EXCITATION OF HYDROGEN AND CaII UNDER CHROMOSPHERIC CONDITIONS

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Summary

Methods are described for calculating excitation conditions in a hot gas in the absence of local thermodynamic equilibrium, taking self-absorption into account, in the approximation that excitation is uniform throughout the medium. The equilibrium excited-state populations and the emission, attenuation, and scattering properties derived from them are tabulated for a variety of physical conditions appropriate to the solar chromosphere ($7500 \leqslant T \leqslant 50\,000°$K; $10^{10} \leqslant N_e \leqslant 10^{12}$ cm^{-3}) for some of the main spectral lines of hydrogen and CaII.

I. Introduction

Because of their importances in relation to the detailed structure of the chromosphere, excitation conditions in hydrogen, helium, and ionized calcium for situations other than of local thermodynamic equilibrium have been the subject of a number of discussions. General details and references are given by Thomas and Athay (1961). The most recent account for hydrogen appears to be that of Jefferies and Giovanelli (1954), for calcium II that of Athay and Zirker (1962), and for helium that of Athay (1963). To render the problems tractable, model atoms have been used, with only three bound states and the continuum in the case of hydrogen and CaII and a larger though still small number of states for helium. Even so, calculations for these models are uncertain because of uncertainties in the transition rates and, particularly, in the extent and significance of self-absorption in the resonance and subordinate lines (Thomas 1960). No satisfactory account has been available for dealing with self-absorption in a multilevel atom.

An approximate method for handling this problem is described below and applied to hydrogen and CaII. (The helium lines λ 10830 and D_3, according to Athay, would seem to be rather insensitive to self-absorption in the resonance lines and are not in such urgent need of reinvestigation.) The ultimate aim is the prediction of the intensities of the relevant spectral lines, which will be the subject of a later paper (see Section III et seq.). This involves calculating the properties of the medium required for insertion into the equation of radiative transfer, e.g. the emission, attenuation, and scattering coefficients; tables of these properties are given in the present paper for a range of temperatures and densities. A necessary prerequisite is a calculation of the distributions of atoms among the various excited states from which they may be brought by collision or otherwise into an emitting level; these are also tabulated here.

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II. Approximate Value of the Net Radiative Bracket

In this section we develop an approximate method for dealing with a uniform slab of gas surrounded by a radiation field.

In the statistically steady state the excitation of a gas is such that the sum of the rates of departure $\Sigma_k P_{jk} N_j$ from a given atomic level j of population density N_j by all processes equals that of the rates of arrival. Then

$$N_j \sum_k P_{jk} = \sum_k N_k P_{kj}. \tag{2.1}$$

Transitions between states will usually be due either to radiative processes at a rate $R_{jk} N_j$ or to electron collisions at a rate $C_{jk} N_{\mathrm{e}} N_j$, so that

$$P_{jk} = R_{jk} + C_{jk} N_{\mathrm{e}}. \tag{2.2}$$

For downward transitions the R_{jk} are in many cases vastly greater than the $C_{jk} N_{\mathrm{e}}$. Yet, if the medium is optically opaque to the emitted radiation, the rate of radiative absorption to a level may very nearly balance the rate of emission, so that radiative transitions of this type may have a much reduced or even negligible influence on the population of the level. To describe this, Thomas (1960) wrote an equation equivalent to

$$N_j R_{jk} - N_k R_{kj} = \alpha_{jk} N_j R_{jk}, \tag{2.3}$$

where α_{jk} might be a very small quantity, particularly for the resonance lines; α_{jk} is Thomas's net radiative bracket. If the values of the various α_{jk} were known, the calculation of the population N_j under any given conditions would be straightforward, coming directly from the solution of the set of simultaneous equations (2.1). However, the various α_{jk} have been obtained only with great difficulty after the solution of various simultaneous differential or integral equations of transfer for the various spectral lines, a problem of such complexity that most published calculations (e.g. those of Athay and Zirker and of Athay) have been for assumed values of α_{jk}, usually 0 or 1 but in a few cases for intermediate values.

Since the intensity of radiation in a spectral line varies throughout a medium, the α_{jk} and hence the degree of excitation vary throughout the medium also. However, as mentioned at the end of Section I, our main concern is the radiation emitted by the medium, to calculate which the various emission, attenuation, and scattering coefficients are needed. In the level of approximation adopted here these coefficients are taken as constant throughout the medium, the radiation intensities then being found from the equation of radiative transfer. If so desired, improved values of the various coefficients could then be obtained, these varying with position, but the solution to the equation of transfer in such a higher stage of approximation becomes unwieldy.

It will be noted that the assumed uniformity of the various coefficients implies that only the average values of the α_{jk} throughout the medium are needed. In such a case a minor change in definition, which allows radiation incident on the medium to be considered separately, enables approximate values of α_{jk} to be calculated very simply from the geometry. Consider an isolated self-luminous medium exposed also to external radiation, and separate the rate of radiative absorption, R_{kj} per k-state atom, into R'_{kj} due to radiation coming directly from atoms within the medium and $R_{kj}-R'_{kj}$ due to the direct penetration of external radiation into the medium (i.e. not involving scattering). The redefinition of α_{jk} is

$$N_j R_{jk} - N_k R'_{kj} = \alpha_{jk} N_j R_{jk}. \tag{2.4}$$

Since any inequality between $N_j R_{jk}$ and $N_k R'_{kj}$ is due only to the escape of radiation from the medium, the average value of α_{jk} for the medium as a whole is the fraction of photons liberated in downward transitions that escapes directly from the medium without undergoing any intervening absorption or scattering process.

To evaluate the rate of upward transition due directly to the penetration of incident radiation, consider a constant-temperature enclosure surrounding the medium. In the external radiation field then existing the rate of radiative absorption per k-state atom is the same as within the medium, namely, R_{kj}. Since the rate of emission within the medium then equals the rate of absorption, but only the fraction $1-\alpha_{jk}$ of the radiation emitted within the medium is absorbed there, the difference $\alpha_{jk} R_{kj}$ must be due to the penetration of externally incident radiation into the medium. It follows that in the absence of a constant-temperature enclosure the average rate of absorption within the medium is also $\alpha_{jk} \mathscr{R}_{kj}$ per k-state atom, where $\mathscr{R}_{kj}$ is now the corresponding rate in the external field, subject to the proviso that the external field is uniformly diffuse and of uniform distribution across the spectral line.

Thus, in the general case,

$$N_j R_{jk} - N_k R_{kj} = \alpha_{jk}(N_j R_{jk} - N_k \mathscr{R}_{kj}). \tag{2.5}$$

The value of α_{jk} depends on the geometry. For illustration we consider a uniform plane-parallel layer of thickness t. If ϵ is the energy emitted in unit time from unit volume into unit solid angle and in a given wavelength or frequency interval, and if κ is the attenuation coefficient (the sum of the scattering and absorption coefficients) so that $\tau = \kappa t$ is the optical thickness of the layer, then the flux escaping across unit area of one surface is readily found to be

$$F = \frac{\pi\epsilon}{\kappa}\{1-(1-\tau)\,\mathrm{e}^{-\tau} + \tau^2\,\mathrm{Ei}(-\tau)\},$$

where $-\mathrm{Ei}(-x) = \int_x^\infty \frac{\mathrm{e}^{-u}}{u}\,\mathrm{d}u$ is the exponential integral. But the total emission in a column of unit section and thickness t is $4\pi\epsilon t$, so that the fraction escaping across both surfaces, i.e. the average value of α_{jk} for the medium, is

$$\alpha_{jk} = \frac{1}{2\tau}\{1-(1-\tau)\,\mathrm{e}^{-\tau} + \tau^2\,\mathrm{Ei}(-\tau)\}. \tag{2.6}$$

For large τ this becomes $1/2\tau$.

