THE DYNAMIC COLLECTIVE MODEL INTERPRETATION OF THE
PHOTONEUTRON CROSS SECTION OF $^{181}$Ta†

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Abstract

The cross section for photoneutron production in $^{181}$Ta has been measured from threshold to 28.8 MeV using bremsstrahlung and direct neutron detection. Integrated between these limits, the absolute value of the cross section has been determined to be $2.47 \pm 0.35$ MeV b. An examination of the cross section variation with excitation energy reveals the existence of the giant quadrupole resonance lying on the high excitation edge of the dipole peak. This provides additional evidence for the validity of the dynamic collective model. The present data do not support the existence of extensive fine structure below 17 MeV, as proposed by Ishkhanov et al. (1969).

I. INTRODUCTION

In the hydrodynamic representation of Okamoto (1956, 1958) and Danos (1957) the giant dipole resonance was envisaged as the resonant oscillations of interpenetrating neutron and proton fluids within a rigid nuclear boundary. Although this static collective model met with some success, particularly in the prediction of the split giant resonances of deformed nuclei, its interpretations of the observed photoabsorption cross sections of spherical and near-spherical nuclei were often clearly deficient. However, the removal of the restriction of a rigid nuclear surface by Danos and Greiner (1964) paved the way towards an explanation of these shortcomings. In their formulation of the dynamic collective model (DCM), Danos and Greiner proposed that the giant resonance oscillations should be coupled with the low energy collective motion of the nuclear surface. The consequence of such a coupling is a dispersion of the dipole strength from the principal resonance energy, the extent of the spreading being characterized largely by the ratio $\beta_0/E_2$, where $\beta_0$ is the average deformation of the surface vibrations which have an energy level spacing of the order of the phonon energy $E_2$. It has been verified experimentally that as the coupling strength increases so does the width and amount of structure in the photoabsorption cross section (Huber et al. 1967). Because the giant resonances of spherical nuclei can only be broadened and split through this dynamic coupling of the giant dipole and surface motions, it is these nuclei which provide the clearest evidence for the validity of the DCM.

In contrast to spherical nuclei, the DCM descriptions of the photoabsorption cross sections of heavy deformed nuclei seem to be only marginally better than the simple hydrodynamic predictions (Spicer 1969). Moreover, the DCM predictions

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for these nuclei are somewhat unconvincing owing to the number of adjustable parameters in the theory; these parameters are the energies $E_\beta$ and $E_\gamma$ of the $\beta$ and $\gamma$ vibrations respectively, the equilibrium deformation parameter $\beta_0$, the nuclear rotational energy $E_r$, the energy $E_0$ of the lower dipole peak, the fractional increase $\alpha$ in the dipole sum rule when exchange interactions are considered, and the widths $\Gamma_i$ of the various theoretical states. The first four of these parameters can be determined, in principle, from the low-energy spectrum of the nucleus, although the value of $\beta_0$ obtained in this way is frequently inaccurate so that this quantity is usually considered to be a free parameter. As a consequence of this and the fact that the values of the remaining three parameters are usually adjusted to fit the experimental results, the DCM affords a fit to the experimental data rather than a prediction of it.

Motivation for the present experiment arose from the desire to observe the giant quadrupole resonance. This provides a further test of the validity of the DCM interpretation of photoabsorption by heavy deformed nuclei. The DCM theory of the giant quadrupole resonance has been advanced by Ligensa et al. (1966). In the case of the giant quadrupole spectrum, all parameters are taken from the low-energy spectrum and dipole resonance, so that the observation of the quadrupole resonance as predicted would constitute far more convincing evidence for the validity of the DCM interpretation than does the shape of the dipole resonance alone. In the case of tantalum 181, the nucleus selected for investigation, since the DCM predicts that the giant quadrupole resonance will lie in the 20–27 MeV region of excitation and will contain only $\sim 7\%$ of the integrated magnitude of the dipole cross section, very careful experimental work is demanded for its observation, and for this reason almost all of our initial experimental endeavour was directed to this energy range. However, in the course of the work a new measurement of the $^{181}$Ta($\gamma$, n) cross section was published by Ishkhanov et al. (1969). This measurement showed much fine fragmentation of the split dipole resonance (see Fig. 1(b)) and so represented a considerable departure from previous results (Fuller and Weiss 1958; Spicer et al. 1958; Bramblett et al. 1963; Bergère et al. 1968), which were unanimous in their findings that the split giant resonance contained little or no evidence of fine fragmentation. It therefore seemed worth while to extend our attention to the dipole resonance, in the hope that some light could be shed upon this disagreement.

