PARAMETERS OF TWO LEVELS OF THE ^{17}F NUCLEUS

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Summary

The resonance structures in the $^{16}O(p,p)^{16}O$ excitation curve at the bombarding energies of 10·52 and 12·40 MeV as found by a University of Wisconsin group have been analysed. Spin and parity for the levels of ^{17}F at 10·50 and 12·26 MeV are believed to be $\frac{7}{2}^-$ and $\frac{3}{2}^-$ respectively.

I. Introduction

Information on states in ^{13}N was extracted from $^{12}C(p, p)^{12}C$ excitation curves and angular distributions by a Melbourne group (Shute *et al.* 1962; Robson and Toutenhoofd 1963). The method used by Robson is applicable for the elastic scattering of spin-$\frac{1}{2}$ probe particles by spin-0 target nuclei, provided the absorption cross section is relatively small and the excitation curve shows a fairly smooth "background" in between the resonance structures.† A concise description of the method is given in Section II of the present paper, and results of its application to the extraction of ^{17}F-level parameters from $^{16}O(p, p)^{16}O$ excitation curves and angular distributions are then given in Section III.

II. Method

The differential elastic scattering cross section $\sigma(\theta)$ in the centre-of-mass system for an unpolarized beam of particles of spin $\frac{1}{2}$, scattered by spinless target nuclei is given by

$$\sigma(\theta) = |A|^2+|B|^2, \tag{1}$$

where

$$\left.\begin{aligned} A(\theta) &= f_c(\theta)+\frac{1}{k}\sum_{l=0}^{\infty}\sum_{j=|l\pm\frac{1}{2}|}(j+\tfrac{1}{2})\exp(2i\sigma_l)\frac{\exp(2i\delta_{lj})-1}{2i}\,\mathrm{P}_l(\cos\theta), \\ B(\theta) &= -\frac{1}{k}\sum_{l=0}^{\infty}\sum_{j=|l\pm\frac{1}{2}|}\pm\exp(2i\sigma_l)\frac{\exp(2i\delta_{lj})-1}{2i}\,\mathrm{P}_l^1(\cos\theta), \end{aligned}\right\} \tag{2}$$

in which $f_c(\theta)$ is the Rutherford scattering amplitude (Messiah 1961, p. 422), k the wave number, σ_l the Coulomb phase shift (Messiah 1961, p. 426), P_l a Legendre polynomial, $\mathrm{P}_l{}^1$ an associated Legendre function, and δ_{lj} the "nuclear phase shift" associated with l and j, which is in principle determined by the Hamiltonian. The derivation of (1) and (2) is given by Lepore (1950) (and is presented by Mather and Swan 1958, p. 271, who corrected a few misprints).

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† The condition that the reaction cross section be small is a *sufficient* requirement for this method to be useful. It is pointed out by Robson (1963) that only the submatrix of **U** involving the resonance need be diagonal (see also Robson and Toutenhoofd 1963).

It is possible to express $A(\theta)$ and $B(\theta)$ in terms of the diagonal elements $U_{lj,lj}$ of the collision matrix $\mathbf{U}$, as defined by Lane and Thomas (1958, p. 285). Indeed it can be shown that

$$U_{lj,lj} = \exp\{2\mathrm{i}(\sigma_l - \sigma_0 + \delta_{lj})\}. \tag{3}$$

If the excitation curve shows a fairly smooth background in between the resonance structures and if the background can be satisfactorily analysed in terms of the optical model, we can obtain the background phase shifts δ^0_{lj} from the optical model wave functions.

A derivation of the formula, valid in the vicinity of an isolated level, expressing the dependence of $U_{lj,lj}$ (and thus of δ_{lj}, see (3)) on the energy E of the probe particle in the centre-of-mass system, is given in terms of the $\mathbf{R}$ matrix theory by Lane and Thomas (1958, pp. 323 and 324). Making use of the one-level approximation and the assumption that the background part $\mathbf{R}^0$ of the $\mathbf{R}$ matrix is diagonal, they obtain

$$U_{lj,lj} = U^0_{lj,lj}\{1+\mathrm{i}\Gamma_{lj}/(E_\lambda+\Delta-E-\tfrac{1}{2}\mathrm{i}\Gamma)\}, \tag{4}$$

where

$$U^0_{lj,lj} = \exp\{2\mathrm{i}(\sigma_l-\sigma_0-\phi'_{lj})\}. \tag{5}$$

We will treat the partial width Γ_{lj}, the total width Γ, and the "resonance energy" $E_\lambda+\Delta = E_{\mathrm{R}}$ as adjustable parameters, independent of the energy in the vicinity of a resonance. The background phase shifts $-\phi'_{lj}$ are real quantities in the approximation that $\mathbf{R}^0$ is diagonal. It is assumed that under the slightly more general assumption, that the off-diagonal terms of $\mathbf{R}^0$ are much smaller than the diagonal terms, (4) and (5) still hold in first approximation and that ϕ'_{lj} will have a small imaginary part. Complex background phase shifts δ^0_{lj} can be obtained with an optical model analysis of the background and it seems to be reasonable, provided the imaginary part of δ^0_{lj} is small, to substitute δ^0_{lj} for $-\phi'_{lj}$.