Expression (2.6) applies to monochromatic radiation and varies with τ across a spectral line. If, as usual, scattering is noncoherent, it is the weighted average value of α_{jk} that is required for a spectral line. Thus, it is necessary to do an appropriate integration, for which, however, no simple exact expression appears to be

TABLE 1

HYDROGEN: TRANSITION-RATE COEFFICIENTS

	T (°K)						
	$7 \cdot 5\times 10^3$	$1 \cdot 0\times 10^4$	$1 \cdot 25\times 10^4$	$1 \cdot 5\times 10^4$	$2 \cdot 0\times 10^4$	$3 \cdot 0\times 10^4$	$5 \cdot 0\times 10^4$
C_{12}	8·36 (−14)*	4·52 (−12)	4·81 (−11)	2·39 (−10)	1·82 (−9)	1·37 (−8)	7·10 (−8)
C_{13}	7·48 (−16)	8·62 (−14)	1·46 (−12)	9·60 (−12)	1·04 (−10)	1·12 (−9)	7·80 (−9)
C_{1i}	3·33 (−18)	7·42 (−16)	1·96 (−14)	1·78 (−13)	2·8 (−12)	4·77 (−11)	5·00 (−10)
C_{23}	1·60 (−8)	3·49 (−8)	5·60 (−8)	7·50 (−8)	1·102 (−7)	1·54 (−7)	1·98 (−7)
C_{21}	1·52 (−7)	1·57 (−7)	1·61 (−7)	1·65 (−7)	1·71 (−7)	1·78 (−7)	1·91 (−7)
C_{2i}	1·172 (−9)	4·40 (−9)	1·080 (−8)	1·890 (−8)	3·77 (−8)	7·49 (−8)	1·255 (−7)
C_{31}	1·14 (−8)	1·18 (−8)	1·21 (−8)	1·24 (−8)	1·28 (−8)	1·34 (−8)	1·43 (−8)
C_{32}	1·32 (−7)	1·39 (−7)	1·425 (−7)	1·437 (−7)	1·46 (−7)	1·42 (−7)	1·36 (−7)
C_{3i}	8·72 (−8)	1·618 (−7)	2·32 (−7)	2·95 (−7)	3·92 (−7)	5·16 (−7)	5·91 (−7)
C_{i1}	2·94 (−30)	2·20 (−30)	1·76 (−30)	1·47 (−30)	1·10 (−30)	7·33 (−31)	4·40 (−31)
C_{i2}	5·76 (−28)	3·83 (−28)	3·02 (−28)	2·36 (−28)	1·59 (−28)	8·90 (−29)	4·10 (−29)
C_{i3}	5·19 (−27)	3·48 (−27)	2·51 (−27)	1·92 (−27)	1·24 (−27)	6·63 (−28)	2·80 (−28)
R_{i1}	2·5 (−13)	1·95 (−13)	1·71 (−13)	1·542 (−13)	1·30 (−13)	1·020 (−13)	7·30 (−14)
R_{i2}	1·013 (−13)	8·43 (−14)	7·29 (−14)	6·43 (−14)	5·23 (−14)	3·84 (−14)	2·53 (−14)
R_{i3}	5·92 (−14)	4·80 (−14)	4·05 (−14)	3·28 (−14)	2·77 (−14)	1·95 (−14)	1·21 (−14)

R_{12}	1·56 (+2)	R_{13}	0·189	R_{1i}	8·4 (−3)
R_{21}	4·68 (+8)	R_{23}	2·25 (+5)	R_{2i}	1·35 (+4)
R_{31}	5·54 (+7)	R_{32}	4·39 (+7)	R_{3i}	1·76 (+5)

* The number in parentheses is the common logarithm of the multiplier: 8·36 (−14) means $8 \cdot 36\times 10^{-14}$.

available. But, for optically thin media in a given line, equation (2.6) gives an adequate value if τ is taken as the opacity at the line centre. It also applies adequately to continua if τ is taken at the band head.

We may note that the value of α_{jk} is the same for a given κ, no matter whether the medium scatters or not. But in the latter case the spectral flux density escaping per unit area per unit time is rather uniform for opacities exceeding unity, having a value πB (or $\pi\epsilon/\kappa$), B being the Planckian function, and falling off rapidly for $\tau \ll 1$. Thus, in the case of a spectral line of high opacity at the line centre, the flux escaping per unit time across the two ends of a column of unit section is approximately $2\pi B.2(\Delta\lambda)$, where $\Delta\lambda$ corresponds to $\tau = 1$. In the common case where line-broadening is Doppler, $\tau = \tau_0 \exp\{-(\Delta\lambda/\Delta D)^2\}$, where τ_0 is the opacity at the line centre and ΔD the Doppler width. Thus, for $\tau = 1$,

$$\Delta\lambda = (\ln \tau_0)^{\frac{1}{2}}\,\Delta D.$$

Since the corresponding internal emission is

$$4\pi \int_0^\infty B\tau \,\mathrm{d}\lambda = 4\pi B\tau_0 \int_0^\infty \exp\{-(\Delta\lambda/\Delta D)^2\}\,\mathrm{d}(\Delta\lambda)$$

$$= 4\pi^{3/2} B\tau_0\,\Delta D\,,$$

the fraction escaping is

$$\alpha_{jk} = \frac{(\ln \tau_0)^{\frac{1}{2}}}{\pi^{\frac{1}{2}}\tau_0} \qquad (\tau_0 \gg 1). \tag{2.7}$$

This is larger than the value given by (2.6) by a factor $2(\ln \tau_0)^{\frac{1}{2}}\pi^{-\frac{1}{2}}$ for large τ; this factor varies only slowly with τ_0 and, for instance, has a value of about $3{\cdot}4$ for $\tau_0 = 10^4$.

Relations (2.6) and (2.7) give equal values of α_{jk} for $\tau_0 = 2$. In many cases it is a good working rule to use (2.6) for $\tau_0 < 2$ and (2.7) for $\tau_0 > 2$ when calculating the net radiative bracket for spectral lines. Relation (2.6) is quite suitable for continua where τ is taken at the band head, though a relation similar to (2.7) can be derived for high continuous opacities if so desired.

III. Excitation Conditions and Radiation Parameters

With the redefined values of α_{jk} evaluated above, we may now rewrite the equation of statistical equilibrium (2.1) as

$$N_j \sum_k Q_{jk} = \sum_k N_k Q_{kj}\,, \tag{3.1}$$

where

$$\left.\begin{aligned} Q_{jk} &= C_{jk} N_{\mathrm{e}} + \alpha_{jk} R_{jk} \\ Q_{kj} &= C_{kj} N_{\mathrm{e}} + \alpha_{jk} \mathscr{R}_{kj} \end{aligned}\right\} \qquad j > k, \quad R_{jk} \equiv A_{jk}, \tag{3.2}$$

$\mathscr{R}_{kj}$ being the rate of excitation per k-state atom to the j state in the field of incident radiation outside our medium.

