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Photoproduction of Pion Pairs at Moderate Energies

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Abstract

The problem of the production of pion pairs in photon–nucleon collisions is studied with the help of a new Lagrangian model. Using a relativistic covariant and gauge-invariant scattering amplitude based on a phenomenological Lagrangian, the differential and total cross sections are calculated for unpolarized and linearly polarized photons. The production amplitude incorporates terms involving $\varepsilon(700)$, $\rho(770)$, f(1260) and $\Delta(1236)$ resonances in addition to the nucleon and pion exchanges.

Introduction

A study of photoproduction of pion pairs from nucleons throws much light on the pion–nucleon system, and its importance in this respect is only next to the study of single pion photoproduction. Much theoretical and experimental work has been carried out on this process (Davier *et al.* 1970; Luke and Söding 1971; Luke 1972). Since the theory of multiparticle production presents great difficulties both kinematically and dynamically, it is necessary to use various approximations even at low photon energies.

In the energy range below 700 MeV, Cutkosky and Zachariasen (1956) calculated the cross sections for the process

$$\gamma \mathrm{p} \to \pi^+ \pi^- \mathrm{p} \tag{1}$$

by means of a static model developed by Chew and Low (1956). In a series of papers by one of the present authors (Srinivasan and Venkatesan 1959*a*, 1959*b*, 1962, 1963), a detailed study of multiple pion production was made in the sub-GeV region and it was found that, while a cutoff theory by itself explains photoproduction of pion pairs up to 600 MeV, it is necessary to introduce, in a meaningful way, the pion–pion interaction for higher energies.

At photon energies below the threshold for ρ-meson production, the reaction (1) is dominated by the process

$$\gamma \mathrm{p} \to \Delta^{++} \pi^- , \tag{2}$$

where Δ^{++} denotes the well-known πN resonance $\Delta(1236)$. But at high energies, the process (1) is known (CBCG 1966; Pumblin 1970; Park 1971) to be completely dominated by the diffractive production of the ρ meson. Hence reaction (1) is particularly suitable for testing the basic ideas of diffraction dissociation as well.

It is the aim of the present paper to study the reaction (1) using a suitable Lagrangian model. Our Lagrangian includes terms involving $\varepsilon(700)$, $\rho(770)$, f(1260) and $\Delta(1236)$ resonances, in addition to the nucleon and pion exchanges. The details of the calculation are given in the subsequent sections.

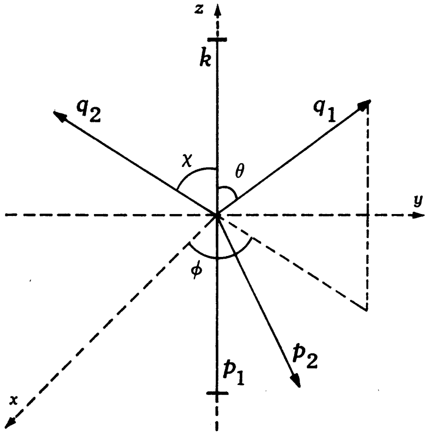

Fig. 1. Definition of momenta and scattering angles in the c.m. system of the incident photon and target proton.

Kinematics

We deal with the process

$$\gamma(k) + \mathrm{p}(p_1) \rightarrow \mathrm{p}(p_2) + \pi^+(q_1) + \pi^-(q_2), \tag{3}$$

where the quantities in parentheses denote the four-momenta of the corresponding particles. The scattering angles θ, ϕ and χ are defined in Fig. 1. The calculations are done in the c.m. system of the incident photon and target proton, i.e. where

$$\boldsymbol{p}_1 + \boldsymbol{k} = \boldsymbol{p}_2 + \boldsymbol{q}_1 + \boldsymbol{q}_2 = 0. \tag{4}$$

The total c.m. energy W $(=\sqrt{s})$ and the laboratory energy E_γ of the photon are related by

$$W^2 = 2mE_\gamma + m^2. \tag{5}$$

Let K, Q_1 and Q_2 be the magnitudes of the three-momenta $\boldsymbol{k}$, $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ respectively. By choosing $\boldsymbol{q}_2$ to lie in the xz plane, we have the relations:

$$q_{1x} = Q_1 \sin\theta \cos\phi, \quad q_{1y} = Q_1 \sin\theta \sin\phi, \quad q_{1z} = Q_1 \cos\theta, \tag{6a}$$

$$q_{2x} = Q_2 \sin\chi, \quad q_{2y} = 0, \quad q_{2z} = Q_2 \cos\chi. \tag{6b}$$

Also, the energies of the initial and final nucleons are given respectively by

$$E_1 = (K^2 + m^2)^{\frac{1}{2}}, \qquad E_2 = (Q_1^2 + Q_2^2 + m^2 + Q_1\delta)^{\frac{1}{2}}, \tag{7}$$

where m is the mass of the nucleon and

$$\delta = 2Q_2(\sin\theta \cos\phi \sin\chi + \cos\theta \cos\chi). \tag{8}$$

The maximal and minimal energies of the outgoing π^- are given by

$$\omega_2^{\max} = \{s-(m+\mu_\pi)^2+\mu_\pi^2\}/2W\,, \qquad \omega_2^{\min} = 1\,, \tag{9}$$

μ_π being the mass of the pion (taken to be unity in the actual calculations below). Once ω_2 is known, Q_2 can be computed from

$$Q_2 = (\omega_2^2-1)^{\frac{1}{2}}. \tag{10}$$

The energy conservation requires

$$E_1 + K = E_2 + \omega_1 + \omega_2\,, \tag{11}$$

that is,

$$(K^2+m^2)^{\frac{1}{2}}+K = \{(\boldsymbol{q}_1+\boldsymbol{q}_2)^2+m^2\}^{\frac{1}{2}}+(Q_1^2+1)^{\frac{1}{2}}+(Q_2^2+1)^{\frac{1}{2}}. \tag{12}$$

