Neutron and Proton Capture by $^6$Li

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Abstract

The $^6$Li(n,$\gamma$) and $^6$Li(p,$\gamma$) cross sections at low energies are calculated by means of a direct-capture potential model and the results are compared with experimental data. Potential depths chosen to fit $^6$Li(n,n) and $^6$Li(n,$\alpha$) data are also used for the $^6$Li+p system. Standard values of the potential parameters and spectroscopic factors give cross sections that are too small for both the (n,$\gamma$) and (p,$\gamma$) reactions. However, modified values of these parameters that fit the (n,$\gamma$) cross section also give good agreement with the (p,$\gamma$) cross section.

1. Introduction

Various properties of mirror systems in light nuclei can be interpreted in terms of a simple potential model, with the potentials for the mirror systems differing only by the Coulomb interaction (Lane 1953; Tombrello 1966). The properties generally considered have been energies of corresponding levels or phase shifts for neutron and proton scattering. In a rather different application of this approach, Tombrello (1965) calculated the $^7$Be(p,$\gamma$)$^8$B direct-capture cross section on a potential model, assuming that the potential parameters were the same as those that fitted data for the mirror reaction $^7$Li(n,$\gamma$)$^8$Li; knowledge of the $^7$Be(p,$\gamma$)$^8$B cross section at low energies is of great importance in the calculation of the detection rate for solar neutrinos. This assumption, that the properties of mirror direct-capture reactions can be well described by potentials that use the same parameter values for the two reactions, apart from the Coulomb interaction, has not been tested previously in any precise way. As a test of the assumption in a case where adequate data were available for each reaction of a mirror pair, it seemed appropriate to study the reactions $^6$Li(p,$\gamma$)$^7$Be and $^6$Li(n,$\gamma$)$^7$Li. Transitions are observed to both the ground and first excited states of $^7$Be and $^7$Li, giving data on branching ratios, angular distributions, and energy dependence and absolute values of integrated cross sections. Owing to disagreements between previous values of the absolute cross sections for $^6$Li(p,$\gamma$), further measurements have been made and are reported by Switkowski et al. (1979).

The $^6$Li(p,$\gamma$) and $^6$Li(n,$\gamma$) reactions at low bombarding energies are interpreted as direct capture except for a resonant contribution in $^6$Li(n,$\gamma_0$), which has been observed in measurements on the inverse reaction $^7$Li($\gamma$,n$_0$) and is treated separately. In direct capture, the initial and final states are each described by a simple single-particle model of a nucleon in a potential well that represents the interaction with the $^6$Li ground state. For the final bound states, the potentials are real and the depths
are obtained by fitting nucleon binding energies. For the initial continuum states, the potentials are complex (since the \( z \) channels are open) and the depths are determined by fitting the \( ^6\text{Li} + \text{n} \) scattering lengths and assuming the same values for \( ^6\text{Li} + \text{p} \). Shell model values are used for the spectroscopic factors.

The next section contains formulae for the direct-capture cross sections, in terms of the spectroscopic factors and radial integrals involving the initial and final single-particle wavefunctions. Also given are calculated and experimental values of the spectroscopic factors, the forms of the potentials used for the initial and final states, and the formula used for the resonant cross section contribution. Section 3 gives the experimental data, including the nucleon-capture data from \( ^6\text{Li}(n, \gamma) \) and \( ^6\text{Li}(p, \gamma) \) and from the inverse reaction \( ^7\text{Li}(\gamma, n_0) \), and also the data from \( ^6\text{Li} + \text{n} \) elastic scattering and the \( ^6\text{Li}(n, \alpha) \) reaction that are used for determining potential depths for the continuum systems. Section 4 contains results, firstly parameter values obtained from fitting the \( ^7\text{Li}(\gamma, n_0) \) data, then parameter values from fitting \( ^6\text{Li}(n, \gamma) \) data and in particular the thermal-neutron cross section, and finally predictions of cross sections and related quantities for the \( ^6\text{Li}(p, \gamma) \) reaction. The calculation and results are discussed in Section 5.

Fig. 1. Energy levels of \(^7\text{Li}\) and \(^7\text{Be}\) (Ajzenberg-Selove 1979).

2. Cross Section Formulae

For low energy nucleon capture by \( ^6\text{Li} (E_p, E_n \ll 1\text{ MeV}) \), the \( \gamma \)-ray energy \( E_\gamma \) does not exceed about 8 MeV and the direct capture can be assumed to involve at most E1, M1 and E2 radiation. Since the ground state of \(^6\text{Li}\) is \( 1^+ \), and \(^7\text{Li}\) and \(^7\text{Be}\) ground
and first excited states are $3/2^-$ and $1/2^-$ respectively, the captured nucleons are in p states. Then s-wave and d-wave incident nucleons may contribute to E1 capture, and we include only p-wave nucleons for M1 and E2 capture. The $A = 7$ energy level diagrams of Fig. I show no well-defined positive-parity states in the regions corresponding to low energy nucleons, so that a direct-capture treatment of the E1 contribution seems reasonable.† There are negative-parity states in these regions, and these could contribute to M1 and E2 capture. In a shell model description (Barber 1966), these states are mainly spin quadruplets and would not give appreciable E2 transitions to the low-lying states, which are mainly spin doublets. Thus the E2 contribution is also assumed to be direct capture. On the other hand, appreciable M1 resonant contributions are expected and these are assumed to be given by R-matrix formulae, and M1 direct capture is neglected.

With the preceding assumptions, the differential cross section for direct capture to a final state of spin $J$ can be written, in the long-wavelength approximation (Tombrello and Parker 1963),

$$\frac{d\sigma}{d\Omega}_J = \frac{M \kappa}{2 \pi \hbar^2 k} \frac{2J+1}{18} \sum \xi_{s l l'} \xi_{s l l'} (-1)^{l'-1}(2L+1)^{-\frac{1}{2}} \times \mathcal{P}_A (\cos \theta_p) (2A+1)^{\frac{1}{2}} U(\mathcal{A}L' \mathcal{A}l'; \mathcal{A}l') (l' \mathcal{A}00 \mid \mathcal{A}0) (L'P - P \mid \mathcal{A}0), \tag{1}$$

where the summation is over $s, l, l', L, L', \mathcal{A}$ and $P$ (with $P = \pm 1$ only), and

$$\xi_{s l l'} = \eta_l (4\pi)^{\frac{1}{2}} e^{\kappa} \mathcal{G}_{kl} r_i \exp [i(\sigma_l - \sigma_k) \exp (i \delta_{ls}) (1L00 \mid 10) \times k^{-1} \int_0^\infty U_j (r) R_{ls}(r) r^l dr, \tag{2}$$

with

$$\eta_1 = -\left(-\frac{\sigma}{4\sqrt{14}}i\right), \quad \eta_2 = -\left(-\frac{3\sigma}{\bar{\sigma}}\frac{\sqrt{42}}{14}\right) \kappa \quad \text{for } ^6\text{Li}(p, \gamma); \tag{3a}$$

$$\eta_1 = \left(-\frac{\sigma}{4\sqrt{14}}i\right), \quad \eta_2 = -\left(-\frac{3\sigma}{\bar{\sigma}}\frac{\sqrt{42}}{14}\right) \kappa \quad \text{for } ^6\text{Li}(n, \gamma). \tag{3b}$$

