A METHOD FOR DETERMINING THE ELECTRON DENSITY DISTRIBUTION ABOUT THE $F_2$ PEAK OF THE IONOSPHERE

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Abstract

A method is given for extending the analysis of topside ionograms to yield ionospheric electron density profiles down to and below $h_{\text{max}}$, the peak of the $F_2$ layer, by analysis of the ground echoes. Calculations using model ionospheres indicate that the accuracy of the method is not seriously affected by the assumptions made or by the limited accuracy with which ionograms can be scaled.

I. INTRODUCTION

The problem of determining electron density distributions in the ionosphere from bottomside and topside ionograms has received considerable attention in recent years (see e.g. Wright and Smith 1967; Jackson 1969). Although the basic methods give the electron density profile over a large range in height, the profiles usually do not extend to $h_{\text{max}}$, the peak of the $F_2$ layer. This is unfortunate because a knowledge of the variations of $h_{\text{max}}$ provides a critical test of $F$-region theory. Current practice is to estimate $h_{\text{max}}$ by extrapolating the calculated profile according to a model layer: usually a parabolic layer for the bottomside of the $F_2$ layer and a Chapman layer for the topside.

In the case of topside ionograms the basic methods of analysis can in principle be extended to obtain the complete electron density profile by utilizing the ground echo traces (Dyson 1967). In practice it is more realistic to limit the analysis to a determination of the shape of the $F_2$ layer, and hence determine $h_{\text{max}}$, rather than to attempt a calculation of the complete electron density profile. This paper presents a method for obtaining the shape of the $F_2$ layer in the vicinity of the $F_2$ peak by using the virtual-height–frequency characteristics of topside ionogram ground echoes. Results are also compared with calculations using model ionospheric layers to give an indication of the accuracy of the method.

II. METHOD

For a radio wave propagating vertically through the ionosphere from height $h_1$ to height $h_2$, the group path is given by

$$P' = \int_{h_1}^{h_2} \mu' (f, f_H, \theta, N) \, dh,$$

where the group refractive index $\mu'$ is a function of the frequency of propagation $f$, the gyrofrequency $f_H$, the dip angle $\theta$, and the electron density $N$. Both $f_H$ and $N$ are functions of $h$.

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Figure 1 shows a schematic diagram of a topside ionogram and the corresponding electron density profile. In the usual analysis of topside ionograms the extraordinary ray trace is used to determine the electron density profile from the height of the satellite down to some lower height $h_L$ (above $h_{\text{max}}$), which corresponds to the observed high frequency cutoff for the extraordinary trace. The gyrofrequency is a known function of height so that, once the electron density profile is determined from the satellite height down to $h_L$, equation (1) can be used to determine, for any frequency, the group path $P'_t$ due to the ionization between the satellite height and $h_L$. If this is done for frequencies at which ground echoes occur, the group path due to the ionization below $h_L$ is given by the difference between the measured group path $P'_m$ and $P'_t$ as

$$P'_t = P'_m - P'_t = \int_{h_b}^{h_L} \mu' \, dh + h_b = \int_{h_b}^{h_{\max}} \mu' \, dh + \int_{h_b}^{h_{\max}} \mu' \, dh + h_b,$$

where $h_b$ is the base height of the ionosphere.

Since $h_L$ is known, equation (2) can be simplified by writing it in terms of the group retardation $R$ rather than the group delay. Equation (2) then becomes

$$R = \int_{h_b}^{h_L} (\mu' - 1) \, dh + \int_{h_b}^{h_{\max}} (\mu' - 1) \, dh.$$  

The variation of electron density with height may be written as $h = F(X)$, where $X = (f_N f)^2$, $f_N$ being the plasma frequency. Making this substitution in equation (2a) gives

$$R = \int_{X_C}^{X_L} (\mu' - 1) \frac{dh}{dX} \, dX + \int_{X_C}^{X_{\max}} (\mu' - 1) \frac{dh}{dX} \, dX,$$

where $X_C$ and $X_L$ are the values of $X$ at $h_{\max}$ and $h_L$ respectively. The advantage of making this substitution is that the limits of the integrals are now known since
$f_0 F_2$ can be read from the topside ionogram to give $X_C$. It is, however, necessary to postulate the form of the function $h = F(X)$ in order to proceed further.

