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Application of the Frahn Scaling Formula to the Analysis of the Excitation of 1^- States in ^{40}Ca by ^{9}Be Scattering

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Abstract

The simple scaling formula derived by Frahn for the heavy-ion inelastic cross section of excitation of 1^- states in even–even nuclei has been applied to the excitation of the 1^- states in ^{40}Ca at 5·91 and 6·94 MeV by 45 and 60 MeV ^{9}Be projectiles.

Introduction

Recently Frahn (1976) has developed a closed form expression for the inelastic scattering amplitude for excitation of low-lying collective states by heavy-ion scattering. This formalism makes use of the Austern–Blair (1965) relation and other approximations appropriate to a strongly absorptive interaction. The resulting inelastic partial wave amplitude is given in terms of the S-matrix elements in the initial and final channels. Frahn (1976) further showed that the dominant mechanism in elastic heavy-ion collisions is Fresnel-type diffractive scattering.

For the specific case of the inelastic excitation of spin 1 states in even–even nuclei when $\xi \to 0$, where ξ is the adiabaticity parameter, the Frahn formalism yields the particularly simple scaling relation (Frahn 1976)

$$\sigma_1(\theta) = (4\pi)^{-1}\delta_1^2\{k\theta\}^2\sigma_{\mathrm{el}}(\theta), \tag{1}$$

where $\sigma_1(\theta)$ is the differential cross section for excitation of the spin 1 state, k is the wave vector in the incident channel, $\sigma_{\mathrm{el}}(\theta)$ is the elastic scattering cross section and δ_1 is the deformation length. In equation (1) it has been assumed that the Coulomb and nuclear deformation lengths are equal (that is, $\delta_1^{\mathrm{C}} = \delta_1^{\mathrm{n}} = \delta_1$).

It has been shown by Sellschop (1977) that the simple relationship given by equation (1) can be expanded to the case where $\xi \geqslant 0$ (with the Coulomb term set to zero). Then

$$\sigma_1(\theta) = (4\pi)^{-1}(\delta_1^{\mathrm{n}})^2\{k(\theta-\theta_{\mathrm{r}})\}^2\sigma_{\mathrm{el}}(\theta) \qquad (\theta > \theta_{\mathrm{r}}), \tag{2}$$

where θ_{r} is the rainbow scattering angle. This expression ignores any contribution from Coulomb excitation to the $\sigma_1(\theta)$ cross section. For the case of high energies (that is, $\xi \to 0$), $\theta_{\mathrm{r}} \to 0$ and equation (2) reduces to (1).

Tests of Frahn Formula

Some preliminary testing of the Frahn formalism and in particular of the simple scaling relationship given by equation (2) has been carried out by Sellschop (1977), although the data used for the test (excitation of the 1^- states at 4·45 MeV in ^{18}O from ^{16}O and ^{18}O scattering) only marginally satisfy the conditions for the validity of the simple Frahn formalism (Frahn 1976).