II. Evidence for the Giant Quadrupole Resonance

From comparison of the measured photonuclear cross sections of heavy nuclei with the predictions of the DCM it was apparent that above the giant resonance there is generally some excess strength which cannot be accounted for by giant dipole modes. This has been taken as evidence for giant quadrupole excitation. Perhaps the best examples of this so far presented lie in the $^{165}$Ho and $^{159}$Tb photoneutron cross sections measured at Saclay by Bergère et al. (1968). In both cases excess strength was observed in the 20–26 MeV region of excitation, where the giant quadrupole resonance is expected to lie. However, there is little or no evidence of the fragmentation of this strength which is to be expected if the DCM description is valid.

The E2 assignment to the excess strength is supported by the observed asymmetric angular distributions of fast photonucleons from this region of excitation.
(Shevchenko and Yuryev 1962; Quirk and Spicer 1964). This asymmetry can arise through interference between E1 and E2 giant resonance states (Eichler and Weidenmuller 1958). Unfortunately, the angular distribution data are not as definitive as may be hoped, since measurements have been made using bremsstrahlung $\gamma$-ray sources at one or two energies only and consequently are not able to indicate the magnitude of the E1–E2 interference as a function of energy.

![Graph](image)

Fig. 1.—Comparison of (a) the present results with (b) those of Ishkhanov et al. (1969) for the cross section for photoneutron production in $^{181}$Ta.

III. Experimental Techniques and Data Analysis

(a) Procedure

The cross section for production of photoneutrons in $^{181}$Ta was measured using bremsstrahlung from the Melbourne betatron. The target consisted of a series of 5.2 cm diameter pure metal discs and was situated within a $4\pi$ Halpern-type neutron detector. The incident bremsstrahlung flux was monitored using a relatively thin-walled ionization chamber which was interposed between the betatron and the neutron detector. The response of this chamber had been previously calibrated against that of a standard NBS type P2 chamber (Pruitt and Domen 1962). Measured between the energies of 7.3 and 29.5 MeV in 0.1 MeV increments, the final derived yield curve had a statistical uncertainty of $\lesssim 0.1\%$, except near threshold.
(b) Yield Curve Analysis and Neutron Multiplicity Correction

The observed cross section

\[ \sigma_{\text{obs}}(\gamma, n) = \sigma(\gamma, n) + \sigma(\gamma, np) + 2\sigma(\gamma, 2n) + 3\sigma(\gamma, 3n) \]

obtained from the experimental yield data with the variable-bin Penfold–Leiss (VBPL) analysis technique (Bramanis et al. 1972) is shown in Figure 1(a). In order to obtain the more physically meaningful cross section

\[ \sigma_n(\gamma, n) = \sigma(\gamma, n) + \sigma(\gamma, np) + \sigma(\gamma, 2n) + \sigma(\gamma, 3n), \]

the additional weighting of the \((\gamma, 2n)\) and \((\gamma, 3n)\) channels must be evaluated. These reactions have respective thresholds of 14·4 and 22·3 MeV (Howerton et al. 1964). The probability that a nucleus with initial excitation \(E\) will emit \(k\) neutrons has been calculated to be (Jackson 1956)

\[ P(E, k) = I(\Delta_k, 2k-3) - I(\Delta_k+1, 2k-1), \quad (1) \]

where Pearson’s incomplete gamma function is defined as

\[ I(z, n) = (n!)^{-1} \int_0^z x^n \exp(-x) \, dx \quad (2) \]

and

\[ \Delta_k = \left( E - \sum_{i=1}^k B_i \right) \theta^{-1}. \quad (3) \]

In equation (3), \(B_i\) is the binding energy of the \(i\)th neutron and \(\theta\) is the nuclear temperature, which is assumed to be given by

\[ \theta = \left( (E - B_i)/a \right)^{\frac{1}{4}}, \quad (4) \]

where \(a\) (MeV\(^{-1}\)) characterizes the nuclear level density:

\[ \rho(E - B_i) \propto \exp[2\{a(E - B_i)\}^{\frac{1}{2}}]. \quad (5) \]

