

DETAILED CALCULATIONS OF CORRELATIONS OCCURRING IN LIGHT-PARTICLE-ACCOMPANIED SPONTANEOUS FISSION

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Abstract

Detailed trajectory calculations are made for α -particle-, triton-, and proton-accompanied fission of ^{252}Cf . Comparison between experimental values of the quantity $\Delta\bar{E}_k/\Delta E_p$ and those deduced from the classical three point charge model suggests either that the initial Coulomb potential energy at scission has a narrow distribution (a result found by Feather 1971) or that a high degree of anticorrelation occurs between the initial potential energy and the initial kinetic energy of the fissioning system. In the latter case, the initial potential energy may be much broader.

I. INTRODUCTION

In a recent paper Feather (1971) studied the final state correlations occurring between fragment kinetic energy, α -particle energy, and fragment excitation energy for the α -particle-accompanied mode of ^{252}Cf fission. The ternary fission mode was assumed to develop out of a corresponding intermediate binary mode and attention was focused on a single mode of mass and charge division. The model employed was somewhat simpler than those normally used for trajectory computations (e.g. Boneh, Fraenkel, and Nebenzahl 1967; Raisbeck and Thomas 1968; Musgrove 1971*a*), as no reference was made to angular relationships. Further, it was assumed that the point of materialization of the α -particle and the kinetic and excitation energies of the heavy fragments were uniquely determined (for a particular mass and charge division) by the interfragment separation distance. Implicitly assumed, of course, was a classical three point charge model for the nucleus at the scission point, since Feather's (1971) treatment was essentially a formalization of the previous trajectory calculations which all depend on this model.

The scission point configuration assumed by Feather (1971) was that found by Musgrove (1971*a*), which was intermediate between the configurations deduced by Boneh, Fraenkel, and Nebenzahl (1967) and Raisbeck and Thomas (1968) in their calculations. The interfragment separation at scission found by Musgrove (1971*a*) was 23.7 f where the fragments have acquired some 25 MeV of their final kinetic energy and the average initial α -particle energy was found to be 2.75 MeV. Fong (1970) has also made trajectory calculations for the α -particle-accompanied mode of ^{252}Cf using initial conditions at scission in agreement with his statistical theory of fission (Fong 1956). For this statistical theory to be a valid description of fission, the heavy fragment kinetic energy must be small (~ 0.5 MeV) at the moment of scission. However, since Fong (1970) made no attempt to fit the detailed shapes of

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the final α -particle kinetic energy distribution and angular distribution his calculations must be discounted. Musgrove (1971a) showed that *average* quantities could be fitted with a wide range of initial scission configurations (including those used by Fong 1970) and therefore no physical significance can be attached to such a fit.

On the basis of his simple model, Feather (1971) derived various formal relationships connecting the final α -particle energy with prompt neutron number and with the final fragment kinetic energy. In particular, he found an expression relating the quantity $\Delta\bar{E}_k/\Delta E_\alpha$ (where \bar{E}_k is the average heavy fragment kinetic energy and E_α the α -particle energy in the final state) to conditions at the scission point. Using a number of partial derivatives obtained from the calculations of Boneh, Fraenkel, and Nebenzahl (1967) and Raisbeck and Thomas (1968), Feather (1971) discovered that the experimental value for $\Delta\bar{E}_k/\Delta E_\alpha$ could be obtained only if, at the moment of scission in the α -particle-accompanied mode, the heavy fragments had a nearly unique separation. Such a small variation of the nuclear configuration is not found in the corresponding binary mode from which the α -particle-accompanied mode is assumed to develop. The result deduced by Feather therefore appears to be at variance with the large body of experimental evidence (Nardi and Fraenkel 1970) that shows that binary and ternary fissions have basically very similar scission configurations.

In the present paper, all the partial derivatives required in Feather's (1971) analysis for α -particle-accompanied fission of ^{252}Cf have been calculated explicitly and his calculation has been extended to include the proton- and triton-accompanied modes.

II. FISSION MODEL

We examine now a model of the ternary fission process used in trajectory calculations. The one point of departure from the model employed by Musgrove (1971a) is that for a specified mass and charge division of the heavy nucleus we assume that the light particle is emitted from a unique point between the heavy fragments. We take this point to be the position of minimum potential energy.

