

ANISOTROPIC DIFFUSION AND THE TOWNSEND-HUXLEY EXPERIMENT

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[Manuscript received 27 February 1973]

Abstract

The relationship between current ratios and electron diffusion coefficients for the Townsend-Huxley experiment is reanalysed with the assumption that diffusion can be represented by two coefficients D_T and D_L for diffusion transverse and parallel respectively to the applied electric field. When the new formula is used to interpret previous experimental data obtained with a diffusion tube of length 2 cm, the derived values of D_T/μ become independent of pressure (μ being the electron mobility). For longer diffusion tubes (≥ 6 cm), current ratios are insensitive to D_L and the results differ insignificantly from those obtained using the formula previously derived on the assumption that diffusion is isotropic.

I. INTRODUCTION

Recent accurate measurements of D_T/μ , the ratio of the transverse diffusion coefficient to the mobility, together with measurements of the drift velocity W have enabled low energy elastic and inelastic cross sections to be determined for many gases (see e.g. Phelps 1968). Almost all measurements of D_T/μ have been obtained using the method initiated by Townsend and modified by Huxley (Huxley and Crompton 1962). In this method, the diffusion of electrons drawn from a source hole by an electric field is determined by measuring, for example, the fraction R of the total current falling on the central disc of a divided electrode.

To determine D_T/μ from a measurement of R , the functional relationship of R and D_T/μ must be known accurately. The relationship that has been used in the most accurate determinations of D_T/μ is (Crompton and Jory 1962)

$$R = 1 - (h/d)\exp\{-\lambda(d-h)\}, \quad (1)$$

where h is the distance between the source hole and the collector, $\lambda = W/2D_T$, and $d^2 = b^2 + h^2$, with b the radius of the central disc. This relationship is obtained by using the so-called "pole" solution of the electron continuity equation, assuming isotropic diffusion, with the inclusion of an image term to ensure that the electron density n is zero over the collector. The pole solution for the electron density distribution in the drift region is $n = \exp\{\lambda(z-r)\}/r$, where z and r are coordinates as indicated in Figure 1. However, this solution does not give $n = 0$ over the electrode containing the source hole. A dipole solution with image terms is necessary to give $n = 0$ over both cathode and anode. Although the resulting expression for R differs insignificantly from equation (1) at large chamber lengths, when the chamber is short,

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e.g. $h = 2$ cm, this dipole solution gives experimental values of D_T/μ which vary with the pressure p for a given ratio of E/p (E being the electric field strength and $\mu = W/E$), whereas values obtained from equation (1) are independent of pressure to within the experimental accuracy (Crompton and Jory 1962).

In the present paper it is shown that the above discrepancy is resolved if the experimental current ratios are reanalysed with the assumption that electron diffusion is anisotropic in the presence of an electric field. In the new analysis $n = 0$ over both electrodes. Brief accounts of this work have been reported earlier (Lowke 1971; Crompton 1972).

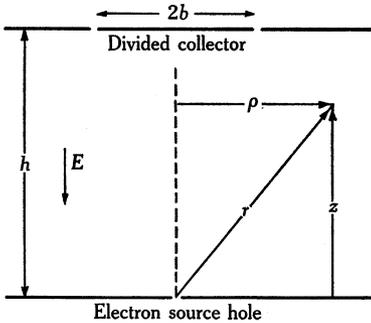


Fig. 1.—Definition of coordinates for the Townsend-Huxley experiment, in which measurement is made of the fraction of the total current falling on the central disc of a divided collector.

II. THEORY

The electron distribution function f , as determined from the Boltzmann equation, is a function of spatial derivatives of n (Parker 1963). For example, if a gradient exists in the direction of an applied electric field, the average velocity of the electrons in that direction, i.e. the convective velocity, differs from the value of the drift velocity W which is found in the absence of the gradient. As a consequence the power derived by the electrons from the field differs from the value in the absence of gradients and f becomes a function of the density gradient and thus, in general, of position. Since the drift velocity and diffusion coefficients at a given position are determined by the local value of f , these quantities also become functions of position. One consequence of the spatial dependence of the drift velocity is that the spread in the field direction of a narrow pulse of electrons drifting in an electric field is described by an apparent diffusion coefficient D_L which may differ considerably from the diffusion coefficient D_T that describes the spread in a direction transverse to the field. Values of D_L determined from the spread of the pulses in drift tube experiments have been found to differ by as much as a factor of eight from D_T (Wagner *et al.* 1969), and these values are consistent with theoretical predictions which account for the effect of density gradients (Parker and Lowke 1969; Lowke and Parker 1969; Skullerud 1969).