Here $s$ is the channel spin, which must be the same for initial and final states for E1 and E2 transitions (denoted by $L = 1$ and 2 respectively), $l$ is the orbital angular momentum in the initial state and $\sigma$, the corresponding Coulomb phase shift, $Mc^2$ is the atomic mass unit, $k$ is the initial wave number, and $\kappa$ and $P$ are the wave number and circular polarization of the radiation. The initial radial wavefunction is normalized so that asymptotically

$$R_{ls}(r) \sim F_l (\eta, \rho) \cos \delta_{ls} + G_l (\eta, \rho) \sin \delta_{ls}, \tag{4}$$

where $F_l$ and $G_l$ are the regular and irregular Coulomb functions and $\delta_{ls}$ is the (complex) nuclear phase shift; the final radial wavefunction is normalized by

† The evidence for the uncertain $3/2^+$ level of $^7\text{Li}$ shown in Fig. 1 at 9·5 MeV with a width of about 3·7 MeV is not very strong. Broad positive-parity levels probably do occur in these energy regions, with widths of the order of single-particle widths, so that interpretation of the capture data in terms of such levels becomes more or less equivalent to the assumption of direct capture.
(Rolfs 1973)

\[ \int_{0}^{\infty} U_{Js}^{2}(r) \, dr = 1. \quad (5) \]

Then \( \mathcal{S}_{Js} \) is the spectroscopic amplitude.

In deriving equation (1), we have made the simplifying assumption that the \( R_{ls} \) (and therefore the \( \delta_{ls} \)) are independent of the total angular momentum \( J_{i} \) in the initial state. This may be justified as follows. For \( l = 0 \), no approximation is involved since \( J_{i} = s \). For \( l = 2 \) and also for \( l = 1, s = \frac{1}{2}, \) the phase shifts for all \( J_{i} \) values are negligible, at any rate for \( ^{6}\text{Li} + p \) (Petitjean et al. 1969), so that the \( R_{ls} \) are independent of \( J_{i} \). For \( l = 1, s = 3/2, \) the \( \delta_{ls} \) do depend on \( J_{i} \) (Petitjean et al. 1969) so that the \( R_{ls} \) do also, but the factors \( \mathcal{S}_{Js} \) with \( s = 3/2 \) are small (see Table 1 below) so that the errors in the cross section introduced by neglecting the dependence of \( R_{1,3/2} \) on \( J_{i} \) in the E2 matrix elements should be small.

Equation (1) can be rewritten

\[ \frac{d \sigma}{d \Omega}_{J} = (4\pi)^{-1} \sigma_{tot,J} W_{J}(\theta_{J}), \quad (6a) \]

with

\[ W_{J}(\theta_{J}) = \sum_{\lambda} B_{J \lambda} P_{\lambda}(\cos \theta_{J}), \quad B_{J0} = 1, \quad (6b) \]

where

\[ \sigma_{tot,J} = \frac{16\pi M e^{2} \kappa^{3}(2J+1)}{343 \hbar^{2} k^{3}} T_{J}, \quad (7a) \]

and

\[ T_{J} = \sum_{s} \mathcal{S}_{Js} Z_{Js0}, \quad B_{JA} = T_{J}^{-1} \sum_{s} \mathcal{S}_{Js} Z_{JsA}. \quad (7b) \]

For \( ^{6}\text{Li}(p,\gamma), \)

\[ Z_{Js0} = |\gamma_{Js0}^{1}|^{2} + 2 |\gamma_{Js2}^{1}|^{2} + \frac{5}{2} \gamma_{Js1}^{2} |\gamma_{Js1}^{2}|^{2}, \quad (8a) \]

\[ Z_{Js1} = \frac{3}{2} \kappa \text{ Re}\{\gamma_{Js1}^{2} (\gamma_{Js0}^{1} - \frac{1}{2} \gamma_{Js2}^{1})\}, \quad (8b) \]

\[ Z_{Js2} = -|\gamma_{Js2}^{1}|^{2} + 2 \text{ Re}(\gamma_{Js0}^{1} \gamma_{Js2}^{1}) + \frac{5}{2} \gamma_{Js1}^{2} |\gamma_{Js1}^{2}|^{2}, \quad (8c) \]

\[ Z_{Js3} = \frac{3}{2} \kappa \text{ Re}(\gamma_{Js1}^{2} \gamma_{Js2}^{1}), \quad (8d) \]

with

\[ \gamma_{Jsl}^{2} = \exp\{i(\delta_{ls} + \sigma_{l} - \sigma_{0})\} \int_{0}^{\infty} U_{J}(r) R_{ls}(r) r^{L} \, dr. \quad (9) \]

For \( ^{6}\text{Li}(n,\gamma), \) the quantities \( \gamma_{Js1}^{2} \) in equations (8) are replaced by \( -\frac{1}{13} \gamma_{Js1}^{2}. \) Thus one requires values of the spectroscopic factors \( \mathcal{S}_{Js} \) and of the initial and final radial wavefunctions \( R_{ls}(r) \) and \( U_{J}(r). \)

Values of \( \mathcal{S}_{Js} \) and of \( \mathcal{S}_{J} = \sum_{s} \mathcal{S}_{Js} \) from some shell model calculations are given in Table 1. Experimental values of \( \mathcal{S}_{J} \) have been obtained from DWBA analyses of data on direct reactions involving single-nucleon transfer between \( ^{6}\text{Li} \) and \( ^{7}\text{Li} \) or \( ^{7}\text{Be}, \) and some of the more recent values are given in Table 1. There have also been some measurements of the fraction \( f_{J}(3/2) \) of \( p_{3/2} \) nucleons in the transfer, and calculated and experimental values of this fraction are included in Table 1. There is
satisfactory agreement for both $\mathcal{F}_J$ and $f_J(3/2)$. It is seen that in all cases we have $\mathcal{F}_{13/2} \ll \mathcal{F}_{J1/2}$, as mentioned above.

In order to obtain the bound-state wavefunctions $U_j(r)$, we take the nuclear potential to be real, central and of Woods–Saxon shape:

$$V_J(r) = -V_J f(x), \quad f(x) = (1 + \exp x)^{-1}, \quad x = (r - R)/a,$$

and the Coulomb potential to be that of a uniformly charged sphere of radius $R$. The radius $R \equiv r_0 A^{1/3}$ and the diffuseness $a$ are assumed to be independent of $J$, the depth $V_J$ being chosen for given $R$ and $a$ to fit the binding energy of the state with respect to the $^6Li$(g.s.) plus nucleon channel.