Below $h_{\text{max}}$ the variation of $X$ with height may be quite complicated during the daytime when substantial $D$, $E$, and $F_1$ layers exist. However, if we initially consider the night-time case, underlying layers may be neglected and the $F_2$ layer can be considered to be parabolic, i.e. it can be represented by the equation

$$X = X_C (1 - (h - h_{\text{max}})^2/4H^2),$$

where $H$ is the scale height of the layer. Then equation (3) becomes

$$R = -H \int_{X_c}^{X_L} \frac{(\mu' - 1) \, dX}{X_C (1 - X/X_C)^{\frac{3}{2}}} + H \int_{X_c}^{X_L} \frac{(\mu' - 1) \, dX}{X_C (1 - X/X_C)^{\frac{1}{2}}}.$$  \hspace{1cm} (5a)

The integrals in equation (5a) can be evaluated if the gyrofrequency is considered constant up to $h_L$, an assumption that is made in the analysis of topside ionograms. Thus the only unknown in equation (5a) is $H$ and we can write the equation as

$$R = AH.$$  \hspace{1cm} (5b)

If the retardation due to the ionization below $h_L$ is determined for a number of frequencies on the ordinary $(o)$ and extraordinary $(x)$ ground echo traces then we may write the series of simultaneous equations

$$R_o(f_1) = A_1 H,$$
$$R_x(f_j+1) = A_{j+1} H,$$
$$R_o(f_2) = A_2 H,$$
$$R_x(f_{j+2}) = A_{j+2} H,$$
$$\vdots$$
$$\vdots$$
$$\vdots$$
$$R_o(f_l) = A_l H,$$
$$R_x(f_{j+n}) = A_{j+n} H.$$

From these equations a least squares solution for $H$ can be obtained. Once $H$ is determined, $h_{\text{max}}$ can be calculated from equation (4).

As demonstrated by equation (5b), $H$ can be determined even if only one measurement of the retardation is available. However, there are two main advantages to be gained from using more than one measurement and finding the least squares solution: (1) minimization of the effect of rounding errors, which arise when an ionogram is scaled, and (2) optimization of the value of $f_0 F_2$, and hence $X_C$, (already known to within definite limits) for the given set of data points.

During the daytime when considerable underlying ionization occurs, the same method may still be used since most of the group retardation occurs near the $F_2$ peak. There is, of course, a decrease in accuracy (see Section III).

In outlining the method, the electron density profile of the $F_2$ layer has been assumed to be parabolic because this is generally believed to be approximately its basic shape. However, the method is not limited to this assumption and other forms of the function $h = F(X)$ can be used. By this means it may be possible to determine if the assumption of a parabolic shape for the $F_2$ layer is reliable.
III. Test of Method

The method has been tested in the following way. A model for the $F_2$ layer was chosen which consisted of a Chapman layer for the topside portion and a parabolic layer for the bottomside. The last known point on the topside ionosphere was taken to be half a scale height above the $F_2$ peak and the layer was considered to be at the equator. The ordinary wave retardation was calculated at 10 chosen frequencies for which the values of $X_C$ lay between 0.7 and 0.925 and the results were rounded to the nearest 10 km to simulate the scaling accuracy of ionograms. Values of $H$ and $h_{\text{max}}$ were then calculated by the method outlined in the previous section. Differences between the calculated and model values therefore indicate the errors due to scaling inaccuracies and the assumption that the layer is purely parabolic. The resulting values which are listed below show that the errors are quite small.

<table>
<thead>
<tr>
<th>Model</th>
<th>$H$ (km)</th>
<th>$h_{\text{L}}-h_{\text{max}}$ (km)</th>
<th>Calculated</th>
<th>$H$ (km)</th>
<th>$h_{\text{L}}-h_{\text{max}}$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>40.0</td>
<td>20.0</td>
<td>41.5</td>
<td>80.0</td>
<td>40.0</td>
</tr>
</tbody>
</table>

The effect of underlying ionization was also determined by using the same approach as outlined above but with the addition of parabolic $F_1$ and $E$ layers to the model. Values of the parameters for two daytime models are given in Table 1 along with values for the scale and peak heights of the $F_2$ layer calculated on the assumption that the retardation is due solely to a parabolic $F_2$ layer. For model 1, the calculated value of $H$ is over 25% too high, and that of $h_{\text{max}}$ is 4 km too low. In this model the ratio of the critical frequency of each lower layer to $f_0F_2$ is quite high, a situation which only occurs at very low sunspot numbers (Davies 1965). For model 2, the calculated value of $H$ is about 12% too high and $h_{\text{max}}$ is in error by 1 km; this model is typical of the mid-latitude ionosphere at noon when the sunspot number is 40 (Davies 1965). These results indicate that even during the daytime the analysis of ground echoes will give a good description of the $F_2$ layer near its peak height, provided that the sunspot number is greater than 40.

IV. Conclusions

A method has been outlined for extending the analysis of topside ionograms to yield electron density profiles to give the shape of the $F_2$ layer and hence $h_{\text{max}}$ by including the analysis of ground echoes. Tests with model ionospheres have
indicated that the assumptions made and the rounding errors which occur in scaling ionograms do not introduce large errors. The method should prove most useful in determining latitudinal variations of $h_{\text{max}}$ and changes in the shape of the $F_2$ layer during magnetic storms.

V. Acknowledgment

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VI. References