The initial parameters defining the state of the system when the light particle is first freed from the nuclear force are: E_0 , the initial energy of the particle; T_0 , the initial energy of the heavy fragments; V_0 , the initial coulomb potential energy of the system; and θ_0 , the initial angle between the particle direction and the light fragment direction. After calculating the particle trajectory to "infinity" we obtain the final particle energy E_p , the final fragment energy E_k , and the final angle θ_f between the particle and the light fragment direction. We write therefore

$$E_p = E_p(E_0, V_0, T_0, \theta_0), \quad E_k = E_k(E_0, V_0, T_0, \theta_0), \quad \theta_f = \theta_f(E_0, V_0, T_0, \theta_0), \quad (1a, b, c)$$

and further

$$E_k + E_p = E_0 + V_0 + T_0. \quad (2)$$

In most trajectory calculations, the light particle is assumed to be emitted isotropically with a Maxwellian distribution for initial energy. The angle θ_0 therefore has a $\sin \theta$ distribution, and in the remainder of this work we assume that an average over θ_0 has been made and no further reference is made to this quantity.

The following standard values for the initial parameters of the system are those derived from the trajectory calculations of Musgrove (1971*a*, 1971*b*) which gave good agreement with the measured energy and angular distribution data:

Nucleus	\bar{E}_0 (MeV)	R_0 (f)	V_h (cm s ⁻¹)
α -particle	2.75	23.7	3.75×10^8
Triton	1.75	22.5	3.25×10^8
Proton	1.00	29.0	5.50×10^8

Here R_0 is the separation of the heavy fragment centres and V_h is the initial velocity of the heavy fragment. We now wish to examine the effect on the final particle and fragment energies of incremental changes in the initial parameters about their average values. From equations (1*a*) and (1*b*) we obtain

$$\Delta E_p = \frac{\partial E_p}{\partial E_0} \Delta E_0 + \frac{\partial E_p}{\partial V_0} \Delta V_0 + \frac{\partial E_p}{\partial T_0} \Delta T_0, \quad (3a)$$

$$\Delta E_k = \frac{\partial E_k}{\partial E_0} \Delta E_0 + \frac{\partial E_k}{\partial V_0} \Delta V_0 + \frac{\partial E_k}{\partial T_0} \Delta T_0, \quad (3b)$$

which simplify to

$$\Delta E_p = \frac{\partial E_p}{\partial E_0} \Delta E_0 + \left(\frac{\partial E_p}{\partial V_0} + \frac{\partial E_p}{\partial T_0} \frac{dT_0}{dV_0} \right) \Delta V_0, \quad (4a)$$

$$\Delta E_k = \frac{\partial E_k}{\partial E_0} \Delta E_0 + \left(\frac{\partial E_k}{\partial V_0} + \frac{\partial E_k}{\partial T_0} \frac{dT_0}{dV_0} \right) \Delta V_0. \quad (4b)$$

From the expression for total energy release in ternary fission for a particular mode of mass division, namely

$$Q_t = T_0 + V_0 + E_0 + D_0, \quad (5)$$

where D_0 is the deformation-excitation energy of the system at the scission point, we find that the quantity dT_0/dV_0 is given by

$$\frac{dT_0}{dV_0} = -1 - \frac{dE_0}{dV_0} - \frac{dD_0}{dV_0}. \quad (6)$$

All derivatives in equation (6) are essentially negative quantities and are therefore numerically less than unity. We assume in the following analysis that dT_0/dV_0 is approximately constant for a specified ternary fission mode and we examine a number of possible values for dT_0/dV_0 in the range $-1 < dT_0/dV_0 < 0$.

From equations (4) the quantity $\Delta E_k/\Delta E_p$ is given by

$$\frac{\Delta E_k}{\Delta E_p} = \frac{\left\{ \frac{\partial E_k}{\partial E_0} \Delta E_0 + \left(\frac{\partial E_k}{\partial V_0} + \frac{\partial E_k}{\partial T_0} \frac{dT_0}{dV_0} \right) \Delta V_0 \right\}}{\left\{ \frac{\partial E_p}{\partial E_0} \Delta E_0 + \left(\frac{\partial E_p}{\partial V_0} + \frac{\partial E_p}{\partial T_0} \frac{dT_0}{dV_0} \right) \Delta V_0 \right\}}. \quad (7)$$

We are interested in comparing the predictions of equation (7) with the experimentally measured values for proton-, triton-, and α -particle-accompanied fission of ²⁵²Cf

obtained by Nardi, Gazit, and Katcoff (1969). Their results were:

$$\begin{aligned}\Delta\bar{E}_k/\Delta E_\alpha &= -0.41 \pm 0.05 && \text{for } \alpha\text{-particles,} \\ \Delta\bar{E}_k/\Delta E_{\text{tr}} &= -0.37 \pm 0.10 && \text{for tritons, and} \\ \Delta\bar{E}_k/\Delta E_{\text{pr}} &= -0.04 \pm 0.20 && \text{for protons.}\end{aligned}$$

