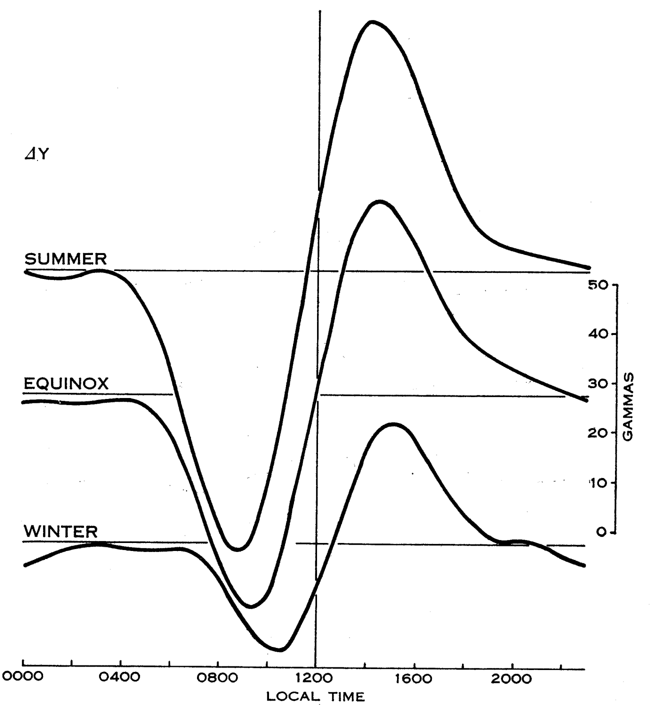

Fig. 4.—Solar daily variation on quiet days in Y for years of high sunspot activity.

then f(x) is a maximum for values of x given by

$$x = \frac{a_1 A_1' + 4a_2 A_2' + 9a_3 A_3'}{a_1 + 4a_2 + 9a_3},$$

where

$$A_1' = 450° - A_1,$$

$$A_2' = \tfrac{1}{2}(450° - A_2),$$

$$A_3' = \tfrac{1}{3}(450° - A_3),$$

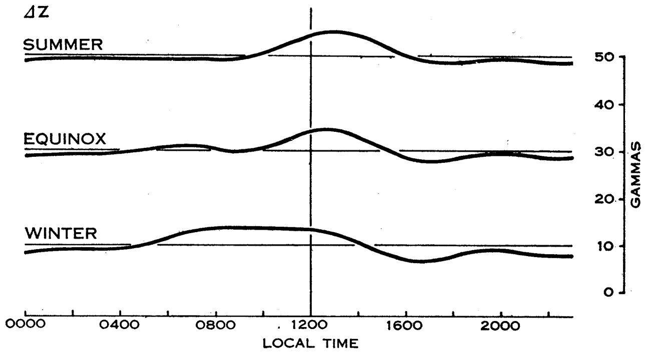

Fig. 5.—Solar daily variation on quiet days in Z for years of low sunspot activity.

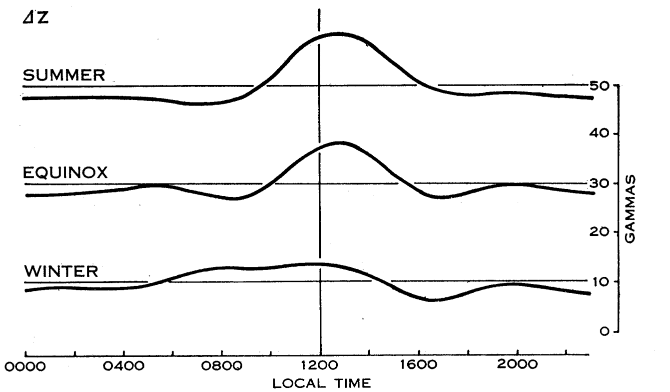

Fig. 6.—Solar daily variation on quiet days in Z for years of high sunspot activity.

and f(x) is a minimum for values of x given by

$$x = \frac{a_1 A_1'' - 4a_2 A_2'' + 9a_3 A_3''}{a_1 + 4a_2 + 9a_3},$$

where

$$A_1'' = 270° - A_1,$$

$$A_2'' = \tfrac{1}{2}(90° - A_2),$$

$$A_3'' = \tfrac{1}{3}(270° - A_3).$$

An example taken from Table 2 gives

$$f(x) = 14 \cdot 8\gamma \sin(275 \cdot 8° + x) + 12 \cdot 8\gamma \sin(253 \cdot 0° + 2x) + 7 \cdot 2\gamma \sin(244 \cdot 5° + 3x)$$

and the method gives the following results:

maxima at 16·2 hr L.T.,
minima at 11·0 hr L.T.