Table 1. Values of spectroscopic factors between $^6Li$ ground state and ground or first excited state of $^7Li$ or $^7Be$

<table>
<thead>
<tr>
<th>$\mathcal{F}$</th>
<th>$J$</th>
<th>$s$</th>
<th>$\text{Calculated values}^A$</th>
<th>$\text{Experimental values}^A$</th>
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<tr>
<td>$\mathcal{F}_{15/2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$0.719$</td>
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<td>$0.980$</td>
</tr>
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<td>$\frac{1}{2}$</td>
<td>$0.045$</td>
<td>$0.003$</td>
</tr>
<tr>
<td>$\mathcal{F}_J$</td>
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<td>$\frac{1}{2}$</td>
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<td>$0.80$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$0.89$</td>
<td>$0.98$</td>
</tr>
<tr>
<td>$\mathcal{F}<em>{11/2}/\mathcal{F}</em>{3/2}$</td>
<td>$1.72$</td>
<td>$1.23$</td>
<td>$1.07$</td>
<td>$1.28$</td>
</tr>
<tr>
<td></td>
<td>$1.55$</td>
<td>$1.39$</td>
<td>$1.35$</td>
<td></td>
</tr>
<tr>
<td>$f_J(\frac{3}{2})$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$0.60$</td>
<td>$0.69$</td>
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<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$0.95$</td>
<td>$0.92$</td>
</tr>
</tbody>
</table>

$^A$ References: (a) Cohen and Kurath (1965, 1967); (b) Barker (1966); (c) Kumar (1974); (d) $^6Li(d,p)^7Li$, Schiffer et al. (1967); (e) $^7Li(p,d)^6Li$, Li and Mark (1969); (f) $^7Li(p,d)^6Li$, Towner (1969); (g) $^7Li(p,d)^6Li$, Fagerström et al. (1976); (h) $^6Li(^3He,d)^7Be$, Lüdecke et al. (1968); (i) $^6^3Cu(^6Li,^3Be)^6^2Ni$, Hudson et al. (1975); (j) $^6Li(d,p)^7Li$, Robson (1966); (k) $^6Li(d,p)^7Li$, Fick et al. (1970); (l) $^{12}C(^6Li,^6Li)^{13}C$, Zeller et al. (1979).

The continuum wavefunctions $R_{is}(r)$ should be complex because the $s$ channels are open as well as the nucleon channels (see Fig. 1). Thus we assume for the continuum states that the nuclear potential is complex and central, with surface absorption:

$$V_{is}(r) = -V_{is} f(x) + i W_{is} f'(x),$$

$R$ and $a$ being the same as for the bound states. The depths $V_{is}$ and $W_{is}$ are chosen to fit scattering and reaction data, which are given in the next section.

Initially, conventional values of the radius parameter and diffuseness are used, namely $r_0 = 1.25$ fm, giving $R = 2.27$ fm, and $a = 0.65$ fm (Bjorklund and Fernbach 1958), and variations of these are then considered in order to improve the fits to the neutron-capture data. Also, variations in some of the potential depths and in the values of the spectroscopic factors are considered.

A formula is also required for the resonant contribution observed in the $^7Li(\gamma,n_0)$ integrated cross section. This is attributed to the $5/2^-$ level of $^7Li$ at 7.46 MeV.
No resonant contributions have been observed from the corresponding level at 7.21 MeV in $^7$Be, or from the lower $5/2^-$ level at 6.73 MeV. We assume that the resonant transition is M1, giving an additive contribution to the integrated cross section. There are wide variations in the experimental data involving the $5/2^-$ level of $^7$Li (Derrien and Edvardson 1977), and probably the best values of the resonance parameters are those obtained by Hale (1977) from an R-matrix fit to data from reactions involving both $^6$Li + n and $^4$He + t entrance channels. The resonance parameters are given by Hale in an S-matrix form, in which the $^7$Li($\gamma, n_0$)$^6$Li resonant cross section is written

$$\sigma_{\text{res}}(\gamma, n_0) = \frac{3\pi}{4\kappa^2} \frac{\Gamma_\gamma \Gamma_n}{(E_\gamma - E)^2 + \frac{1}{4}(\Gamma_n + \Gamma_\gamma)^2},$$  \hspace{1cm} (12)$$

where $E = E_\gamma = \kappa hc$ is the $\gamma$-ray energy and

$$\Gamma_c = \{P_c(E)/P_c(E_i)\} \Gamma_c(E_i) \quad (c = \gamma, n, a),$$

with $P_c(E) = E_\gamma^3$, while $P_c (c = n$ or $a)$ is the normal penetration factor of R-matrix theory (Lane and Thomas 1958) evaluated at the channel radius $a_c$. Hale's fit gives the parameter values

$$a_n = 4.20 \text{ fm}, \quad a_z = 4.02 \text{ fm}, \quad E_\gamma = 7.46 \text{ MeV}, \quad (13a)$$

$$\Gamma_n(E_i) = 0.054 \text{ MeV}, \quad \Gamma_\gamma(E_i) = 0.023 \text{ MeV}. \quad (13b)$$

Thus the only adjustable parameter in fitting the $^7$Li($\gamma, n_0$) resonant cross section is $\Gamma_\gamma(E_i)$.

3. Experimental Data

(a) Nucleon Capture Data

For $^6$Li($n, \gamma$), the measured total thermal-neutron cross section is

$$\sigma_{n\gamma}(E_{th}) = 38.5 \pm 3.0 \text{ mb}, \quad (14)$$

with a ground-state branching ratio

$$\beta_n = 61 \pm 1 \% \quad (15)$$

(Ajzenberg-Selove and Lauritsen 1974). For higher neutron energies, the $^7$Li($\gamma, n_0$)$^6$Li integrated cross section measured by Bramblett et al. (1973), using monoenergetic photons produced by positron annihilation in flight, may be used. Their experimental points for $E_\gamma \lesssim 8.5 \text{ MeV}$ (B. L. Berman, personal communication) are shown in Fig. 2. Other experimental values shown in Fig. 2 were obtained earlier by Green and Donahue (1964), using monoenergetic neutron-capture $\gamma$ rays. The curves in Fig. 2 are discussed in Section 4 below.