TABLE 2

HYDROGEN: EXCITATION PROPERTIES

Log N_e	N_1	N_2	N_3	Lα τ_{12}	Lα α_{12}	Lβ τ_{13}	Lβ α_{13}	L_{cont} τ_{1i}	Hα τ_{23}	Hα α_{23}
	$T = 7 \cdot 5 \times 10^3$ °K		$V^2 = 0$*							
10	3·14(+9)†	1·10(+3)	5·80	4·19(+4)	4·40(−5)	6·71(+3)	2·50(−4)	5·08	1·22(−1)	8·12(−1)
11	1·97(+11)	1·08(+5)	7·51(+2)	2·63(+6)	8·24(−7)	4·22(+5)	4·81(−6)	3·20(+2)	1·20(+1)	7·40(−2)
12	9·97(+12)	5·51(+6)	3·20(+5)	1·33(+8)	1·83(−8)	2·13(+7)	1·09(−7)	1·61(+4)	6·11(+2)	2·34(−3)
			$V^2 = 1 \cdot 2 \times 10^{12}$							
10	3·17(+9)	1·10(+3)	5·78	3·02(+4)	6·00(−5)	4·84(+3)	3·40(−4)	5·13	8·73(−2)	8·52(−1)
11	1·99(+11)	1·09(+5)	7·11(+2)	1·90(+6)	1·13(−6)	3·24(+5)	6·60(−6)	3·22(+2)	8·62	9·60(−2)
12	1·03(+13)	5·71(+6)	3·09(+5)	9·84(+7)	2·46(−8)	1·58(+7)	1·46(−7)	1·67(+4)	4·52(+2)	3·08(−3)
	$T = 1 \cdot 0 \times 10^4$ °K		$V^2 = 0$							
10	1·10(+9)	1·09(+3)	5·74	1·27(+4)	1·36(−4)	2·04(+3)	7·63(−4)	1·79	1·05(−1)	8·31(−1)
11	5·96(+9)	9·33(+4)	6·54(+2)	6·88(+4)	2·74(−5)	1·10(+4)	1·56(−4)	9·65	8·97	9·32(−2)
12	1·04(+11)	3·00(+6)	2·42(+5)	1·20(+6)	1·76(−6)	1·92(+5)	1·03(−5)	1·68(+2)	2·89(+2)	4·65(−3)
			$V^2 = 1 \cdot 2 \times 10^{12}$							
10	1·19(+9)	1·07(+3)	5·62	1·05(+4)	1·63(−4)	1·69(+3)	9·12(−4)	1·94	7·84(−2)	8·63(−1)
11	6·45(+9)	9·32(+4)	6·22(+2)	5·68(+4)	3·29(−5)	9·10(+3)	1·87(−4)	1·04(+1)	6·83	1·15(−1)
12	1·08(+11)	3·14(+6)	2·35(+5)	9·54(+5)	2·19(−6)	1·53(+5)	1·27(−5)	1·76(+2)	2·30(+2)	5·72(−3)

	$T = 1\cdot25\times10^4$ °K			$V^2 = 0$						
10	4·12(+8)	1·23(+3)	6·55	4·25(+3)	3·83(−4)	6·82(+2)	2·11(−3)	6·67(−1)	1·06(−1)	8·31(−1)
11	1·42(+9)	9·27(+4)	6·89(+2)	1·46(+4)	1·19(−4)	2·35(+3)	6·70(−4)	2·30	7·97	1·02(−1)
12	7·05(+9)	1·96(+6)	1·84(+5)	7·29(+4)	2·59(−5)	1·17(+4)	1·48(−4)	1·14(+1)	1·68(+2)	7·59(−3)
				$V^2 = 1\cdot2\times10^{12}$						
10	4·49(+8)	1·20(+3)	6·40	3·69(+3)	4·38(−4)	5·92(+2)	2·41(−3)	7·27(−1)	8·22(−2)	8·58(−1)
11	1·53(+9)	9·14(+4)	6·51(+2)	1·26(+4)	1·37(−4)	2·02(+3)	7·69(−4)	2·49	6·26	1·22(−1)
12	7·42(+9)	2·04(+6)	1·78(+5)	6·11(+4)	3·07(−5)	9·79(+3)	1·75(−4)	1·20(+1)	1·39(+2)	8·99(−3)
	$T = 1\cdot5\times10^4$ °K			$V^2 = 0$						
10	1·93(+8)	1·25(+3)	6·97	1·82(+3)	8·47(−4)	2·92(+2)	4·60(−3)	3·13(−1)	9·85(−2)	8·39(−1)
11	5·97(+8)	9·22(+4)	7·55(+2)	5·63(+3)	2·94(−4)	9·03(+2)	1·63(−3)	9·67(−1)	7·24	1·10(−1)
12	1·58(+9)	1·62(+6)	1·59(+5)	1·49(+4)	1·17(−4)	2·39(+3)	6·57(−4)	2·56	1·27(+2)	9·77(−3)
				$V^2 = 1\cdot2\times10^{12}$						
10	2·10(+8)	1·23(+3)	6·87	1·62(+3)	9·44(−4)	2·60(+2)	5·11(−3)	3·40(−1)	7·94(−2)	8·62(−1)
11	6·47(+8)	9·08(+4)	7·16(+2)	5·01(+3)	3·28(−4)	8·04(+2)	1·82(−3)	1·05	5·86	1·28(−1)
12	1·67(+9)	1·66(+6)	1·54(+5)	1·30(+4)	1·34(−4)	2·08(+3)	7·50(−4)	2·71	1·07(+2)	1·14(−2)
	$T = 2\cdot0\times10^4$ °K			$V^2 = 0$						
10	6·71(+7)	1·09(+3)	7·33	5·48(+2)	2·58(−3)	8·79(+1)	1·36(−2)	1·09(−1)	7·39(−2)	8·69(−1)
11	1·98(+8)	8·11(+4)	8·90(+2)	1·62(+3)	9·48(−4)	2·59(+2)	6·01(−3)	3·14(−1)	5·52	1·34(−1)
12	3·44(+8)	1·31(+6)	1·44(+5)	2·81(+3)	5·65(−4)	4·51(+2)	3·09(−3)	5·58(−1)	8·92(+1)	1·34(−2)
				$V^2 = 1\cdot2\times10^{12}$						
10	7·15(+7)	1·07(+3)	7·31	5·01(+2)	2·81(−3)	8·03(+1)	1·47(−2)	1·16(−1)	6·23(−2)	8·84(−1)
11	2·12(+8)	8·03(+4)	8·55(+2)	1·48(+3)	1·03(−3)	2·38(+2)	5·55(−3)	3·43(−1)	4·68	1·50(−1)
12	3·65(+8)	1·33(+6)	1·39(+5)	2·56(+3)	6·18(−4)	4·10(+2)	3·38(−3)	5·92(−1)	7·76(+1)	1·52(−2)

* Units of V^2 are cm²/sec².

† The number in parentheses is the common logarithm of the multiplier: 3·14(+9) means $3\cdot14\times10^9$.

TABLE 2 (*Continued*)

Log N_e	N_1	N_2	N_3	Lα		Lβ		L_{cont}	Hα	
				τ_{12}	α_{12}	τ_{13}	α_{13}	τ_{1i}	τ_{23}	α_{23}
	$T = 3 \cdot 0 \times 10^4$ °K		$V^2 = 0$							
10	1·61(+7)	4·51(+2)	6·26	1·08(+2)	1·13(−2)	1·73(+1)	5·52(−2)	2·61(−2)	2·50(−2)	9·42(−1)
11	5·56(+7)	4·66(+4)	8·82(+2)	3·71(+2)	3·70(−3)	5·95(+1)	1·92(−2)	9·01(−2)	2·59	2·13(−1)
12	8·45(+7)	8·14(+5)	9·22(+4)	5·64(+2)	2·52(−3)	4·52(+1)	1·33(−2)	1·37(−1)	4·52(+1)	2·44(−2)
			$V^2 = 1 \cdot 2 \times 10^{12}$							
10	1·66(+7)	4·35(+2)	6·24	9·92(+1)	1·22(−2)	1·59(+1)	5·90(−2)	2·68(−2)	2·17(−2)	9·48(−1)
11	5·81(+7)	4·60(+4)	8·68(+2)	3·48(+2)	3·92(−3)	5·58(+1)	2·03(−2)	9·42(−2)	2·29	2·24(−1)
12	8·88(+7)	8·21(+5)	9·00(+4)	5·39(+2)	2·62(−3)	8·64(+1)	1·38(−2)	1·42(−1)	4·19(+1)	2·60(−2)
	$T = 5 \cdot 0 \times 10^4$ °K		$V^2 = 0$							
10	1·97(+6)	3·91(+1)	2·77	1·02(+1)	8·43(−2)	1·63	2·77(−1)	3·20(−3)	1·68(−3)	9·94(−1)
11	1·30(+7)	1·23(+4)	4·20(+2)	6·70(+1)	1·73(−2)	1·07(+1)	8·10(−2)	2·10(−2)	5·30(−1)	5·43(−1)
12	2·53(+7)	3·91(+5)	4·88(+4)	1·31(+2)	9·54(−3)	2·09(+1)	4·70(−2)	4·09(−2)	1·68(+1)	5·64(−2)
			$V^2 = 1 \cdot 2 \times 10^{12}$							
10	1·98(+6)	3·73(+1)	2·74	9·56	8·87(−2)	1·53	2·91(−1)	3·21(−3)	1·50(−3)	9·94(−1)
11	1·32(+7)	1·20(+4)	4·10(+2)	6·36(+1)	1·81(−2)	1·02(+1)	8·43(−2)	2·13(−2)	4·82(−1)	5·65(−1)
12	2·60(+7)	3·91(+5)	4·81(+4)	1·26(+2)	9·86(−3)	2·02(+1)	4·85(−2)	4·22(−2)	1·57(+1)	5·95(−2)