Considering only the \((\gamma, n)\), \((\gamma, 2n)\), and \((\gamma, 3n)\) reactions and introducing (in the manner of Thies and Spicer 1960) the parameter \(x\) to express the fraction of non-statistical decay processes, we derive from equation (1)

\[ P(E, 1) = 1 - (1-x)\{1-(1+\Delta_2)\exp(-\Delta_2)\}, \quad (1a) \]

\[ P(E, 2) = (1-x)\{(1+\Delta_3+\frac{1}{2}\Delta_2^2+\frac{1}{6}\Delta_2^3)\exp(-\Delta_3)-(1+\Delta_2)\exp(-\Delta_2)\}, \quad (1b) \]

\[ P(E, 3) = (1-x)\{1-(1+\Delta_3+\frac{1}{2}\Delta_2^2+\frac{1}{6}\Delta_2^3)\exp(-\Delta_3)\}, \quad (1c) \]

where by definition

\[ P(E, k) = \sigma(\gamma, kn)/\sigma_n(\gamma, n). \quad (6) \]
Fig. 2.—Theoretical curve for the neutron multiplicity correction function $1/M(E)$ with the parameter values $a = 15.7$ MeV$^{-1}$ and $x = 0.40$. The curve is compared with experimental points from Bergère et al. (1968) which have been raised in energy by 0.3 MeV to give better agreement with the $^{181}$Ta$(\gamma,2n)$ threshold.

Fig. 3.—Multiplicity corrected $^{181}$Ta photoneutron cross section as derived using the VBPL technique with analysis bin widths close to 1.5 times the Thies (1961) optimum value. The full curve represents the DCM predicted form, an incoherent sum of calculations by Arenhövel (1965) and Ligensa et al. (1966) for the E1 and E2 giant resonances respectively. The dashed curve is an extrapolation of the predicted dipole resonance.
With the neutron multiplicity defined in the usual fashion, namely

\[ M(E) = \frac{\sigma_{\text{obs}}(\gamma, n)}{\sigma_n(\gamma, n)}, \]

we obtain

\[ M(E) = 1 + (1 - x)\left\{ 2 - (1 + A_2)\exp(-A_2) - (1 + A_3 + \frac{1}{2}A_3^2 + \frac{1}{6}A_3^3)\exp(-A_3) \right\}. \]

At excitations below the \((\gamma, 3n)\) threshold equation (8) reduces to the familiar expression

\[ M(E) = 1 + (1 - x)\left\{ 1 - (1 + A_2)\exp(-A_2) \right\}, \]

(8a)
in agreement with Blatt and Weisskopf's (1952) evaluation.

The neutron multiplicity function of \(^{181}\)Ta has been measured both at Saclay (Bergère et al. 1968) and Livermore (Bramblett et al. 1963) using a technique which involves the coincident counting of neutrons. These measurements indicate the validity of the values \(a = 15.7\) MeV\(^{-1}\) and \(x = 0.40\) which determine the correction used in the present case. Figure 2 compares this correction, namely the reciprocal of the neutron multiplicity \(1/M(E)\), with the Saclay measurements, which have been raised in energy by 0.3 MeV to give better agreement with the \(^{181}\)Ta\((\gamma, 2n)\) threshold of 14.42 MeV as derived from atomic mass tables (Howerton et al. 1964).

Figure 3 shows the multiplicity corrected photoneutron cross section of \(^{181}\)Ta as obtained using the VBPL method with an analysis bin width close to 1.5 times the “optimum” bin defined by Thies (1961).

IV. DISCUSSION OF RESULTS

(a) Comparison with Other Measurements

Although prior measurements of the \(^{181}\)Ta photoneutron cross section by Fuller and Weiss (1958), Spicer et al. (1958), Bramblett et al. (1963), and Bergère et al. (1968) have established the splitting of the dipole peak, it was not until the recent publication of a measurement by Ishkhanov et al. (1969) that serious proposals of additional structure were made. As shown in Figure 1(b), Ishkhanov and his co-authors found that the photoabsorption cross section below 17 MeV contained a series of fine resonances of widths 0.3–0.5 MeV whilst, at higher excitations, broader structure was seen to exist. A broad envelope around the fine structure closely resembles the split dipole resonance shape observed elsewhere.

The present measurement also suggests the existence of additional structure, although not nearly to the extent reported by Ishkhanov et al. (1969). Despite some evidence of fine structure at 11 and 16.7 MeV in the cross section shown in Figure 3, we cannot support any proposals of extensive fine fragmentation of the dipole resonance. To ensure that our failure to observe this fine structure was not a result of using an excessively broad analysis resolution function, the data were re-analyzed using bin widths close to the Thies (1961) optimum bin. This meant that over the bulk of the dipole resonance the analysis bin width was either 0.2 or 0.3 MeV, and this is sufficiently narrow to ensure the observation of any significant resonances of width 0.3–0.5 MeV. The derived cross section shown in Figure 4 confirms our
belief that the dipole resonance of $^{181}$Ta is not structurally fragmented into a series of fine resonances.

