EFFECT OF THE STATISTICAL COMPOUND NUCLEUS PROCESS IN ELASTIC SCATTERING MEASUREMENTS

By P. J. Dallimore*

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Summary

The errors in elastic scattering cross section calculations due to a statistical compound nucleus component are investigated. It is found that the spin of the target nucleus is a major parameter in the estimation of these errors.

I. Introduction

It is well known that optical model analyses usually improve with increase in both incident bombarding energy and mass of the target. This is because of a decrease in the compound nucleus contribution and an improvement in the nucleon–nucleus interaction approximation (Feshbach 1958; Hodgson 1967).

If it is assumed that some compound nucleus contribution is present then it is also necessary to consider the effects of the energy resolution and the mean level width Γ of the compound nucleus in the method of analysis. As most optical model analyses are done at energies that correspond to $\geqslant 17$ MeV excitation energy in the compound nucleus it is possible to apply the statistical theory of fluctuations to estimate the errors involved.

II. Calculations

The approximate expression for the error is given by an extension of the formula of Dallimore and Hall (1966) to include a direct reaction component. The expression is now

$$\langle\sigma\rangle = \bar{\sigma}[1\pm\{(a/N)(1-y_{\rm d}^2)\}^{\frac{1}{2}}], \tag{1}$$

where

$$a = 2n^{-1}\tan^{-1}n - n^{-2}\ln(1+n^2),$$

$\langle\sigma\rangle$ and $\bar{\sigma}$ are respectively the measured and theoretical mean cross sections in the range ΔE, $n = \Delta E/\Gamma$, N is the fluctuation damping coefficient (Ericson and Mayer-Kuckuk 1966), and $y_{\rm d}$ is the ratio of the mean direct cross section $\sigma_{\rm d}$ to $\langle\sigma\rangle$.

It must be emphasized that (1) gives the r.m.s. error for the theoretical mean cross section due to the statistical nature of the compound nucleus process. It therefore follows that the errors are correlated over angles for which the compound nucleus cross sections are correlated. The above error does not include any contribution due to the experimental measurement of the mean cross section.

* Research School of Physical Sciences, Australian National University, P.O. Box 4, Canberra, A.C.T. 2600.

The value of $1/a$ has been shown to be the effective number of independent cross section measurements in the range ΔE (Gibbs 1965). In the following calculations of ΔE the stopping powers have been taken from Williamson, Boujot, and Picard (1966) while the values of Γ have been taken from the fit given in Figure 7 of Ericson and Mayer-Kuckuk (1966).

Figure 1 shows the dependence of the error on target thickness t and target mass number A for 10 MeV incident protons if one assumes an excitation energy in the compound nucleus of 20 MeV. The energy range ΔE, over which the cross sections have been averaged, has been assumed to be due only to the target thickness and to have a rectangular resolution function. For experiments using tandem Van de Graaff accelerators, the error caused by neglecting the effect of the beam resolution is negligible.

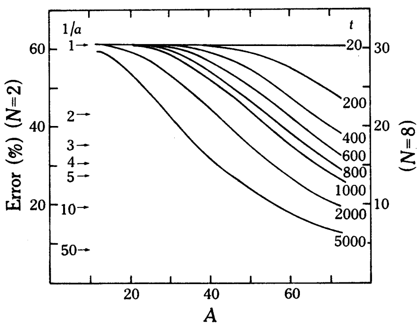

Fig. 1.—Dependence of the percentage error,
$100\{(a/N)(1-y_{\mathrm{d}}^2)\}^{\frac{1}{2}}$,
on the target thickness t (μg cm^{-2}) and mass number A for 10 MeV protons, a compound nucleus excitation energy of 20 MeV, and a 50% direct reaction contribution. The left-hand ($N = 2$) and right-hand ($N = 8$) scales correspond to the elastic scattering of protons measured around 90° from spin 0 and spin $\frac{1}{2}$ target nuclei respectively. The arrows indicate the errors when various numbers of independent cross sections are averaged.