A diagonal element of a matrix, which is unitary and diagonal, is necessarily of modulus one. Thus if we allow δ^0_{lj} to have a small imaginary part, we must also allow $\mathbf{U}^0$ to have small off-diagonal terms. This is reasonable since both the presence of off-diagonal terms in $\mathbf{U}^0$ and the fact that δ^0_{lj} is complex are consequences of the occurrence of non-elastic scattering at "background energies".

Reduced width amplitudes γ^2_{lj} (real quantities) can be estimated with the aid of

$$\Gamma_{lj} = 2\mathrm{P}_l\,\gamma^2_{lj}\cos^2(\phi_l-\phi'_{lj}), \tag{6}$$

in which P_l and $-\phi_l$ are the penetrability and the hard sphere scattering phase shift respectively. In the $\mathbf{R}$ matrix theory, use is made of the "boundary value matrix", the elements of which are arbitrary real numbers. A convenient choice (Robson and Toutenhoofd 1963) leads to (6).

A systematic search for the values of l and j to be associated with a certain resonance was made by substitution in (2) of

$$\exp(2\mathrm{i}\delta^0_{0,\frac{1}{2}})\{1+\mathrm{i}\Gamma_{\mathrm{p}}/(E_{\mathrm{R}}-E-\tfrac{1}{2}\mathrm{i}\Gamma)\}$$

for $\exp(2\mathrm{i}\delta_{0,\frac{1}{2}})$, then

$$\exp(2\mathrm{i}\delta^0_{1,\frac{1}{2}})\{1+\mathrm{i}\Gamma_{\mathrm{p}}/(E_{\mathrm{R}}-E-\tfrac{1}{2}\mathrm{i}\Gamma)\}$$

for $\exp(2\mathrm{i}\delta_{1,\frac{1}{2}})$, and so on. All the values of l and j were tried for which δ^0_{lj} was not negligible. The corresponding theoretical differential cross sections obtained with (1) were compared with the experimental results. In all the cases it was found that only one particular choice l_R and j_R of l and j gave differential cross sections with resonance shapes, similar to those obtained by experiment. The parameters E_R, Γ_p, Γ, and δ^0_{lj} were then varied in order to improve the fits.

The parities of the corresponding levels of the compound nucleus were calculated from

$$\pi_R = (-)^{l_R} \pi_t \, \pi_p, \tag{7}$$

where π_t and π_p are parities of the target nucleus and the probe particle respectively (in the case of ^{16}O(p, p), $\pi_R = (-)^{l_R}$).

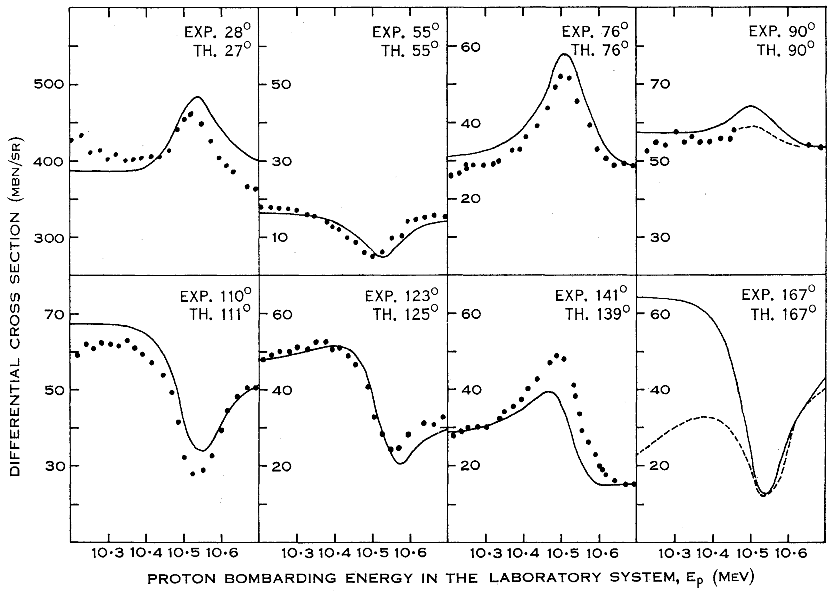

Fig. 1.—The resonance structure at 10·52 MeV in the elastic scattering excitation curves of ^{16}O(p,p)^{16}O. The full circles and the broken lines represent the experimental values of Hardie, Dangle, and Oppliger (1963). The solid lines have been obtained from calculations in which the parameters of Table 1 were used. The scattering angle in the centre-of-mass system is indicated for each curve (EXP. for the experimental points and TH. for the theoretical curve).