Since E_1, K and ω_2 are already known quantities, we define $X = E_1 + K - \omega_2$. Also defining $Y = X^2 + 1 - Q_2^2 - m^2$, after some algebra, we arrive at the following expression for Q_1:

$$Q_1 = \{Y\pm 2X(\delta^2+Y^2-4X^2)^{\frac{1}{2}}\}/(\delta^2-4X^2). \tag{13}$$

In this equation it is found that one of the roots is always negative for all combinations of angles. Since Q_1 is the magnitude of the three-momentum of the outgoing π^+, this solution can be easily discarded thus giving a unique value of Q_1 from equation (13). Once Q_1 is determined, ω_1 can be obtained from the relation

$$\omega_1 = (Q_1^2+\mu_\pi^2)^{\frac{1}{2}}. \tag{14}$$

Finally, the expression for the total cross section σ for unpolarized photons and unpolarized nucleon targets can be written as

$$\sigma = \frac{m^2}{4(2\pi)^5(s-m^2)}\int Q_1^3\,\mathrm{d}\Omega_{q_1}\int_1^{\omega_2^{\max}} Q_2\,\mathrm{d}\omega_2\int \mathrm{d}\Omega_{q_2}$$

$$\times\{Q_1^2(W-\omega_2)+\omega_1\,\boldsymbol{q}_1\cdot\boldsymbol{q}_2\}^{-1}\tfrac{1}{4}\sum_{s_1,s_2,\lambda=1,2}|M_{\mathrm{fi}}|^2. \tag{15}$$

Here

$$\mathrm{d}\Omega_{q_1} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,, \qquad \mathrm{d}\Omega_{q_2} = 2\pi\sin\chi\,\mathrm{d}\chi\,, \tag{16}$$

s_1 and s_2 are the polarizations of the initial and final protons respectively and λ represents the photon polarization. The Lorentz-invariant matrix elements M_{fi} are given in the next section.

Matrix Elements

Lorentz invariance and gauge invariance are taken to be the primary criteria for writing down the matrix elements for the electromagnetic process here. Also the electromagnetic interaction is introduced in a minimal way and considered only up to first order. The interaction Lagrangians used in calculating the various vertices are (Pfeil 1968; Pilkuhn *et al.* 1973):

πNN, $\quad \mathrm{i}g_{\pi\mathrm{NN}}\bar{\psi}\gamma_5\boldsymbol{\tau}\psi\boldsymbol{\phi}$; (17a)

$\varepsilon\pi\pi$, $\quad \tfrac{1}{2}g_{\varepsilon\pi\pi}m_\varepsilon\boldsymbol{\phi}\cdot\boldsymbol{\phi}\varepsilon$; (17b)

εNN, $\quad g_{\varepsilon\mathrm{NN}}\bar{\psi}\psi\varepsilon$; (17c)

$\rho\pi\pi$, $\quad g_{\rho\pi\pi}\boldsymbol{\rho}_\mu(\boldsymbol{\phi}\times\partial^\mu\boldsymbol{\phi})$; (17d)

ρNN, $\quad \tfrac{1}{2}g^{(1)}_{\rho\mathrm{NN}}\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi\boldsymbol{\rho}^\mu-\tfrac{1}{8}m^{-1}g^{(2)}_{\rho\mathrm{NN}}\bar{\psi}\sigma_{\mu\nu}\boldsymbol{\tau}\psi(\partial^\mu\boldsymbol{\rho}^\nu-\partial^\nu\boldsymbol{\rho}^\mu)$; (17e)

f$\pi\pi$, $\quad 2m_\mathrm{f}^{-1}g_{\mathrm{f}\pi\pi}\partial_\mu\boldsymbol{\phi}\,\partial_\nu\boldsymbol{\phi}f^{\mu\nu}$; (17f)

fNN, $\quad 2\mathrm{i}m^{-1}g^{(1)}_{\mathrm{fNN}}\bar{\psi}(\gamma_\mu\partial_\nu+\gamma_\nu\partial_\mu)\psi f^{\mu\nu}+4m^{-2}g^{(2)}_{\mathrm{fNN}}\partial_\mu\bar{\psi}\,\partial_\nu\psi f^{\mu\nu}$; (17g)

ΔNπ, $\quad \mathrm{i}\mu_\pi^{-1}g_{\Delta\mathrm{N}\pi}\bar{\boldsymbol{\Delta}}_\mu\psi\,\partial^\mu\boldsymbol{\phi}$; (17h)

γNN, $\quad -e\bar{\psi}\{\tfrac{1}{2}\gamma_\mu(1+\tau_3)A^\mu-\tfrac{1}{4}\mathrm{i}m^{-1}(\chi_\mathrm{s}+\tau_3\chi_\mathrm{v})\sigma_{\mu\nu}F^{\mu\nu}\}\psi$; (17i)

$\gamma\pi\pi$, $\quad \mathrm{i}e(\boldsymbol{\phi}^\dagger\partial_\mu\boldsymbol{\phi}-\partial_\mu\boldsymbol{\phi}^\dagger\boldsymbol{\phi})A^\mu$. (17j)

In these expressions the g's denote the coupling constants; ψ is the nucleon field, of isospin τ; ϕ is the pion field; ε, ρ_μ, $f^{\mu\nu}$ and Δ_μ are the fields of the corresponding resonances; A^μ and $F^{\mu\nu}$ are the electromagnetic potential and field strength respectively; χ_s and χ_v are the scalar and vector parts of the anomalous magnetic moments of the nucleon; and the γ_μ matrices are given by (we use the metric $a\cdot b = a_0 b_0 - \boldsymbol{a}\cdot\boldsymbol{b}$)

$$\gamma_0=\begin{bmatrix}\mathbf{I} & 0\\ 0 & \mathbf{I}\end{bmatrix},\qquad \gamma_k=\begin{bmatrix}0 & \sigma_k\\ -\sigma_k & 0\end{bmatrix},\qquad \gamma_5=\begin{bmatrix}0 & \mathbf{I}\\ \mathbf{I} & 0\end{bmatrix},$$

where $\mathbf{I}$ is the unit matrix and the σ_k are the well-known Pauli matrices. Also we have $\sigma_{\mu\nu}=\tfrac{1}{2}\mathrm{i}[\gamma_\mu,\gamma_\nu]$.

The forms (17) for the interaction Lagrangians were used to obtain suitable combinations for the matrix elements corresponding to the different particles involved and the associated Feynmann diagrams, as shown in Figs 2–6. The resulting expressions are considered in detail below.