If there are n states then there are n equations (3.1), one of which is redundant. The population ratios may be found from (3.1) alone. For a four-level atom* (three bound levels and the continuum) whose states are j, k, l, m it is found that

$$\frac{N_j}{N_k} = \frac{Q_{kj}(1-q_{lm}q_{ml}) + Q_{kl}(q_{lj}+q_{lm}q_{mj}) + Q_{km}(q_{mj}+q_{ml}q_{lj})}{Q_{jk}(1-q_{lm}q_{ml}) + Q_{jl}(q_{lk}+q_{lm}q_{mk}) + Q_{jm}(q_{mk}+q_{ml}q_{lk})}, \tag{3.3}$$

where $q_{kj} = Q_{kj}/\Sigma_j Q_{kj}$. The N_j are settled by an auxiliary condition, such as a given total density $N = \Sigma_j N_j$.

It remains to determine the various opacities τ_{jk} from which the α_{jk} are derived. This is by an iterative procedure. The τ_{jk} are obtained from the appropriate line-broadening mechanism, the thickness of the medium, and the values of the N_j. We commence by allocating arbitrary values to the α_{jk}, from which the approximate values of N_j are obtained by use of (3.3); these lead to approximate values of τ_{jk} and improved α_{jk}. The iterative procedure converges rapidly in many cases; for slow convergence the procedure is interrupted after five iterations and a new starting point selected on the basis of the trend. With a little experience rapid convergence is readily obtainable.

It might be thought that, with average populations established, the emission from the medium could be most easily obtained from a suitable integration of the source function, itself derived directly from N_j/N_k. However, the source function so obtained is independent of depth, since the N_j are average values, and this procedure is unrealistic. Instead, the emission is to be found from a solution to the equation of radiative transfer, for which the necessary parameters are required. These are the attenuation coefficient κ_{jk}, the albedo for single scattering $\varpi_{jk}(=\sigma_{jk}/\kappa_{jk}$, where σ_{jk} is the scattering coefficient), and the ratio of the emission and absorption coefficients $(1-\varpi_{jk})(\epsilon/\kappa)_{jk}$; κ_{jk} follows immediately from the N_{jk}.

Using an extension of the technique of Giovanelli and Jefferies (1954) we can obtain ϖ_{jk} by a simple artifice. As before, take $j > k$. In place of (3.1) we start from equation (2.1) and introduce the modified net radiative bracket relations (2.4) only for pairs of transitions other than j, k. Then (3.1) is replaced by

$$N_j(P_{jk}+Q_{jl}+Q_{jm}) = N_k P_{kj} + N_l Q_{lj} + N_m Q_{mj},$$

$$N_k(P_{kj}+Q_{kl}+Q_{km}) = N_j P_{jk} + N_l Q_{lk} + N_m Q_{mk},$$

$$N_l \sum_k Q_{lk} = \sum_k N_k Q_{kl},$$

$$N_m \sum_k Q_{mk} = \sum_k N_k Q_{km},$$

and (3.3) is replaced by

$$\frac{N_j}{N_k} = \frac{P_{kj}(1-q_{lm}q_{ml}) + Q_{kl}(q_{lj}+q_{lm}q_{mj}) + Q_{km}(q_{mj}+q_{ml}q_{lj})}{P_{jk}(1-q_{lm}q_{ml}) + Q_{jl}(q_{lk}+q_{lm}q_{mk}) + Q_{jm}(q_{mk}+q_{ml}q_{lk})}. \tag{3.4}$$

* A general solution of the equations of statistical equilibrium is given by White (1961).

TABLE 3

HYDROGEN: RADIATION PARAMETERS

Log N_e	Lα $1-\varpi$	Lα $\frac{\epsilon}{\kappa(1-\varpi)}$	Lβ $1-\varpi$	Lβ $\frac{\epsilon}{\kappa(1-\varpi)}$	Hα $1-\varpi$	Hα $\frac{\epsilon}{\kappa(1-\varpi)}$
	$T = 7 \cdot 5 \times 10^3$ °K		$V^2 = 0$*			
10	7·49(−6)†	2·53(−8)	6·83(−2)	7·55(−11)	3·04(−3)	8·78(−4)
11	3·26(−5)	3·07(−8)	2·39(−2)	1·56(−10)	2·97(−3)	3·24(−5)
12	3·26(−4)	3·06(−8)	6·35(−3)	1·32(−9)	6·83(−3)	4·76(−5)
			$V^2 = 1 \cdot 2 \times 10^{12}$			
10	7·51(−6)	2·55(−8)	8·57(−2)	7·45(−11)	3·16(−3)	8·86(−4)
11	3·26(−5)	3·07(−8)	2·74(−2)	1·46(−10)	2·98(−3)	3·36(−5)
12	3·26(−4)	3·06(−8)	6·85(−3)	1·23(−9)	6·83(−3)	4·75(−5)
	$T = 1 \cdot 0 \times 10^4$ °K		$V^2 = 0$			
10	1·24(−5)	4·57(−7)	1·50(−1)	2·14(−10)	3·92(−3)	6·97(−4)
11	3·60(−5)	1·51(−6)	3·72(−2)	4·51(−9)	3·42(−3)	3·64(−5)
12	3·38(−4)	1·61(−6)	8·02(−3)	9·60(−8)	8·13(−3)	7·75(−5)
			$V^2 = 1 \cdot 2 \times 10^{12}$			
10	1·20(−5)	4·71(−7)	1·66(−1)	1·93(−10)	4·08(−3)	6·96(−4)
11	3·59(−5)	1·52(−6)	4·35(−2)	3·97(−9)	3·45(−3)	3·64(−5)
12	3·38(−4)	1·61(−6)	8·67(−3)	8·90(−8)	8·13(−3)	7·75(−5)
	$T = 1 \cdot 25 \times 10^4$ °K		$V^2 = 0$			
10	1·79(−5)	3·30(−6)	2·46(−1)	6·56(−10)	5·75(−3)	4·81(−4)
11	4·31(−5)	1·36(−5)	5·56(−2)	2·02(−8)	4·42(−3)	3·79(−5)
12	3·55(−4)	1·65(−5)	9·91(−3)	1·09(−6)	9·61(−3)	1·02(−4)
			$V^2 = 1 \cdot 2 \times 10^{12}$			
10	1·77(−5)	3·33(−6)	2·61(−1)	5·88(−10)	6·10(−3)	4·70(−4)
11	4·27(−5)	1·37(−5)	6·46(−2)	1·76(−8)	4·51(−3)	3·80(−5)
12	3·55(−4)	1·65(−5)	1·07(−2)	1·00(−6)	9·63(−3)	1·02(−4)
	$T = 1 \cdot 5 \times 10^4$ °K		$V^2 = 0$			
10	2·22(−5)	1·32(−5)	3·12(−1)	1·49(−9)	8·82(−3)	3·31(−4)
11	5·05(−5)	5·82(−5)	6·89(−2)	5·30(−8)	5·93(−3)	4·03(−5)
12	3·98(−4)	7·34(−5)	1·26(−2)	4·35(−6)	1·19(−2)	1·09(−4)
			$V^2 = 1 \cdot 2 \times 10^{12}$			
10	2·24(−5)	1·32(−5)	3·23(−1)	1·36(−9)	9·42(−3)	3·20(−4)
11	5·02(−5)	5·86(−5)	7·87(−2)	4·63(−8)	6·14(−3)	4·05(−5)
12	3·96(−4)	7·39(−5)	1·36(−2)	3·97(−6)	1·20(−2)	1·09(−4)

* Units of V^2 are cm²/sec².