Notwithstanding the lack of agreement at energies below 17 MeV, at higher excitations there exists a remarkable correlation between structure observed in the present experiment and that found by Ishkhanov et al. (1969). The latter work indicates peaks centred at 18.7, 21.3, 25.0, and 27.5 MeV, together with cross section minima at 20 and 26 MeV, and all these features are readily apparent in the present data. In view of such excellent agreement above 17 MeV and the similarity of the measurement techniques (both experiments used bremsstrahlung with the endpoint energy stepped in 0.1 MeV increments), the discrepancies at lower energies demand further examination.

Although they gave no details of the procedure used to analyse their $^{181}$Ta data, in subsequent publications Ishkhanov et al. (1972a, 1972b) have indicated use
of the Penfold–Leiss technique with the analysis bin width being increased at several points along the yield curve (however, in contradistinction to the VBPL method employed here, no criterion is given for these bin changes). In addition, because of the readily observed strong correlation between adjacent cross section points, it is evident that their $^{181}$Ta data have been smoothed in some way. It is hoped that this smoothing has been applied correctly, since it is known that unless such a process is performed in a consistent fashion the calculated cross section may well be erroneous (Bramanis et al. 1972), e.g. a falsely structured cross section may result from the analysis of smoothed yield data using a narrow resolution function, unless this too has been modified.

If the foregoing presumptions regarding the analysis of Ishkhanov et al. (1969) are in fact valid,* it may be possible to understand why their cross section agrees well with the present result above 17 MeV, but poorly below. Whereas a falsely structured cross section may be produced at lower energies through the combination of unsystematic smoothing and insufficient bin width, the cross section above the dipole peak may be quite realistic provided that the bin width is adequate. The comparison of the cross section derived by Ishkhanov et al. with the present result shows some discrepancy in the average magnitude above the giant resonance peak. This may arise through differences between the shapes of the experimental and analysis bremsstrahlung spectra (Hicks 1972).

(b) Collective Model Interpretation

In Figure 3 the measured cross section is compared with the predictions of the DCM, which are represented by the full curve. This curve is an incoherent sum of the results of specific calculations by Arenhövel (1965) and Arenhövel et al. (1967) for the E1 component and Ligensa et al. (1966) for the E2 component of the $^{181}$Ta giant resonance. With the exception of the giant quadrupole width $\Gamma_Q$, all the parameters have been fixed by the dipole resonance and the low energy vibrational spectrum of neighbouring even–even nuclei. Whilst $\Gamma_Q$ is assumed constant for all E2 levels, the dipole width $\Gamma_D$ is allowed to vary with the excitation energy $E$ in the manner

$$\Gamma_D = \Gamma_0(E/E_0)^\delta,$$

where experimental trends seem to indicate that $\delta$ lies between 1·5 and 2·2 (Danos and Greiner 1965). The particular values of the above parameters used to calculate the photoabsorption cross section of $^{181}$Ta shown in Figure 3 were as follows.

<table>
<thead>
<tr>
<th>$\beta_0$</th>
<th>$E_\gamma$ (MeV)</th>
<th>$E_\gamma$ (MeV)</th>
<th>$E_0$ (MeV)</th>
<th>$\Gamma_0$ (MeV)</th>
<th>$\Gamma_0$ (MeV)</th>
<th>$\delta$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0·21</td>
<td>15·1</td>
<td>0·8</td>
<td>1·4</td>
<td>12·4</td>
<td>1·9</td>
<td>0·8</td>
<td>1·6</td>
</tr>
</tbody>
</table>

Despite some minor discrepancies the DCM description of the present experimental measurement proves to be extremely good. The structure observed in the range 20–26 MeV has been predicted by the extension of the DCM to consider the

* In their recent work Ishkhanov et al. (1971) admit using the Penfold–Leiss technique to analyse a smoothed $^{40}$Ca($\gamma$, n) yield curve, but no mention is made of any modification to the analysis resolution function.
giant quadrupole oscillations. The dashed curve beneath the structure in this region in Figure 3 represents an extrapolation of the predicted dipole resonance. If this is tentatively postulated to characterize the dipole strength underlying the quadrupole structure, it is possible to calculate the ratio of the E2 cross section, integrated (without energy weighting) between 20.7 and 26.5 MeV, and the integrated E1 cross section. With an upper limit of 28.8 MeV on the integration of the dipole component, this ratio is found to be 5±1%, which lies in reasonably good agreement with the theoretical non-energy-weighted value of about 7% (Ligena et al. 1966).