ε-meson Contribution (*Fig.* 2)

The matrix element for the ε meson is

$$\begin{aligned}M^\varepsilon_{\mathrm{fi}} &= em_\varepsilon g_{\varepsilon\pi\pi}g_{\varepsilon\mathrm{NN}}\bar{u}(p_2)\frac{1}{(q_1+q_2)^2-m_\varepsilon^2+\mathrm{i}m_\varepsilon\Gamma_\varepsilon}\,\frac{\gamma\cdot p_1+\gamma\cdot k+m}{(P_1+k)^2-m^2}\varepsilon^\mu C^\mathrm{p}_\mu u(p_1)\\ &+em_\varepsilon g_{\varepsilon\pi\pi}g_{\varepsilon\mathrm{NN}}\bar{u}(p_2)\frac{1}{(q_1+q_2)^2-m_\varepsilon^2+\mathrm{i}m_\varepsilon\Gamma_\varepsilon}\varepsilon^\mu C^\mathrm{p}_\mu\,\frac{\gamma\cdot p_2-\gamma\cdot k+m}{(p_2-k)^2-m^2}u(p_1)\\ &+em_\varepsilon g_{\varepsilon\pi\pi}g_{\varepsilon\mathrm{NN}}\bar{u}(p_2)\frac{\varepsilon\cdot(2q_1-k)}{(q_1-k)^2-\mu_\pi^2}\,\frac{1}{(p_1-p_2)^2-m_\varepsilon^2+\mathrm{i}m_\varepsilon\Gamma_\varepsilon}u(p_1)\\ &-em_\varepsilon g_{\varepsilon\pi\pi}g_{\varepsilon\mathrm{NN}}\bar{u}(p_2)\frac{\varepsilon\cdot(2q_2-k)}{(q_2-k)^2-\mu_\pi^2}\,\frac{1}{(p_1-p_2)^2-m_\varepsilon^2+\mathrm{i}m_\varepsilon\Gamma_\varepsilon}u(p_1),\end{aligned}\qquad (18)$$

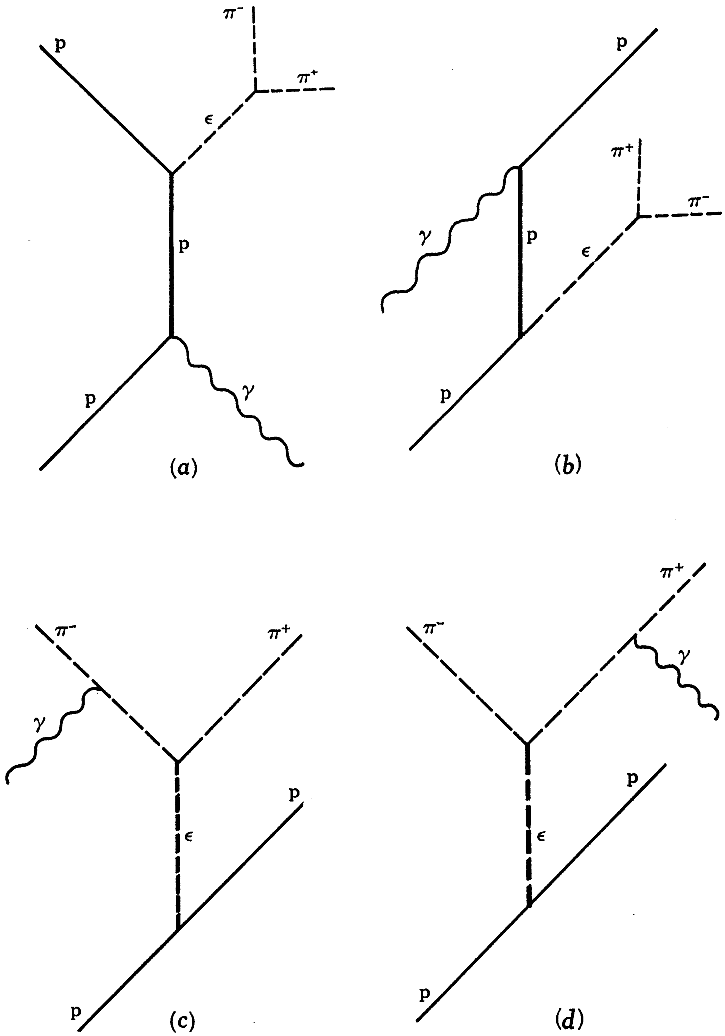

Fig. 2. Feynmann diagrams for ε-meson exchange.

where ε^{μ} is the polarization four vector of the photon, Γ_{ε} represents the width of the ε meson and C^{p}_{μ} describes the electromagnetic–proton vertex:

$$C^{\mathrm{p}}_{\mu} = \gamma_{\mu} - \tfrac{1}{2} i m^{-1} \sigma_{\mu\nu} k^{\nu} \chi_{\mathrm{p}}, \qquad \text{with} \qquad \chi_{\mathrm{p}} = 1\cdot78 .$$

The first two terms and the last two terms in equation (18) form gauge-invariant combinations.