TABLE 1

CALCULATED PARTIAL DERIVATIVES FOR LIGHT-PARTICLE-ACCOMPANIED TERNARY FISSION

The values are for a fragment mass ratio of 1.4

E_0 (MeV)	$\frac{\partial E_p}{\partial E_0}$	$\frac{\partial E_k}{\partial E_0}$	$\frac{\partial E_p}{\partial V_0}$	$\frac{\partial E_k}{\partial V_0}$	$\frac{\partial E_p}{\partial T_0}$	$\frac{\partial E_k}{\partial T_0}$
(a) α -particles						
0.23	11.70	-10.67	0.038	0.96	-0.112	1.113
0.69	5.05	-4.02	0.056	0.95	-0.121	1.121
1.15	3.49	-2.47	0.064	0.94	-0.121	1.120
1.60	2.80	-1.77	0.070	0.93	-0.119	1.120
2.02	2.43	-1.40	0.074	0.93	-0.117	1.116
2.38	2.21	-1.19	0.077	0.93	-0.115	1.125
2.75	2.05	-1.02	0.079	0.93	-0.113	1.113
3.12	1.92	-0.89	0.081	0.93	-0.111	1.111
3.48	1.82	-0.80	0.083	0.92	-0.110	1.110
3.94	1.73	-0.70	0.085	0.92	-0.108	1.108
4.49	1.64	-0.61	0.087	0.91	-0.106	1.106
5.13	1.56	-0.53	0.089	0.91	-0.104	1.104
5.96	1.48	-0.45	0.091	0.91	-0.101	1.101
7.33	1.39	-0.36	0.095	0.91	-0.097	1.097
9.17	1.30	-0.28	0.099	0.90	-0.092	1.093
Averaged over $E_0 \geq 1.15$ MeV	2.23	-1.21	0.079	0.92	-0.112	1.112
(b) Tritons						
0.15	9.45	-8.43	0.014	0.99	-0.054	1.054
0.44	4.56	-3.54	0.023	0.98	-0.066	1.066
0.73	3.26	-2.24	0.028	0.97	-0.069	1.069
1.02	2.67	-1.65	0.031	0.97	-0.069	1.069
1.29	2.34	-1.32	0.032	0.97	-0.069	1.069
1.51	2.14	-1.12	0.035	0.97	-0.068	1.068
1.75	2.00	-0.98	0.037	0.96	-0.068	1.068
1.99	1.88	-0.86	0.038	0.96	-0.067	1.067
2.21	1.79	-0.77	0.039	0.96	-0.067	1.067
2.51	1.70	-0.68	0.040	0.96	-0.066	1.066
2.86	1.62	-0.60	0.042	0.96	-0.065	1.065
3.26	1.54	-0.52	0.043	0.96	-0.064	1.064
3.79	1.47	-0.45	0.044	0.96	-0.062	1.062
4.66	1.38	-0.36	0.047	0.95	-0.060	1.060
5.84	1.30	-0.28	0.049	0.95	-0.057	1.057
Averaged over $E_0 \geq 1.29$ MeV	1.82	-0.80	0.040	0.96	-0.066	1.066

TABLE 1 (Continued)

E_0 (MeV)	$\frac{\partial E_p}{\partial E_0}$	$\frac{\partial E_k}{\partial E_0}$	$\frac{\partial E_p}{\partial V_0}$	$\frac{\partial E_k}{\partial V_0}$	$\frac{\partial E_p}{\partial T_0}$	$\frac{\partial E_k}{\partial T_0}$
(c) Protons						
0.08	12.13	-11.12	0.030	0.97	-0.029	1.029
0.25	5.01	-4.01	0.038	0.96	-0.029	1.029
0.42	3.45	-2.44	0.041	0.96	-0.028	1.028
0.58	2.77	-1.76	0.044	0.96	-0.027	1.027
0.73	2.40	-1.39	0.045	0.96	-0.026	1.026
0.87	2.19	-1.18	0.046	0.95	-0.026	1.026
1.00	2.03	-1.02	0.047	0.95	-0.025	1.025
1.13	1.91	-0.90	0.048	0.95	-0.025	1.025
1.27	1.81	-0.81	0.049	0.95	-0.024	1.024
1.43	1.72	-0.71	0.050	0.95	-0.024	1.024
1.63	1.63	-0.62	0.051	0.95	-0.023	1.023
1.87	1.55	-0.55	0.051	0.95	-0.023	1.023
2.17	1.48	-0.47	0.052	0.95	-0.022	1.022
2.67	1.39	-0.38	0.054	0.95	-0.021	1.021
3.33	1.31	-0.30	0.055	0.94	-0.020	1.020
Averaged over } $E_0 \geq 0.42$ MeV }						
	2.21	-1.20	0.047	0.95	-0.025	1.025