III. Application to ^{16}O(p, p)^{16}O

In the work described in this paper, the resonances seen at 10·52 and 12·39 MeV (proton energy in the laboratory system) in the proton–oxygen differential scattering cross sections, measured by Hardie, Dangle, and Oppliger (1963), were chosen for investigation (Figs. 1 and 2). The optical model phase shifts obtained by these authors were substituted for the background phase shifts δ^0_{lj}. The resonance energies and widths as published by Dangle, Oppliger, and Hardie (1964, Table 3) were used; partial widths were estimated. In the investigation of the level at 10·52 MeV the following combinations of values (l,j) were tried: $(0,\frac{1}{2})$, $(1,\frac{1}{2})$, $(1,\frac{3}{2})$, $(2,\frac{3}{2})$, $(2,\frac{5}{2})$, $(3,\frac{5}{2})$, and $(3,\frac{7}{2})$ of which only the last one reproduced remarkably well the resonance

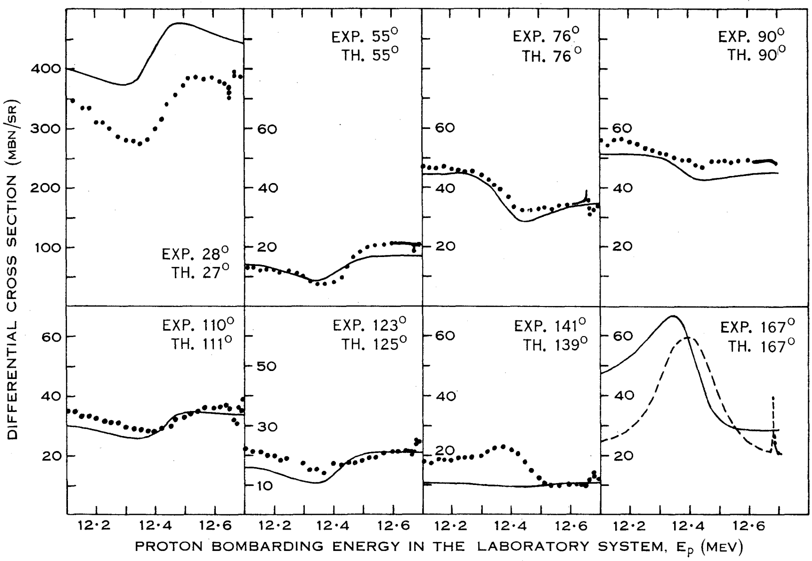

Fig. 2.—The resonance structure at 12·39 MeV; otherwise as in Figure 1.

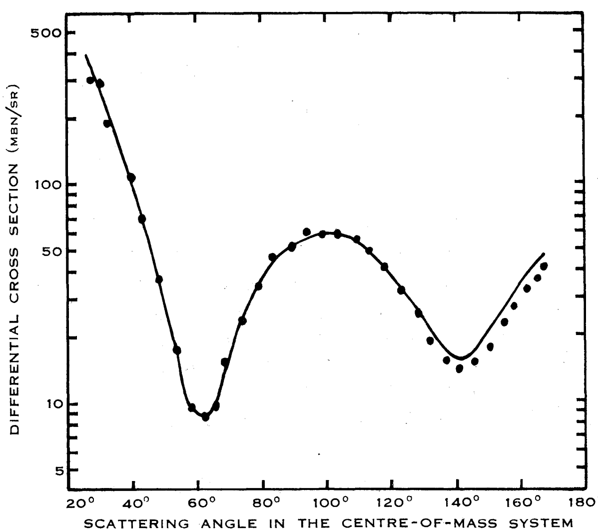

Fig. 3.—The angular distribution at 10·744 MeV. The full circles represent experimental values, the solid line shows the theoretical fit obtained. (See also caption of Figure 5.)

shape at all the eight angles for which differential cross sections were available. The background phases δ^0_{lj} were varied by small amounts away from the optical model values (Fig. 5(a)) in order to improve the fits to the angular distribution at 10·74 MeV (Fig. 3).