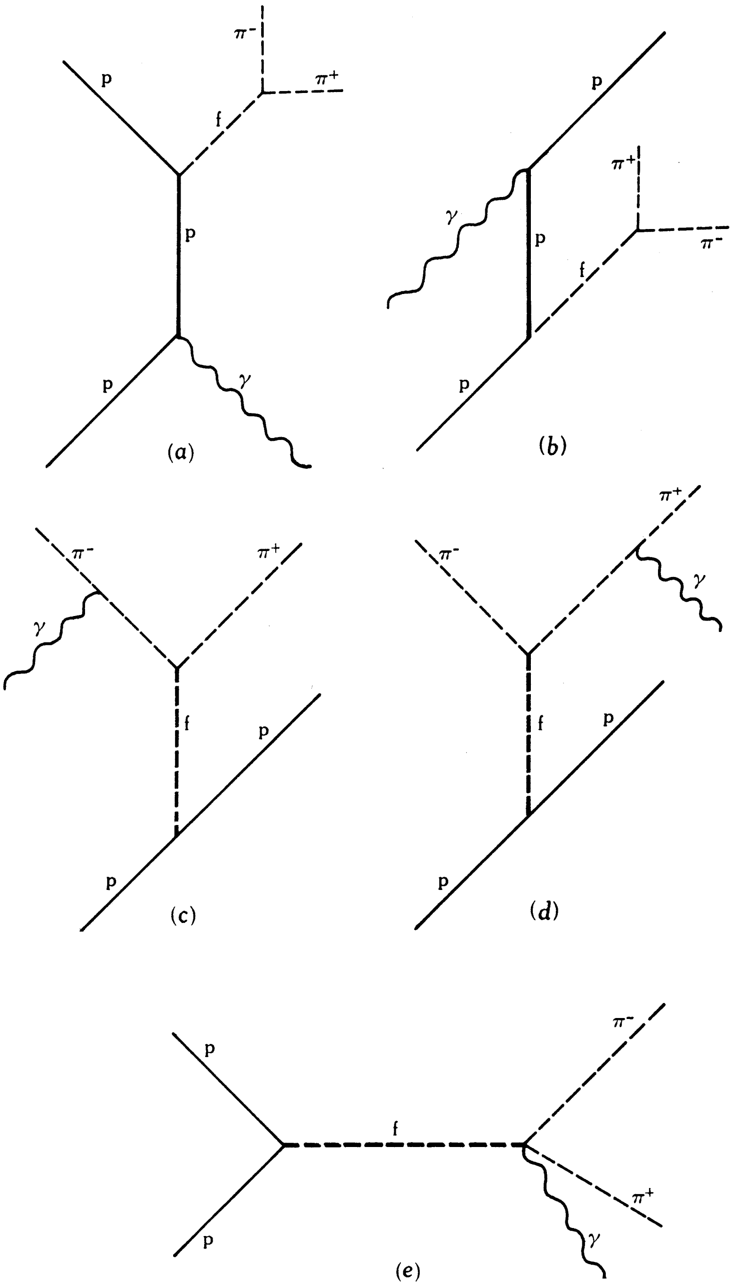

Fig. 3. Feynmann diagrams for f-meson exchange.

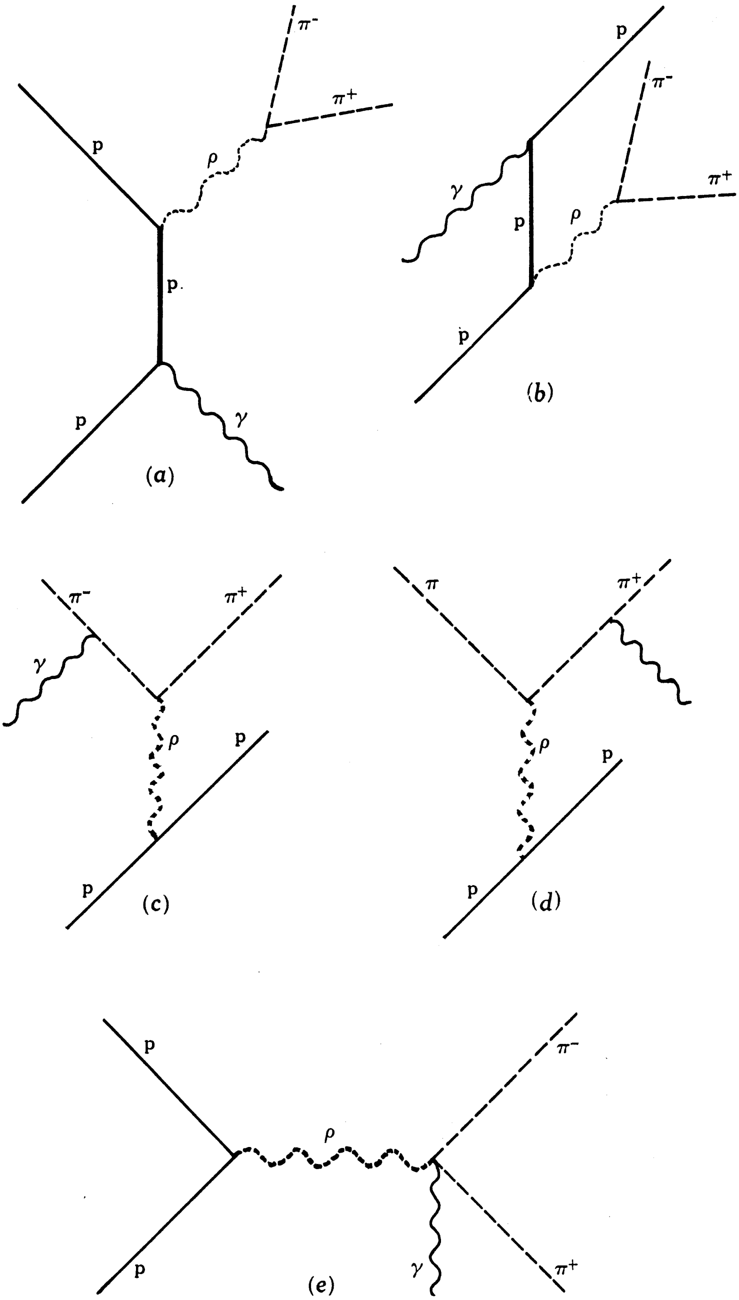

Fig. 4. Feynmann diagrams for ρ-meson exchange.