Although the predictions are generally very good, there is some structure present in the measured cross section for which the DCM does not account. For example, there are indications of a small peak at 11 MeV, lying in a region of excess strength above the extrapolated DCM dipole cross section. This excess strength is commonly observed when comparing experimental results with the predictions of the DCM and is attributed to single-particle excitations to which the model gives no consideration. Above the split dipole peak, further unpredicted structure is observed at 16.7, 18.7, and 28.0 MeV, all of which is confirmed by the Ishkhanov et al. (1969) measurement. A possible source of this additional structure is the coupling of the giant resonance oscillations with anharmonic components of the surface vibrations and harmonic components with multipolarity greater than quadrupole. These are neglected in the DCM.

An alternative explanation of the structure observed at 28 MeV is that it may represent a grouping of 3hw E1 transitions, i.e. it may be a component of the first E1 overtone. Although experimental evidence for the existence of the first dipole overtone is meagre, Danos (1961) has calculated its resonant energy and strength from the theoretical standpoint of the hydrodynamic model. The resonant energy of the (j−1)th dipole overtone $W_j$ of a spherical nucleus is given in terms of the resonant energy of the usual single quantum dipole peak $W_1$ by

$$W_j = (z_j/z_1)W_1,$$  \hspace{1cm} (10)

where $z_j$ is the jth pole in the hydrodynamic wave equation describing nuclear density oscillations. Furthermore, the fraction $F_j$ of the total dipole sum rule contained within the jth mode is given by

$$F_j = 2/(z_j^2 - 2).$$  \hspace{1cm} (11)

With the values $z_1 = 2.08$ and $z_2 = 5.95$ we see that the usual (j = 1) dipole resonance and its first overtone (j = 2) should account for 86% and 6% of the dipole sum respectively. Although equation (10) suggests that the first $^{181}$Ta overtone will lie at ~36 MeV, it is expected that its strength will be fragmented over a reasonably wide energy range because of the large nuclear deformation. If the peak observed at 28 MeV in the present measurement does indeed reflect the first dipole overtone, it will exhaust about 30% of the strength of this mode.

(c) Integrated Cross Section and Moments

The absolute value of the $^{181}$Ta photoneutron cross section, integrated to 28.8 MeV, has been determined by the present experiment to be 2.47±0.35 MeV b.
The first and second moments, given by

\[ \int_{0}^{0.08} (\sigma/E) \, dE \quad \text{and} \quad \int_{0}^{0.08} (\sigma/E^2) \, dE, \]

are 170 mb and 12.3 mb MeV\(^{-1}\) respectively.

V. Conclusions

In addition to the well-known splitting of the giant resonance, the \(^{181}\text{Ta}\) photon-neutron cross section presented here contains suggestions of further structure, mostly lying at energies above the dipole peak energy. With the exception of the work of Ishkhanov et al. (1969) this additional structure has not been indicated by any previous measurements. In spite of the fact that we cannot support the extensive fine fragmentation reported by Ishkhanov et al. below 17 MeV, their determinations of broader structure at higher energies are seen to be in excellent agreement with those obtained in the present measurement.

The observation of structure lying in close agreement with the predicted \(^{181}\text{Ta}\) giant quadrupole resonance constitutes much more convincing evidence for the validity of the DCM description of heavy deformed nuclei than does the solitary observation of the split dipole peak. The E2 assignment to this structure coinciding with the predicted giant quadrupole resonance is supported by the asymmetric angular distributions of nucleons emitted in photoreactions in \(^{181}\text{Ta}\).

Notwithstanding the good description provided by the DCM for the present experimental results, it does not account for all of the observed structure. In particular we have noted excess strength near the threshold which may be attributed to single-particle excitations, ignored in the DCM description. The nature of additional unexplained strength at 28 MeV is uncertain, but it may reflect a grouping of three-quantum dipole transitions. Further unexplained structure may lie at 16.7 and 18.7 MeV. A likely reason for the DCM's failure to account for this structure is its neglect of all but the harmonic quadrupole component of the surface vibrations.

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VII. References

PHOTONEUTRON CROSS SECTION OF $^{181}\text{Ta}$