An absolute measurement of the $^6$Li($p, \gamma$)$^7$Be cross section for $E_p = 200$–1200 keV is reported by Switkowski et al. (1979), where results of earlier measurements on this reaction are also summarized. Most of the experimental values that we need here are taken from this paper.
(b) Elastic Scattering and Particle Reaction Data

Scattering and reaction data are used to determine the depths of the complex potentials for the $^6$Li+n continuum system. The s-wave potential depths $V_{os}$ and $W_{os}$ can be obtained by fitting values of the complex scattering lengths $a_s = x_s + i y_s$ for the two channel spins $s = 1/2$ and $3/2$. The $^6$Li(n,$\alpha$) cross section $\sigma_{th}$ for thermal neutrons is given by

$$-4\pi k_{th}^{-1}(\frac{1}{2}y_{1/2} + \frac{3}{2}y_{3/2}) \equiv \sigma_{th} = 940 \pm 4 \text{ b}$$

(Ajzenberg-Selove 1979), while Glättli et al. (1978) have determined the ratio

$$y_{3/2}/y_{1/2} \equiv \sigma_+^e/\sigma_-^e = 0.127 \pm 0.016,$$

which is consistent with the earlier value of Mahaux and Robaye (1965) of 0.146.
Then equations (16) and (17) give

\[ y_{1/2} = -0.537 \pm 0.014 \text{ fm}, \quad y_{3/2} = -0.068 \pm 0.007 \text{ fm}. \]  

(18)

Asami and Moxon (1970) measured the \(^6\text{Li} + \text{n}\) elastic-scattering differential cross section for \(E_n = 1-110\) keV and analysed their results, assuming real scattering lengths, to obtain

\[ a_{1/2} = 4.12 \pm 0.06 \text{ fm}, \quad a_{3/2} = 0.70 \pm 0.15 \text{ fm}. \]

Approximately the same fits are obtained with complex scattering lengths, using the values (18) for the imaginary parts and (13) for the resonance parameters for the \(5/2^-\) level of \(^7\text{Li}\), if

\[ x_{1/2} = 4.08 \pm 0.06 \text{ fm}, \quad x_{3/2} = 0.72 \pm 0.15 \text{ fm}. \]

The corresponding value of the low-energy scattering cross section \(\sigma_0\) is

\[ \sigma_0 = 4\pi \left\{ \frac{1}{2} (x_{1/2}^2 + y_{1/2}^2) + \frac{3}{2} (x_{3/2}^2 + y_{3/2}^2) \right\} = 0.75 \pm 0.04 \text{ b}. \]

If \(x_{1/2}\) is adjusted to fit the more accurate value \(\sigma_0 = 0.72 \pm 0.01\) b (Sowerby et al. 1970), then

\[ x_{1/2} = 3.98 \pm 0.06 \text{ fm}, \quad x_{3/2} = 0.72 \pm 0.15 \text{ fm}. \]  

(19)

These values are consistent with the measurements of Glättli et al. (1978), who obtained \(b_{3/2} - b_{1/2} = -3.8 \pm 0.5\) fm, where \(b_s = (7/6)x_s\); the values (19) give \(b_{3/2} - b_{1/2} = -3.80 \pm 0.19\) fm. Also, the real part of the coherent scattering length (bound) is \(\frac{1}{2} b_{1/2} + \frac{3}{2} b_{3/2} = 2.11 \pm 0.12\) fm from the values (19), which may be compared with measured values of 7 fm (Shull and Wollan 1951) and 1.8 fm (Peterson and Smith 1962). Thus, for given values of \(R\) and \(a\), we choose \(V_{0s}\) and \(W_{0s}\) to fit the values (18) and (19).

The continuum p-wave and d-wave potentials are initially assumed to be real. \(V_{1,1/2}\) is chosen to be equal to \(V_{3,3/2}\), the depth of the ground-state potential, since the ground state contains mainly channel spin 1/2 (see Table 1), while \(V_{1,3/2}\) is taken to have a smaller value corresponding to a resonance a few MeV above the \(^6\text{Li} + \text{n}\) threshold. The d-wave depth \(V_{2s}\) is assumed to be independent of \(s\), and to be equal to the average of the s-wave depths, since this is approximately true for other cases (Tombrello 1966).

Values of \(V_j\), \(V_{1s}\), and \(W_{Is}\) are given in Table 2 for the conventional values \(r_0 = 1.25\) fm and \(a = 0.65\) fm. It is seen that the values of \(V_j\) for corresponding bound states in \(^7\text{Li}\) and \(^7\text{Be}\) are approximately equal. We assume that the values of \(V_{1s}\) and \(W_{Is}\) for \(^6\text{Li} + \text{p}\) continuum states are the same as those for the corresponding \(^6\text{Li} + \text{n}\) states. In principle, values for \(^6\text{Li} + \text{p}\) could have been obtained by fitting the \(^6\text{Li} + \text{p}\) elastic scattering (complex) phase shifts at low energies; however, different groups using different approaches or approximations have obtained widely different values of the s-wave phase shifts, especially the imaginary parts. Values of a particular
Fig. 3. Comparison of experimental and calculated values for the $^6$Li+p s-wave phase shifts $\delta_{0s}$ as functions of the proton bombarding energy $E_p$:

(a) The quantity defined in equation (20) in terms of the imaginary parts of $\delta_{0s}$. The experimental values are from Spiger and Tombrello (1967), Lejeune (1968) (approximate), Petitjean et al. (1969) and Hardy et al. (1972). The full curve is calculated from the parameter values in Table 2.

(b) Real parts of $\delta_{0s}$. The experimental points are from Petitjean et al. (1969). The curves are calculated from the parameter values in Table 2.
function of the imaginary parts of the s-wave phase shifts,

$$\sum_s (2s+1)\{1-\exp(-4\text{Im}\delta_{0s})\} \equiv (6k^2/\pi)\sigma_{I=0},$$  \hspace{1cm} (20)$$

where $\sigma_{I=0}$ is the s-wave $^6\text{Li}(p,\alpha)$ integrated cross section, are shown in Fig. 3a, as obtained from the analyses of different groups. For the real parts of the s-wave phase shifts, probably the best values are those of Petitjean et al. (1969), who included polarization data in their fits, and these are shown in Fig. 3b. Also shown in Fig. 3 are values calculated from the potential parameters in Table 2. There is satisfactory agreement with the experimental results, in view of their scatter. The parameter values in Table 2 give negligible values of $\delta_{1/2}$ and $\delta_{2s}$ at the energies of interest, in agreement with the results of Petitjean et al. The changes in potential parameters that we consider in Section 4b below do not alter significantly these calculated values of the $^6\text{Li}+p$ phase shifts.