† The number in parentheses is the common logarithm of the multiplier: 7·49(−6) means $7 \cdot 49 \times 10^{-6}$.

But N_j is the sum of two terms, $\bar{N}_j$ proportional to the rate of absorption of j, k radiation (one of the components of P_{kj}), and n_j not involving j, k radiation. The former

TABLE 3 (*Continued*)

Log N_e	Lα $1-\varpi$	Lα $\frac{\epsilon}{\kappa(1-\varpi)}$	Lβ $1-\varpi$	Lβ $\frac{\epsilon}{\kappa(1-\varpi)}$	Hα $1-\varpi$	Hα $\frac{\epsilon}{\kappa(1-\varpi)}$
	$T = 2\cdot0\times10^4$ °K		$V^2 = 0$			
10	2·99(−5)	7·62(−5)	3·73(−1)	4·58(−9)	1·91(−2)	1·92(−4)
11	6·13(−5)	3·70(−4)	9·09(−2)	1·94(−7)	1·03(−2)	5·38(−5)
12	5·00(−4)	4·47(−4)	1·78(−2)	2·00(−5)	1·71(−2)	1·19(−4)
			$V^2 = 1\cdot2\times10^{12}$			
10	3·05(−5)	7·47(−5)	3·79(−1)	4·29(−9)	2·04(−2)	1·86(−4)
11	6·13(−5)	3·71(−4)	1·00(−1)	1·74(−7)	1·08(−2)	5·41(−5)
12	4·97(−4)	4·50(−4)	1·90(−2)	1·82(−5)	1·74(−2)	1·18(−4)
	$T = 3\cdot0\times10^4$ °K		$V^2 = 0$			
10	5·56(−5)	3·14(−4)	4·19(−1)	1·71(−8)	6·37(−2)	1·29(−4)
11	8·08(−5)	2·16(−3)	1·42(−1)	7·24(−7)	2·61(−2)	8·02(−5)
12	6·56(−4)	2·59(−3)	2·92(−2)	6·44(−5)	3·08(−2)	1·23(−4)
			$V^2 = 1\cdot2\times10^{12}$			
10	5·73(−5)	3·05(−4)	4·21(−1)	1·67(−8)	6·70(−2)	1·27(−4)
11	8·13(−5)	2·14(−3)	1·49(−1)	6·83(−7)	2·73(−2)	8·05(−5)
12	6·53(−4)	2·60(−3)	3·08(−2)	5·98(−5)	3·16(−2)	1·22(−4)
	$T = 5\cdot0\times10^4$ °K		$V^2 = 0$			
10	1·52(−4)	6·00(−4)	4·40(−1)	7·79(−8)	2·42(−1)	1·83(−4)
11	1·27(−4)	7·21(−3)	2·99(−1)	1·58(−6)	8·85(−2)	1·40(−4)
12	8·52(−4)	1·04(−2)	5·42(−2)	1·44(−4)	6·61(−2)	1·38(−4)
			$V^2 = 1\cdot2\times10^{12}$			
10	1·56(−4)	5·81(−4)	4·40(−1)	7·77(−8)	2·51(−1)	1·83(−4)
11	1·29(−4)	7·12(−3)	3·07(−1)	1·52(−6)	9·16(−2)	1·41(−4)
12	8·50(−4)	1·04(−2)	5·64(−2)	1·37(−4)	6·76(−2)	1·37(−4)

results in scattered radiation. But from the definition of ϖ_{jk} it follows that

$$R_{jk}\bar{N}_j = \varpi_{jk}R_{kj}N_k.$$

Since $R_{jk} \equiv A_{jk}$, disregarding stimulated emission,

$$\begin{aligned}\varpi_{jk} &= A_{jk}\bar{N}_j/R_{kj}N_k \\ &= A_{jk}(1-q_{lm}q_{ml})/D \qquad (3.5)\end{aligned}$$

from (3.4), where D is the denominator of (3.4).

As to $(\epsilon/\kappa)_{jk}$, we note first that ϵ is the rate of emission from unit volume into unit solid angle per unit frequency interval. In a black body $(\epsilon/\kappa)_{jk}$ is the Planckian function B_ν which, disregarding stimulated emission, is given by

$$B_\nu = \frac{2h\nu^3}{c^2}\cdot\frac{w_k}{w_j}\cdot\frac{n_j}{N_k},$$

TABLE 4

CALCIUM: TRANSITION-RATE COEFFICIENTS

	T (°K)						
	$7 \cdot 5 \times 10^3$	$1 \cdot 0 \times 10^4$	$1 \cdot 25 \times 10^4$	$1 \cdot 5 \times 10^4$	$2 \cdot 0 \times 10^4$	$3 \cdot 0 \times 10^4$	$5 \cdot 0 \times 10^4$
C_{12}	1·42 (−8)*	2·56 (−8)	3·60 (−8)	4·61 (−8)	6·15 (−8)	8·39 (−8)	1·116 (−7)
C_{13}	1·172 (−8)	3·54 (−8)	6·79 (−8)	1·049 (−7)	1·773 (−7)	2·94 (−7)	4·40 (−7)
C_{1i}	9·0 (−17)	1·06 (−14)	1·85 (−13)	1·29 (−12)	1·46 (−11)	1·79 (−10)	1·41 (−9)
C_{23}	9·10 (−8)	1·335 (−7)	1·675 (−7)	1·89 (−7)	2·170 (−7)	2·35 (−7)	2·31 (−7)
C_{21}	3·95 (−8)	3·70 (−8)	3·50 (−8)	3·35 (−8)	3·28 (−8)	3·24 (−8)	3·32 (−8)
C_{2i}	1·035 (−15)	7·06 (−14)	8·85 (−13)	4·84 (−12)	4·44 (−11)	3·92 (−10)	2·51 (−9)
C_{31}	5·05 (−7)	4·54 (−7)	4·20 (−7)	3·97 (−7)	3·64 (−7)	3·30 (−7)	3·04 (−7)
C_{32}	1·41 (−6)	1·203 (−6)	1·068 (−6)	9·87 (−7)	8·36 (−7)	6·84 (−7)	5·39 (−7)
C_{3i}	2·06 (−14)	7·62 (−13)	6·29 (−12)	2·67 (−11)	1·54 (−10)	1·08 (−9)	5·32 (−9)
C_{i1}	5·47 (−30)	4·20 (−30)	3·35 (−30)	2·81 (−30)	2·10 (−30)	1·41 (−30)	8·26 (−31)
C_{i2}	3·57 (−29)	2·69 (−29)	2·15 (−29)	1·79 (−29)	1·402 (−29)	8·99 (−30)	5·29 (−30)
C_{i3}	2·92 (−29)	2·32 (−29)	1·85 (−29)	1·55 (−29)	1·079 (−29)	7·60 (−30)	4·48 (−30)
R_{i1}	4·15 (−15)	3·65 (−15)	3·33 (−15)	3·05 (−15)	2·75 (−15)	2·27 (−15)	1·61 (−15)
R_{i2}	1·27 (−12)	1·1 (−12)	9·85 (−13)	9·0 (−13)	7·8 (−13)	6·35 (−13)	4·92 (−13)
R_{i3}	1·3 (−13)	1·1 (−13)	1·0 (−13)	8·0 (−14)	7·0 (−14)	6·0 (−14)	5·0 (−14)
		R_{12}	0	R_{13}	2·07 (+4)	R_{1i}	1·0 (−4)
		R_{21}	0	R_{23}	4·5 (+4)	R_{2i}	3·6 (−1)
		R_{31}	1·47 (+8)	R_{32}	1·24 (+7)	R_{3i}	7·0 (−2)