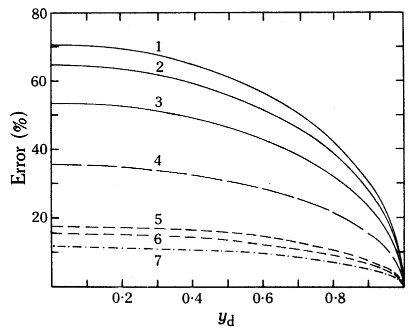

Fig. 2.—Dependence of the percentage error on the fraction of direct reaction y_{d} and on the mass and spin I_{a} of the target nucleus for 10 MeV protons and 200 μg cm^{-2} targets:

Curve	Target	N	I_{a}	$1/a$
1	^{28}Si	2	0	1·00
2	^{58}Ni	2	0	1·20
3	^{74}Ge	2	0	1·75
4	^{19}F	8	$\frac{1}{2}$	1·00
5	^{23}Na	32	$\frac{3}{2}$	1·00
6	^{63}Cu	32	$\frac{3}{2}$	1·31
7	^{27}Al	72	$\frac{5}{2}$	1·00

Figure 2 shows the variation of the error with the fraction of direct reaction y_{d} and with the mass and spin I_{a} of the target nucleus. In calculating the dependence of the error on the spin of the target nucleus the expression for N at 90° has been used (Ericson 1963), namely,

$$N = \tfrac{1}{2}(2i_{\mathrm{a}}+1)(2I_{\mathrm{a}}+1)(2i_{\mathrm{b}}+1)(2I_{\mathrm{b}}+1)\,, \tag{2}$$

where i_{a}, I_{a}, i_{b}, and I_{b} refer to the spins of the incoming particle, target nucleus, outgoing particle, and residual nucleus respectively. Thus, for elastic scattering of particles $i_{\mathrm{a}} = i_{\mathrm{b}}$ and $I_{\mathrm{a}} = I_{\mathrm{b}}$ so that for a given incident particle the error is inversely proportional to $2I_{\mathrm{a}}+1$. Equation (2) gives an upper limit for the value of N and therefore a lower limit for the estimation of the errors. It is found to give a good representation for angles between approximately 50° and 130° (Allardyce *et al.* 1965; Dietzsch *et al.* 1968). For other angles the values of N are smaller, reaching minimum values at 0° and 180°. Therefore, for a fixed value of y_{d}, the error will be a maximum at 0° and 180° and a minimum around 90°. The errors for the integrated cross sections and for the total cross section will be considerably smaller than the above estimates because of the much larger values of N expected. This is because the cross sections are effectively formed by the addition of several independent cross sections.

III. Discussion

From Figures 1 and 2, it is seen that the statistical error to be expected depends largely on the spin of the target nucleus, and, to a lesser extent, on the fraction of direct reaction and the target thickness. For example, elastic scattering measurements with ^{27}Al ($\frac{5}{2}^{+}$) should have errors that are a factor of six smaller than measurements with nearby spin 0 target nuclei (e.g. ^{26}Mg or ^{28}Si). Also, neglecting the approximation of the nucleon–nucleus interaction, it is possible that better representations of the average angular distributions can be obtained for light nuclei of high spin, e.g. ^{27}Al ($\frac{5}{2}^{+}$), than for medium weight nuclei of low spin, e.g. ^{58}Ni (0^{+}). However, the theoretical calculation of the average compound nucleus contribution by the Hauser–Feshbach method (Hauser and Feshbach 1952; Moldauer 1961) depends on the amount of flux that is removed from the shape elastic channel as direct reaction in the inelastic channels (Hodgson and Wilmore 1967). As this is very difficult, if not impossible, to estimate, the theoretical calculation of the average cross section must improve with increase in y_{d}; this generally implies that the calculations improve with increase in A.

The important parameter that may be varied experimentally is the effective number of independent cross sections contributing to the energy-averaged cross section. By using thick targets this number is increased although the removal of impurity peaks then becomes a problem. It is more satisfactory to measure several angular distributions separated by an energy interval δE which is given by the solution of

$$(2\Gamma/\delta E)\tan^{-1}(\delta E/\Gamma)-(\Gamma/\delta E)^{2}\ln\{1+(\delta E/\Gamma)^{2}\} = 1\,.$$

The approximate solution is given by $\delta E = \pi\Gamma$ (Gibbs 1965).

If it is assumed that the direct and average compound nucleus components are slowly varying with energy, then for medium mass nuclei ($\Gamma \approx 3$ keV) the measurement of angular distributions at approximately 10 keV intervals results in independent cross section measurements. By measuring several of these and averaging them it should be possible to obtain a good representation of the average angular distribution over an energy range for which the direct and compound nucleus cross sections are assumed constant.

For light nuclei the mean level widths are much larger (e.g. $A = 20$, $\Gamma \approx 100$ keV; $A = 40$, $\Gamma \approx 15$ keV) and any such averaging will be over an energy range for which the direct and mean compound nucleus cross sections may exhibit significant variations. In these cases it is better to measure excitation functions over large energy ranges and to fit them with smooth curves. It is then possible to estimate the average angular distributions at any energy in the range ΔE from these fits to the excitation functions.

The above arguments also apply for inelastic scattering although in these cases the error is inversely proportional to $\{(2I_a+1)(2I_b+1)\}^{\frac{1}{2}}$.

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