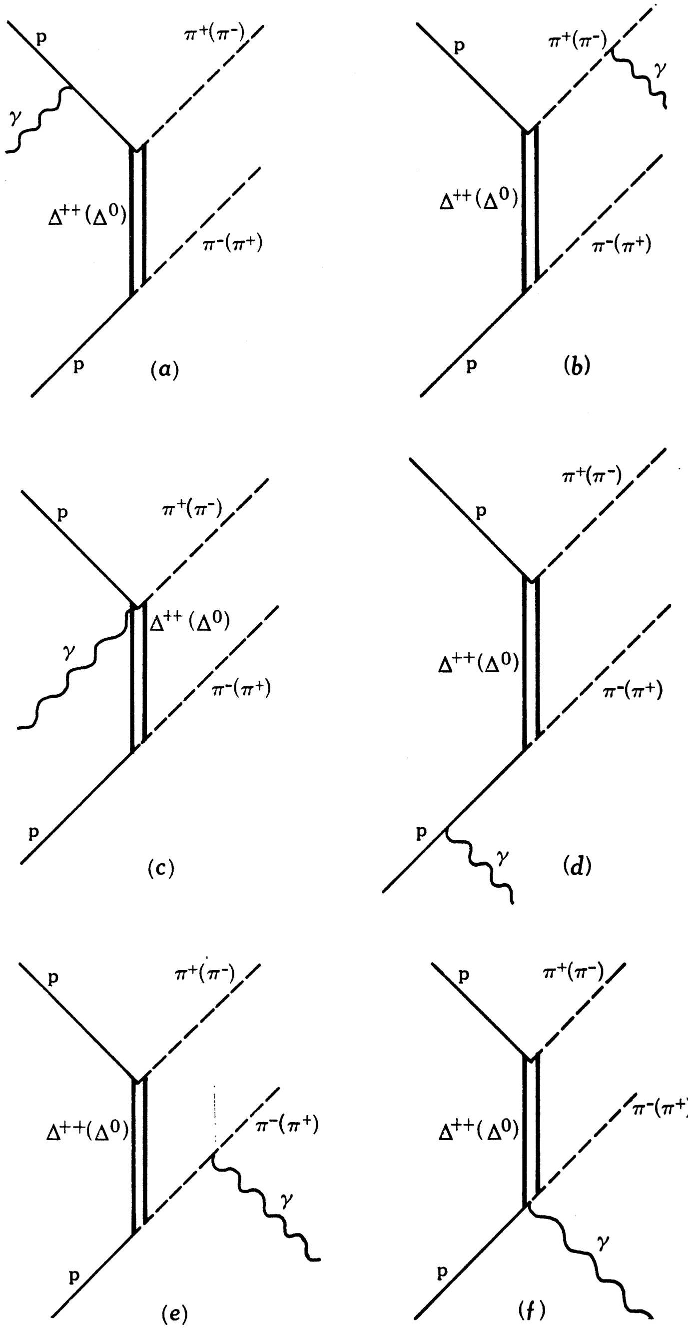

Fig. 5. Feynmann diagrams for Δ-resonance exchange.

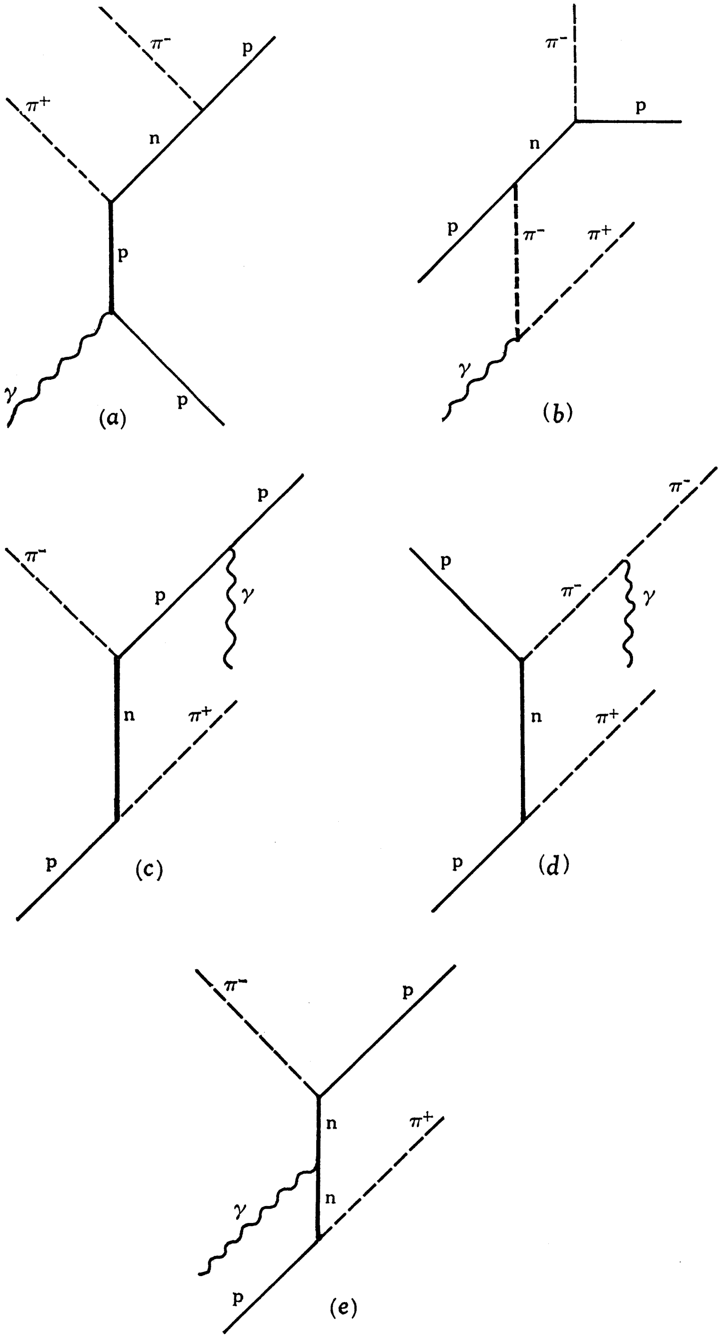

Fig. 6. Feynman diagrams for nucleon and pion exchange.

f-meson Contribution (*Fig.* 3)

The matrix element for the f meson is

$$M_{\mathrm{fi}}^{\mathrm{f}} = \frac{e g_{\mathrm{f}\pi\pi} g_{\mathrm{fNN}}^{(1)}}{m_{\mathrm{f}} m} \bar{u}(p_2)(q_1-q_2)^\mu (q_1-q_2)^\nu \frac{P_{\mu\nu\rho\sigma}(q_1+q_2)}{(q_1+q_2)^2 - m_{\mathrm{f}}^2 + \mathrm{i} m_{\mathrm{f}} \Gamma_{\mathrm{f}}}$$

$$\times \Big((q_1+q_2)^\rho \gamma^\sigma + (q_1+q_2)^\sigma \gamma^\rho\Big) \frac{\gamma . p_1 + \gamma . k + m}{(p_1+k)^2 - m^2} \varepsilon^\mu C_\mu^{\mathrm{p}} u(p_1)$$

$$+ \frac{e g_{\mathrm{f}\pi\pi} g_{\mathrm{fNN}}^{(1)}}{m_{\mathrm{f}} m} \bar{u}(p_2)(q_1-q_2)^\mu (q_1-q_2)^\nu \frac{P_{\mu\nu\rho\sigma}(q_1+q_2)}{(q_1+q_2)^2 - m_{\mathrm{f}}^2 + \mathrm{i} m_{\mathrm{f}} \Gamma_{\mathrm{f}}}$$

$$\times \frac{\gamma . p_2 - \gamma . k + m}{(p_2-k)^2 - m^2} \varepsilon^\mu C_\mu^{\mathrm{p}} \Big((q_1+q_2)^\rho \gamma^\sigma + (q_1+q_2)^\sigma \gamma^\rho\Big) u(p_1)$$