In the present analysis it is assumed that the steady state electron density distribution is governed by the continuity equation $\nabla \cdot \mathbf{j} = 0$, where the electron current density \mathbf{j} is assumed to be given by

$$\mathbf{j} = (nW - D_L \partial n / \partial z) \mathbf{i} - D_T (\partial n / \partial \rho) \mathbf{k},$$

\mathbf{i} and \mathbf{k} being unit vectors in the axial and radial directions respectively. In cylindrical

coordinates, if it is assumed that W , D_T , and D_L are independent of position, the continuity equation becomes

$$\frac{D_T}{D_L} \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial n}{\partial \rho} \right) + \frac{\partial^2 n}{\partial z^2} - 2\lambda \frac{D_T}{D_L} \frac{\partial n}{\partial z} = 0. \quad (2)$$

It should be recognized that in using equation (2) it is assumed that electron diffusion in general can be represented simply by two diffusion coefficients D_L and D_T which are independent of density gradients. This assumption does not follow from the experiments of Wagner *et al.* (1969) or the theory of Parker and Lowke (1969), in which a value of D_L is simply derived in both cases from the width of an electron pulse which is initially a delta function. Although the analysis of Huxley (1972) indicates that equation (2) is exact, it has now been shown that in general the higher order derivatives are not identically zero (Skullerud, personal communication). Thus, referring again to the example of a travelling pulse, equation (2) implies that an initially symmetrical three-dimensional pulse would spread symmetrically about its centroid, whereas it is to be expected that the spread due to diffusion in the leading edge of the pulse would be greater than that in the trailing edge, the density gradient being negative in front of the centroid and positive behind. Nevertheless, equation (2) rather than a continuity equation with an isotropic diffusion coefficient should give a more accurate, although still imperfect, representation of a steady state distribution of electrons in an electric field. It is to be noted that recently a continuity equation for electrons, which involves third-order derivatives of the electron density, has been derived by Virr *et al.* (1972) to analyse their measurements using a Townsend-Huxley apparatus, but the justification of the original equations is obscure.

We proceed to analyse equation (2) by introducing a scaling factor into the radial coordinate, in a manner similar to that used by Huxley (1940) in his analysis of the effect of magnetic fields. With $\phi = s^{\frac{1}{2}} \rho$ and substitution of $\beta = \lambda/s$, where $s = D_L/D_T$, equation (2) becomes identical in form with that used in analyses of isotropic diffusion by previous investigators. The electron density is then given by an infinite series of dipole solutions (Warren and Parker 1962)

$$n = \sum_{N=-\infty}^{\infty} r_N^{-3} (z - 2Nh)(\beta r_N + 1) \exp\{\beta(z - r_N)\}, \quad (3)$$

where $r_N^2 = (z - 2Nh)^2 + s\rho^2$. The current density at the collector plate is entirely due to diffusion and is given by $j = -D_L \partial n / \partial z$. Substituting for n from equation (3) and integrating over the collector plate to determine the fraction R of the total current falling on a central disc of radius b , we obtain

$$R = 1 - \frac{\sum_{N=0}^{\infty} r_N^{-3} \{h^2 r_N (2N-1)^2 - b^2 s^2 / \lambda\} \exp(-\lambda r_N / s)}{\sum_{N=1}^{\infty} \exp\{-(2N-1)\lambda h / s\}}, \quad (4)$$

where $r_N^2 = sb^2 + (2N-1)^2 h^2$. It is because dipoles for $N = 0$ and 1 each contribute the same amount in the summations as do the pair of dipoles with $N = -1$ and 2 , etc., that the sum can be written from $N = 1$ to ∞ in equation (4).

III. NUMERICAL CALCULATIONS

For any given experimental tube geometry, determined by h and b , the current ratio is a function of the two swarm coefficients D_T/μ or $\lambda = W/2D_T$ and $s = D_L/D_T$. To determine D_T/μ at a given E/p it is really necessary to measure two current ratios, either for different values of h or b or for different gas pressures. The second unknown, s , can then also be determined from the measurements, although usually not with great accuracy.