* The number in parentheses is the common logarithm of the multiplier: 1·42 (−8) means $1 \cdot 42 \times 10^{-8}$.

w_j and w_k being appropriate statistical weights. From (3.4),

$$\frac{\epsilon}{\kappa_{jk}} = \frac{2h\nu^3}{c^2} \cdot \frac{w_k}{w_j} \cdot \frac{C_{kj}\, N_{\mathrm{e}}(1-q_{lm}\, q_{ml}) + Q_{kl}(q_{lj}+q_{lm}\, q_{mj}) + Q_{km}(q_{mj}+q_{ml}\, q_{lj})}{D}.$$

The ratio of emission and absorption coefficients $(\epsilon/\kappa)_{jk}/(1-\varpi_{jk})$ follows from (3.5) and (3.6).

IV. The Special Cases of Hydrogen and CaII

Above some level which is at least as close as 1500 km to its base, the chromosphere appears to consist of a variety of structures that can only be analysed using the relatively few spectral lines in which they can be observed. As a first approximation to their geometry we shall subsequently use (with some care) the model structure described in Section III, a uniform plane-parallel layer whose thickness is taken here as 2×10^8 cm. In practice, chromospheric structures are inclined at various angles to the surface, and it seems scarcely warranted as yet to specify too closely the angle which our model structure makes with the solar surface. Nevertheless, I tend to think in terms of a perpendicular structure; a specific model is needed because of the centre-limb variation of the intensities of the lines from the chromosphere as a whole and, therefore, of the angular variation in intensity of radiation incident on the structure.

Atomic models of four levels (three bound and the ionized continuum) have been used both for hydrogen and CaII. For hydrogen such a model is known to give fairly satisfactory results, substates of a given principal quantum number being populated in proportion to their atomic weights because of the high collision rates for transitions between them, even in the case $2\,{}^2S_{1/2} \rightleftarrows 2\,{}^2P_{1/2}$ for $N_{\mathrm{e}} \approx N_{\mathrm{i}} \gtrsim 10^{11}$ cm^{-3} (Purcell 1952). For CaII the bound levels used are the $4s$, $3d$, and $4p$. The levels $4p\ {}^2P_{3/2}$ and $4p\ {}^2P_{1/2}$, from which respectively the K and H lines (and also the λ 8498 + λ 8542 and λ 8662 lines) originate, are relatively widely spaced, and the rate of direct collisional transition between them is much lower than the spontaneous rate from them. Despite this, the two $4p$ levels are lumped together for the purposes of obtaining excited-state populations (not for deriving source functions, but for scattering and emission characteristics), the populations of the substates being assumed proportional to their statistical weights. The ranges over which calculations have been made are $2\times10^9 \leqslant N_{\mathrm{e}} \leqslant 4\times10^{12}$ cm^{-3} and $7\cdot5\times10^3 \leqslant T \leqslant 5\times10^4$ °K. The electron concentration is assumed to be equal to that of the hydrogen ions. In the tables and in the discussion below all units are in the CGS system unless otherwise stated.

(a) *Hydrogen*

The spontaneous transition rates used are theoretical. Two-body recombination rates to these bound states are predominantly spontaneous and are derivable via Gaunt's relation for the atomic absorption coefficient, with g factors that are here taken as unity. Collision rates to and from the ground states are derived from Hummer's (1963) relations and for transitions between the $n = 2$ and $n = 3$ states from Seaton's relation (Allen 1963). Classical rates are used for collision ionization

TABLE 5

CALCIUM: EXCITATION PROPERTIES

Log N_e	N_1	N_2	N_3	K τ_K	K α_{13}	8542 Å τ_{23}	8542 Å α_{23}
	$T = 7\cdot5 \times 10^3$ °K		$V^2 = 0$*				
10	1·21(+4)†	5·30(+2)	1·83	5·30	1·62(−1)	6·85(−2)	8·76(−1)
11	3·60(+5)	6·60(+4)	4·76(+2)	1·57(+2)	1·03(−2)	8·53	9·68(−2)
12	1·53(+7)	5·36(+6)	3·36(+5)	6·70(+3)	3·25(−4)	6·92(+2)	2·08(−3)
			$V^2 = 1\cdot2 \times 10^{12}$				
10	1·24(+4)	5·19(+2)	1·79	8·63(−1)	4·80(−1)	1·07(−2)	9·71(−1)
11	4·16(+5)	5·03(+4)	1·90(+2)	2·90(+1)	4·49(−2)	1·03	3·82(−1)
12	1·58(+7)	5·02(+6)	2·78(+5)	1·10(+3)	1·76(−3)	1·03(+2)	1·18(−2)
	$T = 1\cdot0 \times 10^4$ °K		$V^2 = 0$				
10	8·91(+3)	4·31(+2)	1·42	3·38	2·12(−1)	4·82(−2)	9·04(−1)
11	1·11(+5)	2·47(+4)	1·37(+2)	4·22(+1)	3·27(−2)	2·77	2·06(−1)
12	1·26(+6)	6·27(+5)	5·33(+4)	4·78(+2)	3·78(−3)	7·02(+1)	1·66(−2)
			$V^2 = 1\cdot2 \times 10^{12}$				
10	9·03(+3)	4·18(+2)	1·38	6·44(−1)	5·48(−1)	8·55(−3)	9·76(−1)
11	1·35(+5)	1·90(+4)	6·52(+1)	9·32	1·10(−1)	3·89(−1)	6·13(−1)
12	1·44(+6)	5·29(+5)	2·51(+4)	9·98(+1)	1·55(−2)	1·08(+1)	8·04(−2)
	$T = 1\cdot25 \times 10^4$ °K		$V^2 = 0$				
10	6·95(+3)	3·36(+2)	1·16	2·36	2·64(−1)	3·63(−2)	9·23(−1)
11	8·03(+4)	2·11(+4)	1·20(+2)	2·72(+1)	4·73(−2)	2·11	2·31(−1)
12	7·18(+5)	4·13(+5)	4·17(+4)	2·44(+2)	6·98(−3)	4·13(+1)	2·63(−2)
			$V^2 = 1\cdot2 \times 10^{12}$				
10	7·31(+3)	3·51(+2)	1·11	5·04(−1)	6·03(−1)	7·14(−3)	9·79(−1)
11	9·79(+4)	1·72(+4)	6·08(+1)	6·76	1·38(−1)	3·50(−1)	6·35(−1)
12	8·91(+5)	3·86(+5)	2·04(+4)	6·15(+1)	2·36(−2)	7·88	1·03(−1)
	$T = 1\cdot5 \times 10^4$ °K		$V^2 = 0$				
10	4·40(+3)	2·39(+2)	7·39(−1)	1·36	3·72(−1)	2·18(−2)	9·48(−1)
11	4·08(+4)	1·10(+4)	5·06(+1)	1·26(+1)	8·75(−2)	1·00	3·90(−1)
12	2·83(+5)	1·57(+5)	1·27(+4)	8·74(+1)	1·74(−2)	1·44(+1)	6·40(−2)
			$V^2 = 1\cdot2 \times 10^{12}$				
10	4·55(+3)	2·31(+2)	7·07(−1)	3·13(−1)	6·98(−1)	4·68(−3)	9·85(−1)
11	4·92(+4)	9·03(+3)	2·97(+1)	3·38	2·12(−1)	1·83(−1)	7·54(−1)
12	3·57(+5)	1·59(+5)	6·14(+3)	2·45(+1)	5·16(−2)	3·24	1·89(−1)

* Units of V^2 are cm²/sec².