The background phases at the resonance energy were then changed by the same amounts. The fits to the differential cross sections presented in Figure 1 were obtained

TABLE 1

PARAMETERS USED IN LEVEL ANALYSES

Proton Energy E_p (MeV)	^{17}F-Level Energy E_x (MeV)	Spin and Parity J^π	Orbital Angular Momentum l	Total Width Γ (MeV)	Ratio of Partial to Total Width Γ_p/Γ	$\frac{\gamma^2_{lj}}{3\hbar^2/2Ma^2} = \theta^2_{lj}$
10·52	10·50	$\frac{7}{2}-$	3	0·15	0·3	0·010
12·40	12·26	$\frac{3}{2}-$	1	0·20	0·3$_5$	0·23

using the parameters presented in Table 1, which also gives the fractions θ^2_{lj} of the Wigner sum rule limit $3\hbar^2/2Ma^2$ (Teichmann and Wigner 1952) constituted by γ^2_{lj}. Note that use was made of Lane and Thomas's (1958) definition of the reduced width γ^2, being Wigner's γ^2 divided by the channel radius a. Note also that the

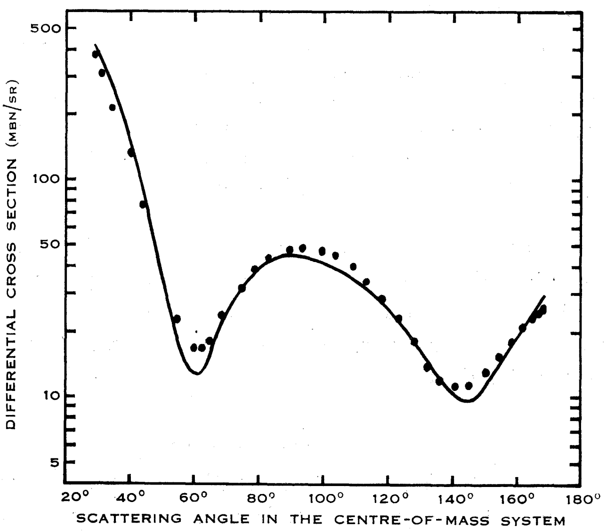

Fig. 4.—The angular distribution at 12·597 MeV. The full circles represent experimental results, the solid line shows the theoretical fit obtained. (See also caption of Figure 5.)

values of l, given in Table 1 are not, of course, quantum numbers of the compound nucleus levels of ^{17}F.

The same procedure was applied to the level at 12·39 MeV (Figs. 2, 4, and 5(b)) except that, in addition to the combinations of values (l,j) tried for the level at $10 \cdot 52$ MeV, $(4, \frac{7}{2})$ and $(4, \frac{9}{2})$ were tried. (It was assumed that $\delta_{4,7/2} = \delta_{4,9/2} = 0$.) The best fit was obtained for $l = 1$ and $j = \frac{3}{2}$. The parameters used are presented in Table 1.

IV. Comparison with Optical Model Calculations

Duke (1963) obtained good fits to the $^{16}O(p, p)$ cross sections at energies between $8 \cdot 66$ and $19 \cdot 2$ MeV in an optical model analysis. No resonance analysis was attempted by Duke but he obtained nevertheless a reasonable fit to the resonance structure at $10 \cdot 5$ MeV by allowing the parameters describing the optical potential

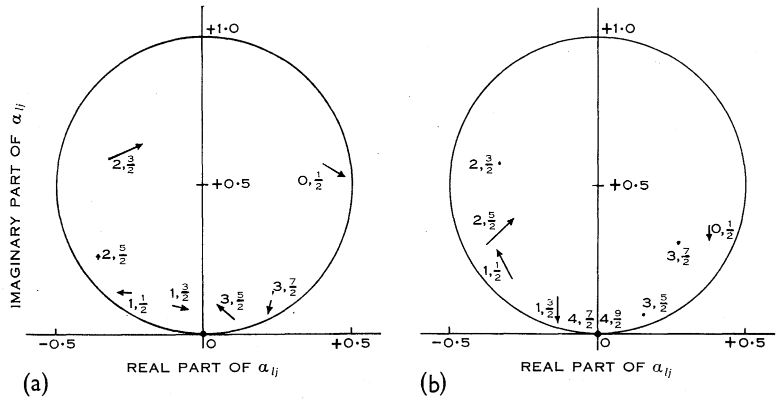

Fig. 5.—The optical model phase shifts at (*a*) $10 \cdot 74$ MeV, (*b*) $12 \cdot 6$ MeV, for the different partial waves as obtained by Hardie, Dangle, and Oppliger (1963) are represented in the complex plane by tails of arrows in the points $\alpha_{lj} = \{\exp(2i\delta_{lj}) - 1\}/2i$. An improved fit to the angular distribution was obtained by small alterations of the phase shifts, the new values being represented by the tips of the arrows. (For the absorption cross section to remain positive, the values of α_{lj} have to be kept inside the circle.)

to vary rapidly with energy. In such a description one would expect a single partial wave to be "suddenly" absorbed at the resonance energy; for the resonance at $E_p = 10 \cdot 52$ MeV this would be the $f_{7/2}$ partial wave. Duke (1963, p. 689) reported such a strong absorption at this energy. This seems to support our spin and parity assignment of the $10 \cdot 52$ MeV level.

V. Acknowledgments

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