$$+ \frac{e g_{\mathrm{f}\pi\pi} g_{\mathrm{fNN}}^{(1)}}{m_{\mathrm{f}} m} (q_1-k-q_2)^\mu (q_1-k-q_2)^\nu \frac{P_{\mu\nu\rho\sigma}(p_1-p_2)}{(p_1-p_2)^2 - m_{\mathrm{f}}^2 + \mathrm{i} m_{\mathrm{f}} \Gamma_{\mathrm{f}}}$$

$$\times \Big((p_1-p_2)^\rho \gamma^\sigma + (p_1-p_2)^\sigma \gamma^\rho\Big) \frac{\varepsilon . (2q_1 - k)}{(q_1-k)^2 - \mu_\pi^2} u(p_1)$$

$$+ \frac{e g_{\mathrm{f}\pi\pi} g_{\mathrm{fNN}}^{(1)}}{m_{\mathrm{f}} m} (q_2-k-q_1)^\mu (q_2-k-q_1)^\nu \frac{P_{\mu\nu\rho\sigma}(p_1-p_2)}{(p_1-p_2)^2 - m_{\mathrm{f}}^2 + \mathrm{i} m_{\mathrm{f}} \Gamma_{\mathrm{f}}}$$

$$\times \Big((p_1-p_2)^\rho \gamma^\sigma + (p_1-p_2)^\sigma \gamma^\rho\Big) \frac{\varepsilon . (2q_2 - k)}{(q_2-k)^2 - \mu_\pi^2} u(p_1), \tag{19}$$

where the numerator $P_{\mu\nu\rho\sigma}$ of the f meson propagator is given by

$$P_{\mu\nu\rho\sigma}(p) = \tfrac{1}{2}(g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho} - \tfrac{2}{3} g_{\mu\nu} g_{\rho\sigma})$$

$$- \tfrac{1}{2} m_{\mathrm{f}}^{-2} (g_{\sigma\nu} p_\mu p_\rho + g_{\nu\rho} p_\mu p_\sigma + g_{\sigma\mu} p_\rho p_\nu + g_{\mu\rho} p_\nu p_\sigma)$$

$$+ \tfrac{1}{3} m_{\mathrm{f}}^{-2} (g_{\mu\nu} p_\rho p_\sigma + g_{\rho\sigma} p_\mu p_\nu) + \tfrac{2}{3} m_{\mathrm{f}}^{-4} p_\mu p_\nu p_\rho p_\sigma .$$

The first two terms in equation (19) correspond to Figs 3*a* and 3*b*, and they form a gauge-invariant combination; it is found that the contributions from these two diagrams cancel. In order to make the last two terms gauge invariant it is necessary to add a contact term (Fig. 3*e*), given by

$$\frac{2 e g_{\mathrm{f}\pi\pi} g_{\mathrm{fNN}}^{(1)}}{m_{\mathrm{f}} m} \Big(\varepsilon^\mu (q_1-q_2)^\nu + \varepsilon^\nu (q_1-q_2)^\mu\Big)$$

$$\times \frac{P_{\mu\nu\rho\sigma}(p_1-p_2)}{(p_1-p_2)^2 - m_{\mathrm{f}}^2 + \mathrm{i} m_{\mathrm{f}} \Gamma_{\mathrm{f}}} \Big((p_1-p_2)_\rho \gamma_\sigma + (p_1-p_2)_\sigma \gamma_\rho\Big) u(p_1).$$

In the actual calculation we make the reasonable approximation $g_{\mathrm{fNN}}^{(2)} = 0$ following the work of Achuthan *et al.* (1970, 1971).

Table 1. Calculated total cross sections for linearly polarized photons

The results give the total cross sections σ_x, σ_y for each contributing resonance when the photon polarization is in the x, y direction

Cross section	σ (μb) for E_γ values (MeV) $E_\gamma = 376$	504	648	808	986	1182	1395
$\sigma_x(\varepsilon)$	$2\cdot9\times10^{-3}$	0·0456	0·122	0·212	0·318	0·436	0·554
$\sigma_y(\varepsilon)$	$6\cdot4\times10^{-4}$	0·0114	0·038	0·082	0·156	0·256	0·360
σ_x(f)	$1\cdot68\times10^{-5}$	0·0009	0·0072	0·032	0·10	0·26	0·56
σ_y(f)	$4\cdot6\times10^{-6}$	0·0003	0·0026	0·012	0·042	0·10	0·26
$\sigma_x(\rho)$	$1\cdot6\times10^{-5}$	0·0026	0·027	0·122	0·38	0·94	2
$\sigma_y(\rho)$	$7\cdot0\times10^{-6}$	0·0011	0·012	0·052	0·18	0·41	0·98
σ_x(N)[A]	$2\cdot7\times10^{-2}$	0·42	1·12	1·8	2·4	2·8	3·2
σ_y(N)[A]	$6\cdot4\times10^{-3}$	0·09	0·25	0·406	0·532	0·626	0·68
$\sigma_x(\Delta^{++})$	$8\cdot1\times10^{-1}$	10·2	31·2				
$\sigma_y(\Delta^{++})$	$3\cdot8\times10^{-1}$	7·3	23·6				

[A] The nucleon exchange contributions were calculated on the electric Born term model.

ρ-meson Contribution (*Fig. 4*)