Accordingly trajectory calculations were made for these three fission modes to obtain explicitly the partial derivatives entering into equation (7). A constant mass ratio of 1.4 corresponding approximately to the most probable mass division was assumed and an average over a $\sin \theta$ distribution of initial particle direction was made. The results of these computations are presented in Table 1 together with the average value of each partial derivative taken over a truncated Maxwellian distribution of initial particle energy which roughly corresponds to the truncated distribution of final energies used by Nardi, Gazit, and Katcoff (1969) in their experiment.

III. RESULTS AND DISCUSSION

The experimental data of Nardi, Gazit, and Katcoff (1969) for $\Delta \bar{E}_k / \Delta E_p$ are the results of best straight line fits to a plot of average fragment kinetic energy versus particle energy. Thus, in order to compare the predictions of equation (7) with experiment, an average over all possible initial conditions must be performed. The right-hand side has therefore been numerically integrated over the initial Maxwellian distribution of particle energy for various values of ΔV_0 and dT_0/dV_0 and the results obtained for α -particles, tritons, and protons are given in Table 2.

Considering first the results of this averaging for α -particles (Table 2(a)) we see that, provided the initial potential energy has a Gaussian distribution, a further averaging of $\Delta E_k / \Delta E_\alpha$ over V_0 (for a constant value of dT_0/dV_0) leads to a value which is independent of the width of the V_0 distribution and of dT_0/dV_0 and which is in acceptable agreement with experiment. Unlike Feather (1971) therefore we find that the classical model imposes no restriction on the variance of the potential energy distribution merely by consideration of the average value of the quantity $\Delta \bar{E}_k / \Delta E_\alpha$.

TABLE 2

CALCULATED VALUES OF $\Delta\bar{E}_k/\Delta E_D$ FOR VARIOUS INCREMENTS IN POTENTIAL ENERGY

The values have been averaged over a Maxwellian distribution of initial energies

ΔV_0 (MeV)	$dT_0/dV_0 = 0$	-0.2	-0.4	-0.6	-0.8	-1.0
<i>(a) α-particles</i>						
-3.0	-2.264	-1.859	-1.457	-1.059	-0.663	-0.270
-2.5	-1.973	-1.636	-1.302	-0.970	-0.639	-0.311
-2.0	-1.681	-1.412	-1.144	-0.878	-0.613	-0.350
-1.5	-1.386	-1.184	-0.984	-0.784	-0.585	-0.387
-1.0	-1.089	-0.955	-0.821	-0.687	-0.554	-0.422
-0.5	-0.790	-0.723	-0.656	-0.589	-0.522	-0.455
0	-0.488	-0.488	-0.488	-0.488	-0.488	-0.488
0.5	-0.183	-0.250	-0.317	-0.384	-0.451	-0.518
1.0	0.122	-0.011	-0.145	-0.279	-0.413	-0.547
1.5	0.430	0.230	0.029	-0.172	-0.373	-0.575
2.0	0.738	0.472	0.205	-0.062	-0.331	-0.601
2.5	1.048	0.717	0.384	0.049	-0.287	-0.625
3.0	1.360	0.963	0.564	0.163	-0.241	-0.649
<i>(b) Tritons</i>						
-3.0	-3.562	-2.857	-2.857	-1.499	-0.843	-0.203
-2.5	-3.053	-2.467	-1.893	-1.331	-0.780	-0.239
-2.0	-2.539	-2.071	-1.611	-1.158	-0.713	-0.275
-1.5	-2.018	-1.668	-1.322	-0.981	-0.644	-0.311
-1.0	-1.491	-1.258	-1.027	-0.799	-0.572	-0.346
-0.5	-0.958	-0.842	-0.726	-0.611	-0.496	-0.382
0	-0.418	-0.418	-0.418	-0.418	-0.418	-0.418
0.5	0.129	0.014	-0.102	-0.219	-0.336	-0.454
1.0	0.684	0.453	0.221	-0.013	-0.249	-0.487
1.5	1.245	0.901	0.553	0.200	-0.156	-0.517
2.0	1.813	1.356	0.892	0.421	-0.058	-0.545
2.5	2.387	1.819	1.239	0.649	0.046	-0.570
3.0	2.968	2.290	1.595	0.884	0.155	-0.594
<i>(c) Protons</i>						
-3.0	-2.123	-1.756	-1.399	-1.052	-0.716	-0.393
-2.5	-1.882	-1.574	-1.272	-0.977	-0.689	-0.409
-2.0	-1.629	-1.380	-1.135	-0.894	-0.658	-0.425
-1.5	-1.364	-1.175	-0.988	-0.804	-0.622	-0.442
-1.0	-1.086	-0.959	-0.832	-0.707	-0.583	-0.459
-0.5	-0.796	-0.732	-0.667	-0.603	-0.540	-0.476
0	-0.493	-0.493	-0.493	-0.493	-0.493	-0.493
0.5	-0.177	-0.244	-0.310	-0.377	-0.443	-0.510
1.0	0.148	0.015	-0.120	-0.255	-0.391	-0.528
1.5	0.483	0.281	0.078	-0.128	-0.336	-0.545
2.0	0.828	0.556	0.282	0.004	-0.278	-0.563
2.5	1.181	0.839	0.493	0.141	-0.217	-0.581
3.0	1.543	1.131	0.711	0.283	-0.154	-0.600