† The number in parentheses is the common logarithm of the multiplier: 1·21(+4) means $1\cdot21 \times 10^4$.

from, and three-body recombination to, the excited states. Rates of radiative excitation in the normal chromosphere are obtained for Hα using a mean central intensity of 0·18 and Allen's value for the mean intensity of the whole disk in the nearby

TABLE 5 (*Continued*)

Log N_e	N_1	N_2	N_3	K τ_K	K α_{13}	8542 Å τ_{23}	8542 Å α_{23}
	$T = 2\cdot0\times10^4$ °K		$V^2 = 0$				
10	8·59(+2)	4·76(+1)	1·39(−1)	2·30(−1)	7·50(−1)	3·77(−3)	9·88(−1)
11	6·66(+3)	1·38(+3)	4·32	1·79	3·11(−1)	1·09(−1)	8·27(−1)
12	4·24(+4)	2·08(+4)	6·66(+2)	1·14(+1)	9·50(−2)	1·64	2·76(−1)
			$V^2 = 1\cdot2\times10^{12}$				
10	8·63(+2)	4·70(+1)	1·37(−1)	5·88(−2)	9·04(−1)	9·45(−4)	9·96(−1)
11	7·30(+3)	1·18(+3)	2·94	4·98(−1)	6·05(−1)	2·37(−2)	9·44(−1)
12	4·70(+4)	2·03(+4)	3·30(+2)	3·21	2·20(−1)	4·09(−1)	6·02(−1)
	$T = 3\cdot0\times10^4$ °K		$V^2 = 0$				
10	6·78(+1)	4·15	1·13(−2)	1·48(−2)	9·67(−1)	2·68(−4)	9·99(−1)
11	5·42(+2)	1·06(+2)	2·40(−1)	1·19(−1)	8·40(−1)	6·83(−3)	9·80(−1)
12	3·81(+3)	1·75(+3)	1·97(+1)	8·33(−1)	4·89(−1)	1·13(−1)	8·22(−1)
			$V^2 = 1\cdot2\times10^{12}$				
10	6·78(+1)	4·14	1·13(−2)	4·55(−3)	9·88(−1)	8·20(−5)	1·00
11	5·47(+2)	1·03(+2)	2·25(−1)	3·67(−2)	9·33(−1)	2·05(−3)	9·93(−1)
12	4·04(+3)	1·66(+3)	1·44(+1)	2·71(−1)	7·24(−1)	3·30(−2)	9·28(−1)
	$T = 5\cdot0\times10^4$ °K		$V^2 = 0$				
10	6·84	4·80(−1)	1·22(−3)	1·16(−3)	9·96(−1)	2·40(−5)	1·00
11	5·33(+1)	1·33(+1)	2·77(−2)	9·04(−3)	9·78(−1)	6·64(−4)	9·97(−1)
12	3·91(+2)	2·10(+2)	1·64	6·62(−2)	8·95(−1)	1·05(−2)	9·71(−1)
			$V^2 = 1\cdot2\times10^{12}$				
10	6·84	4·79(−1)	1·22(−3)	4·45(−4)	9·98(−1)	9·22(−6)	1·00
11	5·34(+1)	1·32(+1)	2·75(−2)	3·48(−3)	9·90(−1)	2·54(−4)	9·99(−1)
12	3·94(+2)	2·09(+2)	1·56	2·56(−2)	9·49(−1)	4·01(−3)	9·87(−1)

continuum. Rates of photoelectric ionization in the Balmer and Paschen continua have been derived analogously. Estimates have been made of the radiative excitation rates in the Lyman lines and continuum using a typical flux of 5·1 erg cm^{-2} sec^{-1} over a 1 Å range in Lα, an Lα/Lβ flux ratio of 85, and an approximate radiation temperature of 6650°K in the Lyman continuum (Tousey 1963). The rates used are listed in Table 1, the $n = 1, 2, 3$ and ionized states being denoted by subscripts 1, 2, 3, and i respectively.

White's (1962) limb-darkening observations have been used in calculating the externally incident flux in Hα; the ratio of the Hα flux incident on unit area of a vertical surface to that passing through unit area of a horizontal surface turns out to be 0·441.

In all cases line-broadening is taken to be Doppler, arising from a combination of thermal and microturbulent velocities. The range of microturbulent velocities V used for both hydrogen and calcium, $0 \leqslant V^2 \leqslant 1\cdot2\times10^{12}$ cm^2/sec^2, probably covers all chromospheric situations. In this range the Doppler widths of the hydrogen

Table 6

CALCIUM: RADIATION PARAMETERS

Log N_e	H,K $1-\varpi$	H,K $\frac{\epsilon}{\kappa(1-\varpi)}$	8542 Å $1-\varpi$	8542 Å $\frac{\epsilon}{\kappa(1-\varpi)}$
	$T = 7 \cdot 5 \times 10^3$ °K		$V^2 = 0$*	
10	7·51 (−4)†	4·97 (−6)	7·12 (−2)	1·62 (−5)
11	2·41 (−3)	1·36 (−5)	7·01 (−2)	1·63 (−4)
12	6·35 (−3)	4·90 (−5)	1·21 (−1)	8·88 (−4)
			$V^2 = 1 \cdot 2 \times 10^{12}$	
10	7·54 (−4)	4·96 (−6)	7·45 (−2)	1·55 (−5)
11	4·66 (−3)	7·46 (−6)	1·86 (−1)	5·40 (−5)
12	6·58 (−3)	4·73 (−5)	1·28 (−1)	8·32 (−4)
	$T = 1 \cdot 0 \times 10^4$ °K		$V^2 = 0$	
10	6·97 (−4)	1·26 (−5)	1·15 (−1)	8·20 (−6)
11	2·86 (−3)	2·90 (−5)	1·36 (−1)	6·69 (−5)
12	5·13 (−3)	1·56 (−4)	1·16 (−1)	8·02 (−4)
			$V^2 = 1 \cdot 2 \times 10^{12}$	
10	6·99 (−4)	1·26 (−5)	1·23 (−1)	7·60 (−6)
11	4·64 (−3)	1·83 (−5)	2·90 (−1)	2·58 (−5)
12	6·23 (−3)	1·28 (−4)	1·59 (−1)	5·57 (−4)
	$T = 1 \cdot 25 \times 10^4$ °K		$V^2 = 0$	
10	6·54 (−4)	2·29 (−5)	1·48 (−1)	5·41 (−6)
11	2·59 (−3)	5·57 (−5)	1·58 (−1)	5·02 (−5)
12	4·47 (−3)	3·16 (−4)	1·12 (−1)	7·41 (−4)
			$V^2 = 1 \cdot 2 \times 10^{12}$	
10	6·56 (−4)	2·29 (−5)	1·60 (−1)	4·95 (−6)
11	4·16 (−3)	3·51 (−5)	3·12 (−1)	2·07 (−5)
12	5·52 (−3)	2·55 (−4)	1·63 (−1)	4·82 (−4)
	$T = 1 \cdot 5 \times 10^4$ °K		$V^2 = 0$	
10	6·24 (−4)	3·50 (−5)	1·81 (−1)	3·95 (−6)
11	3·08 (−3)	6·90 (−5)	2·26 (−1)	2·98 (−5)
12	4·48 (−3)	4·65 (−4)	1·32 (−1)	5·70 (−4)
			$V^2 = 1 \cdot 2 \times 10^{12}$	
10	6·25 (−4)	3·49 (−5)	1·93 (−1)	3·65 (−6)
11	4·08 (−3)	5·24 (−5)	3·76 (−1)	1·44 (−5)
12	5·94 (−3)	3·50 (−4)	2·15 (−1)	3·16 (−4)

* Units of V^2 are cm²/sec².