The matrix element for the ρ meson is

$$
\begin{aligned}
M_{\mathrm{fi}}^{\rho} = {} & e g_{\rho\pi\pi}\bar{u}(p_2)(q_1-q_2)_\mu \frac{P^{\mu\nu}(q_1+q_2)}{(q_1+q_2)^2-m_\rho^2+\mathrm{i}m_\rho\Gamma_\rho} \\
& \times\Big(g_{\rho\mathrm{NN}}^{(1)}\gamma_\nu+\tfrac{1}{2}m^{-1}g_{\rho\mathrm{NN}}^{(2)}\gamma_\nu\gamma_\sigma(q_1+q_2)^\sigma\Big)\frac{\gamma.p_1+\gamma.k+m}{(p_1+k)^2-m^2}\varepsilon^\mu C_\mu^{\mathrm{p}}u(p_1) \\
& + e g_{\rho\pi\pi}\bar{u}(p_2)\varepsilon^\mu C_\mu^{\mathrm{p}}\frac{\gamma.p_2-\gamma.k+m}{(p_2-k)^2-m^2}\Big(g_{\rho\mathrm{NN}}^{(1)}\gamma_\mu+\tfrac{1}{2}m^{-1}g_{\rho\mathrm{NN}}^{(2)}\gamma_\mu\gamma_\sigma(q_1+q_2)^\sigma\Big) \\
& \times\frac{P^{\mu\nu}(q_1+q_2)}{(q_1+q_2)^2-m_\rho^2+\mathrm{i}m_\rho\Gamma_\rho}(q_1-q_2)_\nu u(p_1) \\
& + e g_{\rho\pi\pi}\bar{u}(p_2)\frac{\varepsilon.(2q_1-k)}{(q_1-k)^2-\mu_\pi^2}(q_1-q_2-k)_\mu \\
& \times\frac{P^{\mu\nu}(p_1-p_2)}{(p_1-p_2)^2-m_\rho^2+\mathrm{i}m_\rho\Gamma_\rho}\Big(g_{\rho\mathrm{NN}}^{(1)}\gamma_\nu+\tfrac{1}{2}m^{-1}g_{\rho\mathrm{NN}}^{(2)}\gamma_\nu\gamma_\sigma(p_1-p_2)^\sigma\Big)u(p_1) \\
& - e g_{\rho\pi\pi}\bar{u}(p_2)\frac{\varepsilon.(2q_2-k)}{(q_2-k)^2-\mu_\pi^2}(q_1-q_2+k)_\mu \\
& \times\frac{P^{\mu\nu}(p_1-p_2)}{(p_1-p_2)^2-m_\rho^2+\mathrm{i}m_\rho\Gamma_\rho}\Big(g_{\rho\mathrm{NN}}^{(1)}\gamma_\nu+\tfrac{1}{2}m^{-1}g_{\rho\mathrm{NN}}^{(2)}\gamma_\nu\gamma_\sigma(p_1-p_2)^\sigma\Big)u(p_1) \\
& - 2e g_{\rho\pi\pi}\varepsilon_\mu\frac{P^{\mu\nu}(p_1-p_2)}{(p_1-p_2)^2-m_\rho^2+\mathrm{i}m_\rho\Gamma_\rho}\Big(g_{\rho\mathrm{NN}}^{(1)}\gamma_\nu+\tfrac{1}{2}m^{-1}g_{\rho\mathrm{NN}}^{(2)}\gamma_\nu\gamma_\sigma(p_1-p_2)^\sigma\Big)u(p_1).
\end{aligned}
\tag{20}
$$

As in the case of the f meson, the last term in equation (20) corresponds to the contact diagram (Fig. 4*e*) introduced in order to fulfil the gauge-invariance requirement.

Table 2. Calculated differential cross sections for linearly polarized photons
The results are for the Δ^{++} resonance at $E_\gamma = 504$ MeV

Cross section	$d\sigma/d\chi$ (μb) at angles χ (degrees) $\chi = 0$	30	60	90	120	150	180
$d\sigma_x/d\chi$	0	2·7	4·8	5·7	4·4	2·6	0
$d\sigma_y/d\chi$	0	1·8	3·6	4·8	2·5	1	0

Δ^{++}*-resonance Contribution* (*Fig.* 5)

The matrix element for the Δ^{++} resonance is (Pittner and Urban 1973)

$$
\begin{aligned}
M_{\mathrm{fi}}^{\Delta^{++}} &= e g_{\Delta^{++}\mathrm{p}\pi}^2\, \bar{u}(p_2)\,\varepsilon^\mu C_\mu^{\mathrm{p}} \frac{\gamma.p_2-\gamma.k+m}{(p_2-k)^2-m^2} q_{1\rho} \\
&\quad \times \frac{\gamma.p_1-\gamma.q_2+M}{(p_1-q_2)^2-M^2+\mathrm{i}M\Gamma_\Delta} \Lambda^{\rho\sigma}(p_1-q_2)\, q_{2\sigma} u(p_1) \\
&\quad - e g_{\Delta^{++}\mathrm{p}\pi}^2\, \bar{u}(p_2) \frac{\varepsilon.(2q_1-k)}{(q_1-k)^2-\mu_\pi^2} (q_1-k)_\rho \\
&\quad \times \frac{\gamma.p_1-\gamma.q_2+M}{(p_1-q_2)^2-M^2+\mathrm{i}M\Gamma_\Delta} \Lambda^{\rho\sigma}(p_1-q_2)\, q_{2\sigma} u(p_1) \\
&\quad + e g_{\Delta^{++}\mathrm{p}\pi}^2\, \bar{u}(p_2)\, \varepsilon_\rho \frac{\gamma.p_1-\gamma.q_2+M}{(p_1-q_2)^2-M^2+\mathrm{i}M\Gamma_\Delta} \Lambda^{\rho\sigma}(p_1-q_2)\, q_{2\sigma} u(p_1) \\
&\quad + e g_{\Delta^{++}\mathrm{p}\pi}^2\, \bar{u}(p_2)\, q_{1\rho} \frac{\gamma.p_2+\gamma.q_1+M}{(p_2+q_1)^2-M^2+\mathrm{i}M\Gamma_\Delta} \Lambda^{\rho\sigma}(p_2+q_1)\, q_{2\sigma} \\
&\quad \times \frac{\gamma.p_1+\gamma.k+m}{(p_1+k)^2-m^2} \varepsilon^\mu C_\mu^{\mathrm{p}} u(p_1) \\
&\quad + e g_{\Delta^{++}\mathrm{p}\pi}^2\, \bar{u}(p_2)\, q_{1\rho} \frac{\gamma.p_2+\gamma.q_1+M}{(p_2+q_1)^2-M^2+\mathrm{i}M\Gamma_\Delta} \Lambda^{\rho\sigma}(p_2+q_1)(q_2-k)_\sigma \\
&\quad \times \frac{\varepsilon.(2q_2-k)}{(q_2-k)^2-\mu_\pi^2} u(p_1) \\
&\quad - e g_{\Delta^{++}\mathrm{p}\pi}^2\, \bar{u}(p_2)\, q_{1\rho} \frac{\gamma.p_2+\gamma.q_1+M}{(p_2+q_1)^2-M^2+\mathrm{i}M\Gamma_\Delta} \Lambda^{\rho\sigma}(p_2+q_1)\, \varepsilon_\sigma u(p_1),
\end{aligned}
\tag{21}
$$

where M is the mass of the Δ resonance and $\Lambda^{\rho\sigma}$ is given by

$$\Lambda^{\rho\sigma}(p) = (\gamma.p+M)\{-g^{\rho\sigma} + \tfrac{1}{3}\gamma^\rho\gamma^\sigma + \tfrac{2}{3}M^{-2}p^\rho p^\sigma + \tfrac{1}{3}M^{-1}(\gamma^\rho p^\sigma - \gamma^\sigma p^\rho)\}.$$

In equation (21) the first three terms and the last three terms form gauge-invariant combinations. The same expression is valid for the Δ^0 resonance with the replacements $q_1 \leftrightarrow q_2$, corresponding to $\pi^+ \leftrightarrow \pi^-$.