Table 2. Potential depths for $^6\text{Li}+n$ and $^6\text{Li}+p$ systems
The potential depths listed are for the standard values $r_0 = 1.25$ fm and $a = 0.65$ fm

<table>
<thead>
<tr>
<th>$V$</th>
<th>$J$</th>
<th>$l$</th>
<th>$s$</th>
<th>$^6\text{Li}+n$</th>
<th>$^6\text{Li}+p$</th>
<th>Both reactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_J$</td>
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<td>65.29</td>
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<tr>
<td>$\frac{1}{2}$</td>
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<td>64.27</td>
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<td>$V_{1s}, W_{1s}$</td>
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<td>$\frac{1}{2}$</td>
<td>23-63, 11-11</td>
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<td>0</td>
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<tr>
<td>1</td>
<td>$\frac{1}{2}$</td>
<td>65, 0</td>
<td></td>
<td></td>
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<tr>
<td>1</td>
<td>$\frac{1}{2}$</td>
<td>30, 0</td>
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<tr>
<td>2</td>
<td>$\frac{1}{2}, \frac{3}{2}$</td>
<td>38, 0</td>
<td></td>
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</tr>
</tbody>
</table>

4. Results

(a) $^7\text{Li}(\gamma, n_0)^6\text{Li}$ Reaction

We fit the $^7\text{Li}(\gamma, n_0)^6\text{Li}$ data of Bramblett et al. (1973) assuming a resonant M1 contribution from the $5/2^-$ level of $^7\text{Li}$ at 7.46 MeV and nonresonant E1 contributions from direct photoemission of s-wave and d-wave neutrons. The E2 contribution to the integrated cross section is calculated to be negligible. For the inverse reaction $^6\text{Li}(n, \gamma_0)^7\text{Li}$, the calculated E1 contributions to the integrated cross section have energy dependences that do not depend much on the choice of potential parameters, although the magnitudes do, and to a good approximation we may write

$$\sigma_0(n, \gamma_0) = c_0 E_n^{-0.51}, \quad \sigma_2(n, \gamma_0) = c_2 E_n^{1.45}$$  \hspace{1cm} (21)$$

for the s-wave and d-wave parts respectively, where $E_n$ is the neutron energy in the c.m. system. Thus the direct contribution to the $^7\text{Li}(\gamma, n_0)$ integrated cross section is taken to be

$$\sigma_{d\gamma}(\gamma, n_0) = E_\gamma^{-2}(b_0 E_n^{0.49} + b_2 E_n^{2.45}),$$  \hspace{1cm} (22)$$
n and p Capture by $^6\text{Li}$

where

$$b_1 = \frac{g}{2} M_n c^2 c_1.$$  \hspace{1cm} (23)

The total cross section

$$\sigma(\gamma, n_0) = \sigma_{\text{res}}(\gamma, n_0) + \sigma_{\text{dir}}(\gamma, n_0),$$

with $\sigma_{\text{res}}$ given by equation (12) and the resonance parameters by equations (13), then contains three adjustable parameters: $\Gamma_\gamma(E_\gamma)$, $b_0$ and $b_2$.

Before comparing $\sigma$ with the experimental data, we must smear it over the experimental energy resolution. The shape of the photon spectrum is assumed to be gaussian, the FWHM giving the resolution. As the value of the resolution is not known precisely, we allow it to vary in order to improve the fit. It is found that the calculated peak energy, due to the resonance on top of a rapidly changing background, occurs at $E_\gamma \approx 7.6\text{ MeV}$, whereas the peak observed by Bramblett $et\ al.$ (1973) is at $E_\gamma \approx 7.5\text{ MeV}$. We assume that the experimental energy scale may be slightly in error, and apply a constant shift (increase) to the experimental $E_\gamma$ values and adjust the value of the shift to improve the fit. The best fit is obtained with a resolution of $0.35\text{ MeV}$ and a shift of $0.10\text{ MeV}$, and is shown in Fig. 2. This value of $0.35\text{ MeV}$ for the resolution is comparable with the values given by Bramblett $et\ al.$ (1973) of $3\%$ (about $0.23\text{ MeV}$) and by Berman (personal communication) of about $0.40\text{ MeV}$. The recalibration of the photon energy by $0.10\text{ MeV}$ is greater than the energy uncertainty of $\frac{1}{2}\%$ quoted by the Livermore group (Berman $et\ al.$ 1970), but this was based on calibration points for $E_\gamma \approx 17\text{ MeV}$.

The parameter values corresponding to the best fit are

$$\Gamma_\gamma(E_\gamma) = 0.151\text{ eV}, \quad b_0 = 2.51\text{ MeV}^{1.51}\text{ mb}, \quad b_2 = 1.47\text{ MeV}^{-0.45}\text{ mb}. \hspace{1cm} (24)$$

Although we are interested primarily in the direct-capture cross section, and therefore in the values of $b_0$ and $b_2$, we may compare the value of $\Gamma_\gamma(E_\gamma)$ in equations (24) with shell model values. Since the formula (12) for $\sigma_{\text{res}}(\gamma, n_0)$ does not explicitly contain a level-shift term in the denominator, the widths involved are observed widths rather than formal widths, in the sense of $R$-matrix theory (Lane and Thomas 1958). Thus the experimental value of $\Gamma_\gamma(E_\gamma)$ in (24) is an observed width. The shell model values are also regarded as observed widths, since they involve integrations over all space. They are $\Gamma_\gamma(E_\gamma) = 0.188\text{ eV}$ (Cohen and Kurath 1965), $0.082\text{ eV}$ (Barker 1966) and $0.028\text{ eV}$ (Kumar 1974). These calculated values vary considerably because they depend sensitively on the values of several small components of the initial and final wavefunctions. They encompass the experimental value.

(b) $^6\text{Li}(n, \gamma)^7\text{Li}$ Reaction

From equations (21) and (23), the values of $b_1$ in (24) give for the $^6\text{Li}(n, \gamma_0)$ s-wave and d-wave cross sections at $E_n = 1\text{ MeV}$ the values

$$\sigma_0 = 2.08\text{ }\mu\text{b}, \quad \sigma_2 = 1.21\text{ }\mu\text{b}. \hspace{1cm} (25)$$
We take these values of $\sigma_n$ together with the values of $\sigma_{n\gamma}(E_{th})$ and $\mathcal{R}_n$ given by equations (14) and (15), as experimental data to be fitted by the direct-capture potential calculations.