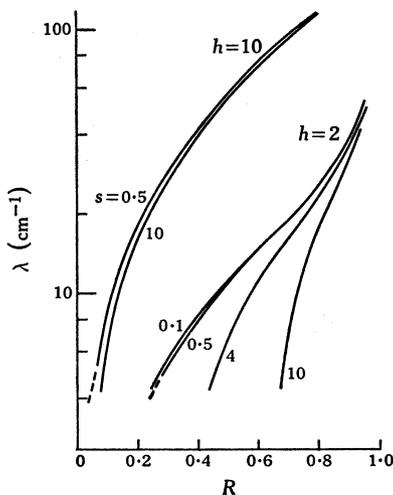


Fig. 2.—Derived theoretical relationships of the current ratio R as a function of $\lambda = W/2D_T$ and $s = D_L/D_T$ for two lengths h (cm) of the diffusion chamber. The results from the Huxley formula (dashed curves) are closely represented by the curves corresponding to $s = 0.5$.

Values of λ as a function of R and s from equation (4) are shown in Figure 2 for $b = 0.5$ cm and $h = 2$ and 10 cm, which are the dimensions used in the experimental investigation of Crompton and Jory (1962). The first term of the infinite series in equation (4) is adequate except for $D_L/D_T \geq 4$. Also shown in Figure 2 are results from equation (1), the Huxley formula. It is seen that for $h = 10$ the curves are insensitive to values of s and, furthermore, that the Huxley formula is closely represented by the curves corresponding to $s = 0.5$. It is coincidental that for the gases helium, hydrogen, and nitrogen, where the most accurate measurements of current ratios have been made, for the range $0.02 < E/p < 2$ V cm⁻¹ Torr⁻¹ the value of D_L/D_T is very close to 0.5 (Wagner *et al.* 1969). Thus, the reason why the Huxley formula gives pressure-independent values of D_T/μ despite the unrealistic boundary conditions is explained.

Experimental results by Crompton and Jory (1962) for $h = 2$ cm taken in hydrogen at $E/p = 0.6$ V cm⁻¹ Torr⁻¹ indicate a strong pressure dependence of derived values of D_T/μ using the dipole formula. These results have been reanalysed using equation (4) and are shown in Figure 3. It is seen that pressure-independent values of D_T/μ are obtained for $D_L/D_T \sim 0.5$. The predicted (Lowke and Parker 1969) value of D_L/D_T at $E/p = 0.6$ is 0.48 . Thus the new formula, which accounts for non-isotropic diffusion and is derived with $n = 0$ at the metal boundaries, gives consistent experimental results. It would be desirable to test equation (4) by taking further experimental measurements for different chamber geometries and for gases other than hydrogen.

Equation (4) has been used to reanalyse the results by Crompton *et al.* (1967, 1968) for helium and hydrogen. Values of D_L/D_T as a function of E/p were taken from Lowke and Parker (1969). It is found that the published values of D_T/μ are correct to 0.3%; the small error is a consequence of the experimental results being taken for a chamber length of 10 cm, in which case the curves for R are insensitive to D_L/D_T .

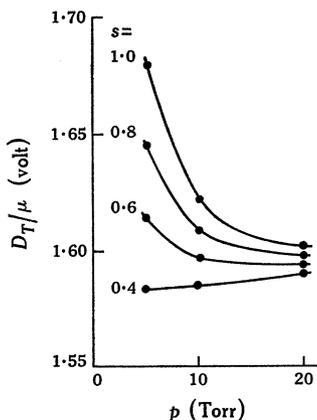


Fig. 3.—Plots of D_T/μ against pressure p , as derived from experimental measurements by Crompton and Jory (1962) of the current ratios in hydrogen for $h = 2$ cm and $E/p = 0.6$ V cm⁻¹ Torr⁻¹. The results give D_T/μ independent of pressure for $s = D_L/D_T \sim 0.5$.

IV. ACKNOWLEDGMENTS

The author gratefully acknowledges many stimulating discussions with Dr. R. W. Crompton, and also the financial support of a Rothman Fellowship.

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