Nucleon Contribution (*Fig. 6*)

Finally, from the nucleon and pion exchange diagrams this matrix element is

$$\begin{aligned} M_{\mathrm{fi}}^{\mathrm{N}} &= 2eg_{\pi\mathrm{NN}}^{2}\,\bar{u}(p_2)\,\gamma_5\frac{\gamma.p_2+\gamma.q_2+m}{(p_2+q_2)^2-m^2}\gamma_5\frac{\gamma.p_1+\gamma.k+m}{(p_1+k)^2-m^2}\varepsilon^{\mu}\,C_{\mu}^{\mathrm{p}}\,u(p_1) \\ &+2eg_{\pi\mathrm{NN}}^{2}\,\bar{u}(p_2)\,\gamma_5\frac{\gamma.p_2+\gamma.q_2+m}{(p_2+q_2)^2-m^2}\gamma_5\frac{\varepsilon.(2q_1-k)}{(q_1-k)^2-\mu_{\pi}^2}u(p_1) \\ &+2eg_{\pi\mathrm{NN}}^{2}\,\bar{u}(p_2)\,\varepsilon^{\mu}\,C_{\mu}^{\mathrm{p}}\frac{\gamma.p_2-\gamma.k+m}{(p_2-k)^2-m^2}\gamma_5\frac{\gamma.p_1-\gamma.q_1+m}{(p_1-q_1)^2-m^2}\gamma_5\,u(p_1) \\ &-2eg_{\pi\mathrm{NN}}^{2}\,\bar{u}(p_2)\frac{\varepsilon.(2q_2-k)}{(q_2-k)^2-\mu_{\pi}^2}\gamma_5\frac{\gamma.p_1-\gamma.q_1+m}{(p_1-q_1)^2-m^2}\gamma_5\,u(p_1) \\ &+2eg_{\pi\mathrm{NN}}^{2}\,\bar{u}(p_2)\,\gamma_5\frac{\gamma.p_2+\gamma.q_2+m}{(p_2+q_2)^2-m^2}\varepsilon^{\mu}\,C_{\mu}^{\mathrm{n}}\frac{\gamma.p_1-\gamma.q_1+m}{(p_1-q_1)^2-m^2}\gamma_5\,u(p_1), \end{aligned} \tag{22}$$

where

$$C_{\mu}^{\mathrm{n}} = -\tfrac{1}{2}im^{-1}\sigma_{\mu\nu}k^{\mu}\chi_{\mathrm{n}}, \qquad \text{with} \qquad \chi_{\mathrm{n}} = -1\cdot91.$$

Masses, Widths and Coupling Constants

The values (in MeV) of the masses and widths used in the calculations were (Particle Data Group 1974)

$$m_{\varepsilon} = 700, \quad m_{\mathrm{f}} = 1260, \quad m_{\rho} = 770, \quad M = 1236, \quad m = 938,$$
$$\Gamma_{\varepsilon} = 400, \quad \Gamma_{\mathrm{f}} = 170, \quad \Gamma_{\rho} = 150, \quad \Gamma_{\Delta} = 120,$$

while the coupling constants were (Pilkuhn *et al.* 1973)

$$e^2/4\pi = 1/137, \qquad g_{\pi\mathrm{NN}}^2/4\pi = 14\cdot5,$$
$$g_{\varepsilon\pi\pi}g_{\varepsilon\mathrm{NN}}/4\pi = 2, \qquad g_{\mathrm{f}\pi\pi}g_{\mathrm{fNN}}^{(1)}/4\pi = 3\cdot0,$$
$$g_{\rho\pi\pi}^2/4\pi = 2\cdot13, \qquad g_{\rho\mathrm{NN}}^{(1)} = \tfrac{1}{2}g_{\rho\pi\pi}, \qquad g_{\rho\mathrm{NN}}^{(2)} = 3\cdot7g_{\rho\mathrm{NN}}^{(1)},$$
$$g_{\Delta^{++}\mathrm{p}\pi}^2/4\pi = 0\cdot26, \qquad g_{\Delta^0\mathrm{p}\pi}^2 = \tfrac{1}{3}g_{\Delta^{++}\mathrm{p}\pi}^2.$$

Numerical Results and Conclusions

With the preceding detailed formulations for the various contributions to the reaction (1) it is possible to make a deep numerical study of this process. As a first step in this direction we have calculated the total and differential cross sections for linearly polarized photons and the results are presented in Tables 1 and 2. The values of E_{γ} chosen correspond to equispaced c.m. photon energies in the range 280–700 MeV in steps of 70 MeV. The calculated results for the total cross section σ_{tot} for unpolarized photons, which include contributions from the Δ^{++} and Δ^0 resonances, are compared in Table 3 with the experimental values of Luke (1972).

The most striking feature of the results in Tables 1 and 2 is that the cross sections for photons polarized in the y direction are consistently less than those for photons polarized in the x direction for all the particles exchanged; such a feature can be checked with experiment when the necessary data become available. We also observe that there are practically no ε and f contributions at low energies. This result is in consonance with the available experimental facts (Ballam *et al.* 1972, 1973). From the present work we find that the ε contribution (for unpolarized photons) reaches a maximum of ~ 1 μb around 3·6 GeV.

Table 3. Comparison of calculated total cross sections with experiment for unpolarized photons

Cross section	$E_\gamma = 376$	504	648 MeV
σ_{tot} (μb) from present work	1·12	12·6	42
σ_{exp} (μb) from Luke (1972)	1	18	70

From the results in Table 1 we see that at high values of energy the present calculated cross sections show a continued tendency to increase. However, this is presumably due to the fact that at these energies additional corrections are necessary to account for such effects as absorption etc. This aspect will be considered in a future publication. In any case, we feel that our work as it stands is a definite step forward in connection with attempts to understand multibody phenomena in hadron physics.

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