Table 3. Experimental and calculated values for $^6\text{Li}(n,\gamma)^7\text{Li}$

<table>
<thead>
<tr>
<th>Parameter set</th>
<th>Parameter modified</th>
<th>Change in parameter$^A$</th>
<th>$\sigma_n(E_{th})$ (mb)</th>
<th>$\mathcal{R}_n$ (%)</th>
<th>$\sigma_0$ (µb)</th>
<th>$\sigma_5$ (µb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>—</td>
<td>—</td>
<td>18.7</td>
<td>62</td>
<td>1.48</td>
<td>1.39</td>
</tr>
<tr>
<td>Standard</td>
<td>$r_0$ (fm)</td>
<td>1.5→1.25</td>
<td>13.0</td>
<td>62</td>
<td>1.01</td>
<td>0.95</td>
</tr>
<tr>
<td>Modified</td>
<td>$\mathcal{I}_{J_5}$</td>
<td>K→CK</td>
<td>14.8</td>
<td>56</td>
<td>1.10</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>$\mathcal{I}_{J_5}$</td>
<td>K→B</td>
<td>17.5</td>
<td>65</td>
<td>1.47</td>
<td>1.44</td>
</tr>
<tr>
<td></td>
<td>$x_{1/2}$ (fm)</td>
<td>3.98→3.92</td>
<td>20.3</td>
<td>62</td>
<td>1.60</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>$x_{3/2}$ (fm)</td>
<td>0.72→0.57</td>
<td>19.0</td>
<td>62</td>
<td>1.49</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>$y_{1/2}$ (fm)</td>
<td>-0.537→-0.551</td>
<td>19.0</td>
<td>62</td>
<td>1.49</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>$y_{3/2}$ (fm)</td>
<td>-0.068→-0.061</td>
<td>18.7</td>
<td>62</td>
<td>1.48</td>
<td>1.39</td>
</tr>
<tr>
<td>Absorption</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$V_{1,3/2}$ (MeV)</td>
<td>20→10</td>
<td>18.7</td>
<td>62</td>
<td>1.48</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>$V_{2s}$ (MeV)</td>
<td>30→20</td>
<td>18.7</td>
<td>62</td>
<td>1.48</td>
<td>1.24</td>
</tr>
<tr>
<td></td>
<td>$W_{1,3/2}$ (MeV)</td>
<td>0→10</td>
<td>18.7</td>
<td>62</td>
<td>1.48</td>
<td>1.39</td>
</tr>
<tr>
<td></td>
<td>$W_{2s}$ (MeV)</td>
<td>0→10</td>
<td>18.7</td>
<td>62</td>
<td>1.48</td>
<td>1.37</td>
</tr>
</tbody>
</table>

Parameter changes (b)

| Modified      | $\mathcal{I}_{J_5}$ | ×2.06                   | 38.5                    | 62                   | 3.04           | 2.86           |
|               | $\mathcal{I}_{J_{3/2}}$ | ×5.97                   | 38.5                    | 70                   | 3.25           | 1.55           |
|               | $r_0$ (fm)          | 1.5→2.02                | 38.5                    | 61                   | 3.12           | 3.53           |
|               | $a$ (fm)            | 0.65→1.0$^b$            | 25.7                    | 60                   | 2.19           | 3.07           |
|               | $r_{bc}$ (fm)       | 0→1.38                  | 38.5                    | 61                   | 2.73           | 2.25           |
|               | $r_{bc,p}$ (fm)     | 0→1.29                  | 38.5                    | 62                   | 3.23           | 2.18           |
|               | $r_{ee}$ (fm)       | 0→3.75                  | 38.0                    | 61                   | 3.27           | 1.08           |

Experimental values: $38.5±3$  $61±1$  $2.08$  $1.21$

$^A$ Change in modified parameter from the value for the basic set; the spectroscopic factors of Kumar (1974), identified as K here, are changed to those of (CK) Cohen and Kurath (1965, 1967), or (B) Barker (1966).

$^b$ Note that the experimental value of $\sigma_{n\gamma}(E_{th})$ is not fitted with this value of $a$.

Initially we calculate these quantities using the spectroscopic factors of Kumar (1974) given in Table 1 and the potential parameters given in Table 2. The resulting values are given in Table 3, where they are referred to as the standard set. Although the calculated branching ratio agrees well with experiment, the calculated cross section values are all lower than the experimental values.

Only small changes in the cross section values are produced by changing the spectroscopic factors from those of Kumar (1974) to those of Cohen and Kurath (1967) or of Barker (1966), which are given in Table 1, or by changing the potential depths by fitting scattering lengths varied within the uncertainties indicated in equations (18) and (19) (see parameter changes (a) in Table 3). Likewise replacing surface absorption by volume absorption has a small effect. Reasonable changes in the values
of $V_{1,3/2}$ and $V_{2s}$ and the corresponding imaginary potential depths $W_{1,3/2}$ and $W_{2s}$ have negligible effects. There are, however, several ways of obtaining appreciable increases in the calculated cross sections; these include increasing the spectroscopic factors either uniformly or selectively, increasing $r_0$ and/or $a$, using a potential well with a repulsive core, and introducing a cutoff radius in the radial integrals.

Firstly, in order to get calculated cross sections somewhat closer to the experimental values, we use the standard set of parameter values except that $r_0 = 1.25$ fm is replaced by $r_0 = 1.50$ fm ($R = 2.73$ fm), and refer to these as the basic set. Then we consider the effects of changing one parameter value at a time; the resultant values are given in Table 3. The small effects of some changes have been mentioned above. We discuss only the parameter changes (b) shown in Table 3, which enable the experimental value of $\sigma_{n}(E_{th})$ to be more or less fitted. Increasing uniformly the values of the spectroscopic factors $\mathcal{F}_{Js}$ renormalizes all cross sections, without changing the branching ratio. In view of the agreement of the unchanged spectroscopic factors $\mathcal{F}_{J}$ with the experimental values given in Table 1, it is difficult to justify a factor of 2 increase. It is possible to retain the agreement in $\mathcal{F}_{J}$ values and still to fit $\sigma_{n}(E_{th})$ by increasing only the $\mathcal{F}_{J3/2}$ values, since these are very small and they multiply large radial integrals in the cross sections: there is much more cancellation between the internal and external contributions to the s-wave radial integrals for $s = 1/2$ than for $s = 3/2$. The difficulty then comes from the calculated values of $f_{J}(3/2)$ (see Table 1), which change to 0.87 and 0.99 for $J = 3$ and 1/2 respectively, so spoiling the agreement with experiment. Increasing $r_0$ requires a value of $r_0 \approx 2$ fm, which is much larger than is normally assumed; on the other hand, conventional values of $r_0$ need not be very appropriate for $^6$Li, which is very diffuse, with a charge r.m.s. radius greater than that of $^{12}$C (Ajzenberg-Selove 1975, 1979). Increasing the value of $a$ is not very effective in increasing $\sigma_{n}(E_{th})$. A repulsive core is introduced most simply by assuming a hard core, of radius $r_{hc}$, additional to the Woods–Saxon potential. The value needed for $r_{hc}$ does not seem to be unreasonable. The hard core could be interpreted as simulating antisymmetrization effects, in which case it might be more justifiable to include the hard core only in the p-wave potentials, with radius $r_{hc,p}$ and not in the potentials for the s-wave and d-wave continuum states. Introducing a cutoff radius $r_{co}$ of the order of $R$ as a lower limit in the radial integrals produces increased s-wave cross sections, due to the large amount of cancellation, particularly for $s = 1/2$ for which $\mathcal{F}_{Js}$ is large. Such cutoff radii have been used in DWBA calculations to simulate finite-range and nonlocal effects (Hodgson 1971). The largest value of $\sigma_{n}(E_{th})$ is obtained for $r_{co} \approx 3.75$ fm, and is consistent with the experimental value. Such a value of $r_{co}$ is of the order used in DWBA calculations. We have considered the changes needed in the values of single parameters in order to fit $\sigma_{n}(E_{th})$, but a fit could also be obtained by smaller simultaneous changes in several parameters.