† The number in parentheses is the common logarithm of the multiplier: 7·51 (−4) means $7 \cdot 51 \times 10^{-4}$.

lines are mainly thermal so that microturbulence has no great effect, particularly at the higher temperatures.

TABLE 6 (*Continued*)

Log N_e	H,K $1-\varpi$	H,K $\frac{\epsilon}{\kappa(1-\varpi)}$	8542 Å $1-\varpi$	8542 Å $\frac{\epsilon}{\kappa(1-\varpi)}$
	$T = 2\cdot0\times10^4$ °K		$V^2 = 0$	
10	6·07(−4)	5·69(−5)	2·34(−1)	2·44(−6)
11	3·94(−3)	8·63(−5)	4·30(−1)	9·81(−6)
12	6·06(−3)	5·49(−4)	2·66(−1)	2·04(−4)
			$V^2 = 1\cdot2\times10^{12}$	
10	6·07(−4)	5·68(−5)	2·38(−1)	2·38(−6)
11	4·13(−3)	8·24(−5)	5·50(−1)	6·08(−6)
12	9·08(−3)	3·66(−4)	4·23(−1)	1·01(−4)
	$T = 3\cdot0\times10^4$ °K		$V^2 = 0$	
10	5·96(−4)	9·18(−5)	2·88(−1)	1·52(−6)
11	4·01(−3)	1·35(−4)	6·04(−1)	4·01(−6)
12	1·00(−2)	5·28(−4)	5·68(−1)	4·63(−5)
			$V^2 = 1\cdot2\times10^{12}$	
10	5·96(−4)	9·18(−5)	2·89(−1)	1·51(−6)
11	4·03(−3)	1·34(−4)	6·20(−1)	3·74(−6)
12	1·08(−2)	4·89(−4)	6·55(−1)	3·22(−5)
	$T = 5\cdot0\times10^4$ °K		$V^2 = 0$	
10	6·08(−4)	1·31(−4)	3·38(−1)	9·61(−7)
11	4·12(−3)	1·92(−4)	6·35(−1)	2·80(−6)
12	1·12(−2)	6·94(−4)	6·82(−1)	2·27(−5)
			$V^2 = 1\cdot2\times10^{12}$	
10	6·08(−4)	1·31(−4)	3·38(−1)	9·61(−7)
11	4·13(−3)	1·92(−4)	6·37(−1)	2·78(−6)
12	1·13(−2)	6·86(−4)	6·94(−1)	2·14(−5)

Table 2 lists a selection of populations, opacities at line centres, and values of α_{jk} for a layer of thickness 2×10^8 cm and microturbulences $V^2 = 0$ and $1\cdot2\times10^{12}$ cm²/sec². Table 3 lists corresponding scattering parameters in the form $1-\varpi_{jk}$ and ratios of emission and absorption coefficients $(\epsilon/\kappa)_{jk}/(1-\varpi_{jk})$ for Lα, Lβ, and Hα.

(*b*) *Calcium II*

Over the temperature range considered ($7\cdot5\times10^3$ to 5×10^4 °K), calcium is predominantly singly or doubly ionized, and the sum of the concentrations of CaII and CaIII is taken equal to the total calcium concentration, assumed to be 2×10^{-6} of that of hydrogen.

The mean spontaneous transition rates between bound states are listed by Athay and Zirker (1962), and I have used their values for the photoelectric ionization rates from the 4*s* and 4*p* states in the normal chromospheric radiation field. For radiative excitation from the $4s$ and $3d$ states to the $4p$ state, rates have been derived from data on absolute intensities at the centre of the disk in H and K and their centre-limb variations given by Goldberg, Mohler, and Müller (1959) and from data on the intensity in λ 8542 shown in the Utrecht Atlas (Minnaert, Mulders, and Houtgast 1940). Two-body recombination rates to the $4s$ and $4p$ states have been derived from Burgess and Seaton (1960). For two-body recombination to the $3d$ state, Allen's (1963) rate has been adopted together with an assumed temperature variation as $T^{-0\cdot 5}$; the corresponding rate of photoelectric ionization from $3d$ in the normal chromospheric radiation field has been modified from Athay and Zirker's value to render it compatible with the recombination rate. Collision rates between bound states have been derived from Seaton's general relations for permitted and for forbidden transitions (Allen 1963), normalized to van Regemorter's (1960, 1961) cross sections. Seaton's general relations for collisional ionization (Allen 1963) have been used for deriving these rates and those of three-body recombination from and to the excited states. The rates used are listed in Table 4, the $4s$, $3d$, $4p$, and ionized states being denoted by subscripts 1, 2, 3, and i respectively.

Because the H and K lines do not overlap, separate account has been taken of them in calculating the modified net radiative brackets associated with the $4s$–$4p$ transitions. We note that $\tau_{\rm K} = 2\tau_{\rm H}$. The individual $\alpha_{\rm H}$ and $\alpha_{\rm K}$ are found from (2.6) or (2.7) depending on whether $\tau_{\rm H}$ or $\tau_{\rm K}$ are less than or greater than 2, and they are then combined by the approximate relation

$$\alpha_{13} = \tfrac{1}{3}\alpha_{\rm H} + \tfrac{2}{3}\alpha_{\rm K},$$

which is certainly valid if the source functions are nearly the same for H and K (since one-third of the emission is then in the H line and two-thirds in the K). Line-broadening is taken to be Doppler (thermal and microturbulent).

Because the Doppler widths are more strongly influenced by microturbulence in calcium than in hydrogen, the calcium populations are rather more sensitive than the hydrogen to microturbulent velocities, to illustrate which a selection of populations, opacities, and α_{jk} values for a layer of thickness 2×10^8 cm and microturbulences $V^2 = 0$ and $1\cdot 2\times 10^{12}$ cm^2/sec^2 is given in Table 5. In general, microturbulence increases the ground-state population (i.e. reduces the degree of double ionization) by a factor of up to $1\cdot 25$ and reduces N_{4p} by a factor of down to $0\cdot 5$. In all cases $N_{4s} > N_{3d} > N_{4p}$, with $N_{4s}/N_{3d} \approx 2$ for the higher $N_{\rm e}$. Calcium is 50% doubly ionized at about 10^4 °K. The various opacities are approximately inversely proportional to ΔD; for $\tau_0 > 2$ the α are approximately proportional to ΔD. Scattering parameters in the form $1-\varpi_{jk}$ and ratios of emission and absorption coefficients $(\epsilon/\kappa)_{jk}/(1-\varpi_{jk})$ are given in Table 6 for the H and K lines (for which the values are identical in the present approximation) and for 8542 Å.

The four-level model for CaII (three bound levels and the continuum) is likely to be deficient mostly in respect of the recombination rate from CaIII. Between the $4p$ level (excitation potential $3\cdot 14$ V) and the continuum (ionization potential $11\cdot 87$ V) there is a large array of levels in the real CaII atom, the transition probabilities down to the $4p$ level from $5S_{1/2}$ and $4D_{5/2}$ being quite high ($1\cdot 3\times 10^8$ and $3\cdot 6\times 10^8$ sec^{-1} respectively). Since recombination can occur to all these excited levels, with a subsequent downward cascade, the total recombination rate may be increased by a significant factor which I assess possibly as high as 3.

The effect of this is to raise the CaII population in those cases where calcium is predominantly doubly ionized, e.g. at temperatures $T \gtrsim 1 \cdot 5 \times 10^4$ °K for all electron densities considered ($N_e \lesssim 4 \times 10^{12}$ cm^{-3}), and for $N_e \lesssim 10^{10}$ cm^{-3} for lower temperatures. For the higher temperatures the CaII population increase is in proportion to the total recombination rate. No great increase occurs in excitation (i.e. N_j/N_1) due to the cascade itself, since $C_{12}/R_{i1} \approx 10^4$–$10^5$, whereas the ratio $N_{\mathrm{CaIII}}/N_{\mathrm{CaII}}$ is much less than this.

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