Nor indeed is a particular choice of dT_0/dV_0 preferred from this analysis. However, if the scatter of values of $\Delta E_k/\Delta E_\alpha$ for a particular mode of mass division is small as Feather assumes (most lying between -0.9 and 0 for example) then we see that for $|dT_0/dV_0| < 0.6$ the potential energy at the scission point is restricted to vary by at most 2 MeV from its average value. This is indeed a rather small variation and forms the basis of Feather's argument for the near uniqueness of the scission configuration in this case.

At present, there is no experimental evidence relating to the spread of values of $\Delta E_k/\Delta E_\alpha$ to be expected for a particular mode of mass division but, even if this spread turns out to be narrow, the classical model offers an alternative explanation to Feather's (1971) result. For values of $|dT_0/dV_0| \gtrsim 0.8$ we see from Table 2(a) that a reasonably narrow range of $\Delta E_k/\Delta E_\alpha$ values is obtained from a much wider spread in initial potential energy than that found by Feather.

Further experimental data are obviously required. In particular there is need for a determination of $\Delta \bar{E}_k/\Delta E_\alpha$ for restricted ranges of the mass splitting ratio. Also a measurement of the spread of values of prompt neutron multiplicity in ternary fission and a determination of whether or not $\Delta \bar{E}_k/\Delta E_\alpha$ depends on the number of prompt neutrons emitted in ternary fission would be of great value in the interpretation of the results found here. It is hoped that information on these questions will be forthcoming from a projected experiment to be performed at the Australian Atomic Energy Commission.

For triton-accompanied fission the results also show that $\Delta E_k/\Delta E_{tr}$ averaged over a Gaussian distribution for the initial potential energy leads to a value that is in quite good agreement with experiment (Table 2(b)). It is therefore disturbing to find that the calculated values for $\Delta E_k/\Delta E_{pr}$ for protons in Table 2(c) are very much smaller than experimental values even allowing for the experimental error. Indeed they are smaller than the results for α -particles. It is possible that the initial conditions obtained by Musgrove (1971b) are greatly in error since the proton energy spectrum is the least well known of the light particle energy spectra. Alternatively, this sharp disagreement with experiment could be caused by using here a model where the proton is emitted from the position of minimum potential energy. In the earlier trajectory calculations a wide distribution of initial positions was assumed for protons with a standard deviation of 7.6 f. The simplification of the model is a much worse approximation to the initial assumed proton-accompanied configuration than to that of either of the other two particles and therefore may be expected to give poorer agreement with experiment.

IV. CONCLUSIONS

For α -particle- and triton-accompanied ternary fission we have found that if the final distribution of $\Delta \bar{E}_k/\Delta E_p$ is to be narrow, as assumed by Feather (1971), then a narrow distribution of initial potential energy is required for values of $|dT_0/dV_0| < 0.6$. This finding agrees with the calculation of Feather (1971). An alternative presented by the classical model would allow a broader distribution in V_0 for values of $|dT_0/dV_0| \gtrsim 0.8$. However, further experimental data are required before a choice can be made between these two possibilities deduced from a classical model of the nucleus at scission.

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