Most of the changes considered here have little effect on the branching ratio, and agreement with experiment is retained. The cross section $\sigma_0$ increases more or less in the same ratio as $\sigma_{n}(E_{th})$, and to a lesser extent $\sigma_2$ does also. For $\sigma_{n}(E_{th})$ fitted, the values of $\sigma_0$ range from about 2·7 to 3·3 $\mu$b, some 50% higher than the experimental value. There is a wide range of $\sigma_2$ values, but they tend to be above the experimental value. We note that we have taken the experimental values from the Livermore measurements (Bramblett et al. 1973), whereas the earlier measurements
of Green and Donahue (1964), also shown in Fig. 2, give cross sections about 50% higher and are consequently consistent with our calculations. A remeasurement of the $^7\text{Be}(\gamma,\text{n})^6\text{Li}$ cross section in this energy range would seem to be desirable.

<table>
<thead>
<tr>
<th>Parameter set</th>
<th>Parameter modified</th>
<th>Change in parameter</th>
<th>$\sigma_p(800)$ (µb)</th>
<th>$\sigma_p(400)$ (µb)</th>
<th>$\sigma_p(800)$/$\sigma_p(400)$</th>
<th>$R_p$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>—</td>
<td>—</td>
<td>1.43</td>
<td>1.65</td>
<td>4.6</td>
<td>61</td>
</tr>
<tr>
<td>Standard</td>
<td>$r_0$ (fm)</td>
<td>1.5→1.25</td>
<td>1.06</td>
<td>1.54</td>
<td>4.1</td>
<td>62</td>
</tr>
<tr>
<td>Modified</td>
<td>$s_{3/2}$</td>
<td>$\times 2.06$</td>
<td>2.95</td>
<td>1.65</td>
<td>4.6</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>$s_{3/2}$</td>
<td>$\times 5.97$</td>
<td>2.26</td>
<td>1.71</td>
<td>5.1</td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>$r_0$ (fm)</td>
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<td>60</td>
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<tr>
<td></td>
<td>$a$ (fm)</td>
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<td>6.0</td>
<td>59</td>
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<tr>
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<td>$r_{ax}$ (fm)</td>
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<td>3.01</td>
<td>1.63</td>
<td>4.7</td>
<td>60</td>
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<tr>
<td></td>
<td>$r_{ax,p}$ (fm)</td>
<td>0→1.29</td>
<td>2.62</td>
<td>1.65</td>
<td>4.7</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>$r_{ce}$ (fm)</td>
<td>0→3.75</td>
<td>2.43</td>
<td>1.60</td>
<td>4.6</td>
<td>60</td>
</tr>
</tbody>
</table>

Experimental values: $2.9±0.3$ 1.6 3.6 61 ±2

<table>
<thead>
<tr>
<th>B₁</th>
<th>B₂</th>
<th>B₃</th>
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<tbody>
<tr>
<td>0.19</td>
<td>0.42</td>
<td>0.08</td>
</tr>
<tr>
<td>0.18</td>
<td>0.42</td>
<td>0.08</td>
</tr>
<tr>
<td>0.19</td>
<td>0.42</td>
<td>0.08</td>
</tr>
<tr>
<td>0.16</td>
<td>0.31</td>
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<tr>
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<tr>
<td>0.16</td>
<td>0.33</td>
<td>0.06</td>
</tr>
<tr>
<td>0.23</td>
<td>0.38</td>
<td>0.09</td>
</tr>
<tr>
<td>0.16</td>
<td>0.27</td>
<td>0.05</td>
</tr>
</tbody>
</table>

A Change in modified parameter from the value for the basic set.
B Note that the experimental value of $\sigma_n(E_{th})$ is not fitted with this value of $a$.

(c) $^6\text{Li}(p,\gamma)^7\text{Be}$ Reaction

Experimental and calculated values of quantities related to the $^6\text{Li}(p,\gamma)^7\text{Be}$ reaction are given in Table 4. The quantities considered are the total integrated cross section at $E_p = 800$ keV, the ratios of the 800 keV cross section to those at 400 and 200 keV, the ground-state branching ratio at 800 keV, and the angular-distribution coefficients $B_{3/2,4} ≡ B_4 (A = 1–3)$ for the ground-state radiation also at 800 keV. The experimental values of $\sigma_p(800)$ and $\sigma_p(400)$ are taken from Switkowski et al. (1979). From the calculations in this paper, the values of $\sigma_p(800)/\sigma_p(400)$ and $\sigma_p(800)/\sigma_p(200)$ are about 1.6 and 6 respectively. Earlier values of these ratios come from the work of Warren et al. (1956), who obtained a ratio of about 1.5 for the 90° yields at 800 and 400 keV, and of Sweeney (1969), who obtained a ratio of 4 for the 0° yields at 800 and 200 keV. From calculated angular distributions at these energies, these results give $\sigma_p(800)/\sigma_p(400) ≈ 1.6$ and $\sigma_p(800)/\sigma_p(200) ≈ 3.3$. The former value agrees well with the results of Switkowski et al., but the latter does not. We also note that Warren et al. fitted their observed angular distribution at $E_p = 800$ keV
n and p Capture by $^6$Li

(for both the ground-state and excited-state transitions) with $B_1 = B_2 = 0$ and $B_3 = 0.52 \pm 0.06$, while Johnston et al. (1969) obtained $B_1 = -0.13$, $B_2 = 0.43$ and $B_3 = 0$. The calculated values in Table 4 include those for the basic and standard sets of parameter values, as in Table 3, and for the modified values $(b)$ given in Table 3. We see that the values of $\sigma_p (E_{th})$ calculated for parameter sets that fit $\sigma_p (E_{th})$ range from about 2.3 to 3.0 $\mu$b, in good agreement with the experimental value. There is reasonable agreement between the other calculated and experimental values in Table 4.