TABLE 4

SUMMARY OF LOCAL TIMES OF OCCURRENCE OF EXTREMA

Element	High Sunspot Activity Years (hr L.T.)	Low Sunspot Activity Years (hr L.T.)
ΔX Summer minimum	11·2	11·4
ΔX Equinox minimum	12·1	12·2
ΔX Winter minimum	13·2	13·6
ΔY Summer minimum	8·7	8·9
maximum	14·3	14·5
ΔY Equinox minimum	9·3	9·5
maximum	14·5	14·5
ΔY Winter minimum	10·4	10·7
maximum	15·0	15·0
ΔZ Summer maximum	12·8	13·0
ΔZ Equinox maximum	12·7	12·7
ΔZ Winter maximum	11·9	10·1

The values calculated by inverse interpolation were

maxima at 16·0 hr L.T.
minima at 10·7 hr L.T.,

hence the formulae show good agreement.

III. DISCUSSION

The smoothed curves representing S_q in X, Y, and Z are given in Figures 1 to 6.

These results are typical of those for a middle latitude observatory and correspond well with results given by Vestine *et al.* (1947) for Toolangi during the Polar Year 1932–1933.

For ΔX and ΔY the difference between winter and summer months, regardless of sunspot activity, is an increase of 10% in the daily range and minima occurring 2 hr earlier, with little or no other change in form. This can be seen both from the graphs, Figures 1 to 6, and from data given in Table 2. The difference between data from low sunspot activity and high sunspot activity years is a 40% increase in daily range and minima occurring approximately 10 min earlier, see Table 4.

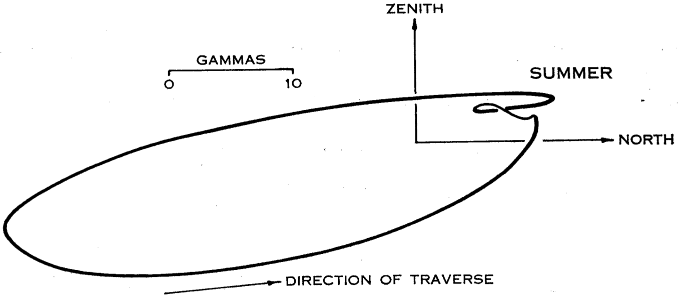

Fig. 7.—$\Delta X - \Delta Z$ vector diagram during summer months of years of high sunspot activity, sunlit portion in heavy line.

For S_q in Z there is considerable change of form from summer to winter and this is shown strikingly in Figures 7 and 8, where the direction of traverse of the Zenith-North vector diagram curve in summer is seen to be in the direction opposite to that taken in winter. Figures 7 and 8 are from data for years of high sunspot activity.

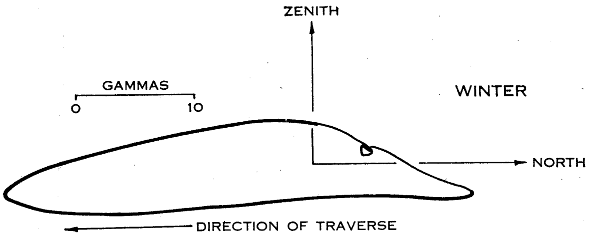

Fig. 8.—$\Delta X - \Delta Z$ vector diagram during winter months of years of high sunspot activity, sunlit portion in heavy line.

An attempt to represent the directions of the overhead current system producing the S_q variation by graphing ΔX as an abscissa and ΔY as a negative ordinate gave good agreement with regard to the location of the focus of the current system with that given by Kazmi (1960).

IV. References

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