Table 5. Energy dependence of calculated values for $^6$Li(p, $\gamma$)$^7$Be

The values listed are for the modified set of parameters in which the $S_{bS}$ values are increased by a factor of 2.06

<table>
<thead>
<tr>
<th>$E_p$ (keV)</th>
<th>$\sigma_{tot}$ ($\mu$b)</th>
<th>$S_{tot}$ (keV b)</th>
<th>$\beta_p$ (%)</th>
<th>Ground state $B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
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</thead>
<tbody>
<tr>
<td>10</td>
<td>$1.43 \times 10^{-6}$</td>
<td>0.106</td>
<td>60.1</td>
<td>0.067</td>
<td>-0.016</td>
<td>0.001</td>
</tr>
<tr>
<td>20</td>
<td>$4.36 \times 10^{-6}$</td>
<td>0.105</td>
<td>60.1</td>
<td>0.073</td>
<td>-0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>40</td>
<td>$1.02 \times 10^{-3}$</td>
<td>0.103</td>
<td>60.2</td>
<td>0.084</td>
<td>0.020</td>
<td>0.003</td>
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<tr>
<td>100</td>
<td>0.0913</td>
<td>0.097</td>
<td>60.2</td>
<td>0.101</td>
<td>0.069</td>
<td>0.006</td>
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<tr>
<td>200</td>
<td>0.636</td>
<td>0.085</td>
<td>60.3</td>
<td>0.123</td>
<td>0.139</td>
<td>0.012</td>
</tr>
<tr>
<td>400</td>
<td>1.79</td>
<td>0.068</td>
<td>60.5</td>
<td>0.157</td>
<td>0.262</td>
<td>0.031</td>
</tr>
<tr>
<td>600</td>
<td>2.48</td>
<td>0.060</td>
<td>60.6</td>
<td>0.179</td>
<td>0.358</td>
<td>0.056</td>
</tr>
<tr>
<td>800</td>
<td>2.95</td>
<td>0.056</td>
<td>60.6</td>
<td>0.188</td>
<td>0.419</td>
<td>0.083</td>
</tr>
<tr>
<td>1000</td>
<td>3.38</td>
<td>0.057</td>
<td>60.5</td>
<td>0.186</td>
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<td>1500</td>
<td>4.63</td>
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<td>60.2</td>
<td>0.155</td>
<td>0.382</td>
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<td>2000</td>
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<td>60.0</td>
<td>0.115</td>
<td>0.249</td>
<td>0.166</td>
</tr>
</tbody>
</table>

In order to allow further comparison with experiment and to provide quantities of potential use in thermonuclear reactor calculations, Table 5 gives the energy dependence of the total cross section and derived $S$ factor, the ground-state branching ratio and the angular distribution coefficients for both ground-state and excited-state transitions, for the particular modified set of parameter values in which the $S_{bS}$ values are increased by a factor of 2.06. The values in Table 5 show the constancy of the branching ratio and the near equality of the angular distributions for the ground and excited-state transitions. The total cross section values of Table 5 are plotted in Fig. 4, which also shows the experimental results of Switkowski et al. (1979), with the error bars now including allowances for systematic errors, and values derived from the 0° measurements of Sweeney (1969). The agreement is good.
Fig. 4. Total cross section of the reaction $^6\text{Li}(p, \gamma)^7\text{Be}$. The experimental points are from Switkowski et al. (1979) and Sweeney (1969). The curve is calculated using values of potential parameters and spectroscopic factors chosen to fit the $^6\text{Li}(n, \gamma)^7\text{Li}$ thermal-neutron cross section.

5. Discussion

The calculation described here was made in order to test the assumption that the properties of mirror direct-capture reactions can be well described by optical potentials that use the same parameter values for each reaction. The present test involves the mirror reactions $^6\text{Li}(p, \gamma)$ and $^6\text{Li}(n, \gamma)$, assumed to proceed by direct capture. The calculated cross sections depend on the potential parameters and spectroscopic factors. Values of these are determined by fitting $^6\text{Li} + n$ elastic scattering and $^6\text{Li}(n, \alpha)$ data, as well as the $^6\text{Li}(n, \gamma)$ thermal-neutron cross section $\sigma_{n\gamma}(E_{\text{th}})$. With the same parameter values the $^6\text{Li}(p, \gamma)$ cross section is calculated and compared with experiment.

A problem is that Woods–Saxon potentials with conventional values of the radius and diffuseness parameters and shell model values of the spectroscopic factors give a calculated value of $\sigma_{n\gamma}(E_{\text{th}})$ that is much too low. Similarly the calculated values of the $^6\text{Li}(p, \gamma)$ cross section $\sigma_{p\gamma}$ are too low. These discrepancies may be attributed
in part to the parameter values, since the conventional values are not necessarily appropriate for a light nucleus like $^6\text{Li}$, and in part to deficiencies of the potential model, such as the assumption of an energy-independent and local optical potential and the neglect of antisymmetrization between the odd nucleon and the nucleons of the core. Various modifications of the parameter values enable the experimental value of $\sigma_{n\gamma}(E_{\text{th}})$ to be fitted, and the significant result for the present test is that in each case the experimental values of $\sigma_{pp}$ are then reasonably well predicted.

It is of interest that previous studies of the $^6\text{Li}(n,\gamma)$ and $^6\text{Li}(p,\gamma)$ reactions using a direct-capture model concluded that very small values of the spectroscopic factors were necessary in order to fit the data, whereas we favour large values. For the ratio of the required spectroscopic factors to the shell model values, Tombrello and Phillips (1961) obtained an upper limit of about 0·02, while Johnston et al. (1969) obtained a value of about 0·0003. Tombrello and Phillips, however, neglected contributions to the E1 matrix elements due to integration over the nuclear volume and assumed that the observed radiation in the $^6\text{Li}(p,\gamma)$ reaction was predominantly M1; also the $^6\text{Li}(p,\gamma)$ cross section that they fitted was only about 10% of the presently accepted value (see Switkowski et al. 1979). Johnston et al. used for the s-wave proton states an optical potential that did not depend on the channel spin $s$; they also fitted a small value of the cross section (Johnston 1968). In the present work, because the $^6\text{Li}+n$ scattering lengths are very different for $s = 1/2$ and 3/2, the corresponding potential depths are very different, and it is found that the E1 matrix elements for $s = 1/2$ are very small due to cancellation between the internal and external contributions. There is less cancellation for $s = 3/2$ but the spectroscopic factors for $s = 3/2$ are very small. Thus small cross sections are obtained even though the total spectroscopic factors are large.

Our result that values of the potential parameters and spectroscopic factors that fit $\sigma_{n\gamma}(E_{\text{th}})$ predict good values for $\sigma_{pp}$ gives support to the use of the assumption under test in other cases; in particular in the following paper (Barker 1980; present issue pp. 177–90) it is used to calculate the $^7\text{Be}(p,\gamma)^8\text{B}$ cross section using parameter values determined by fitting $^7\text{Li}(n,\gamma)^8\text{Li}$ and $^7\text{Li}+n$ scattering data, and the relevance of this to the solar-neutrino problem is discussed.

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References


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