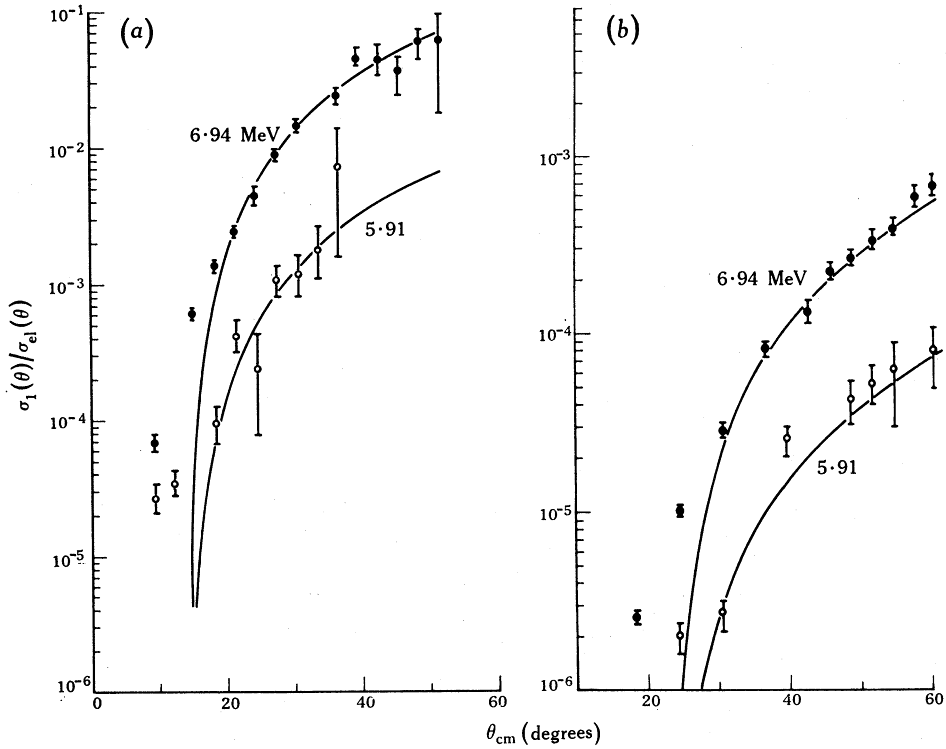


Fig. 1. Ratios of the inelastic scattering cross section σ_1 for excitation of the 1^- states in ^{40}Ca at 5·91 and 6·94 MeV to the elastic scattering cross section σ_{el} for incident ^{9}Be projectiles of energy (*a*) 60 MeV and (*b*) 45 MeV. The points show the experimental ratios while the curves are the predictions from the Frahn scaling formula, for deformation length parameters $\delta_1^{5\cdot91}$ and $\delta_1^{6\cdot94}$ respectively of (*a*) 0·112 and 0·354, and (*b*) 0·121 and 0·373.

As part of a more extensive program to study the interaction of ^{9}Be projectiles with various target nuclei, we have measured the inelastic cross sections for excitation of the 1^- levels in ^{40}Ca at excitation energies of 5·91 and 6·94 MeV by 45 and 60 MeV ^{9}Be projectiles (Eck *et al.* 1980) (E_{cm} = 36·73 and 48·98 MeV respectively). In this note, the relationship given by equation (2) is further tested using these measured inelastic scattering cross sections. For the cases presented here, the condition of strong absorption is well satisfied and a wide angular range of data is available for comparison of the simple scaling formula.

The ratios of the inelastic yields, for scattering of 60 and 45 MeV ^{9}Be to the 5·91 and 6·94 MeV states in ^{40}Ca, to the elastic scattering yield are plotted as a function of scattering angle θ in Figs 1*a* and 1*b*. No centre-of-mass corrections for solid angle or scattering angle difference between ground and excited state are applied as they are negligible for these bombarding energies. The rainbow scattering angles are determined from the experimental elastic cross section by using the relationship that the ratio of the elastic scattering cross section to the Rutherford cross section at the rainbow angle is 0·44 times the same ratio at the maximum (Ford and Wheeler 1959), that is,

$$\sigma_{\mathrm{el}}(\theta_{\mathrm{r}})/\sigma_{\mathrm{R}}(\theta_{\mathrm{r}}) = 0{\cdot}44\,\sigma_{\mathrm{el}}(\theta_{\max})/\sigma_{\mathrm{R}}(\theta_{\max}), \tag{3}$$

where $\theta_{\max}$ is the angle for which $\sigma_{\mathrm{el}}/\sigma_{\mathrm{R}}$ is a maximum. This yields $\theta_{\mathrm{r}} = 15°$ for the 60 MeV ^{9}Be+^{40}Ca scattering and $\theta_{\mathrm{r}} = 23°$ for the 45 MeV case. The quantity $\{k(\theta-\theta_{\mathrm{r}})\}^2/4\pi$ is calculated and scaled to the experimentally determined $\sigma_1(\theta)/\sigma_{\mathrm{el}}(\theta)$ by adjusting the single parameter δ_1^2. The calculated curves are shown in Fig. 1 for the following best-fit (by eye) deformation length parameters.

Parameter	60 MeV	45 MeV	Average
$\delta_1^{5\cdot91}$	$0{\cdot}112\pm0{\cdot}010$	$0{\cdot}121\pm0{\cdot}010$	$0{\cdot}117\pm0{\cdot}007$
$\delta_1^{6\cdot94}$	$0{\cdot}354\pm0{\cdot}030$	$0{\cdot}373\pm0{\cdot}030$	$0{\cdot}364\pm0{\cdot}020$

It is seen from Fig. 1 that there is a good agreement between the simple scaling formula (2) and the experimental data. Furthermore, the scaling formula yields the correct energy dependence for the $\sigma_1/\sigma_{\mathrm{el}}$ ratio, as evidenced by the fits at two energies for essentially the same value of the δ_1 parameter. One difficulty with the present analysis is that the mechanism for excitation of the 1^- states such as those considered here is usually assumed to be a two-step process consisting of an octupole excitation of the 3^- (3·73 MeV) state followed by a quadrupole excitation of the 1^- state (i.e. $0^+(0{\cdot}0) \to 3^-(3{\cdot}73) \to 1^-(E_1)$, where $E_1 = 5{\cdot}91$ or 6·94 MeV), rather than a one-step dipole excitation (see Fig. 2 below). The theoretical interpretation of the success of the Frahn scaling formula is unclear, however, as there can be no direct, collective excitation of a 1^- state in the form

$$R(\theta) = R_0 + \sum \delta_{\lambda,m}\, Y_\lambda^m(\theta, Q),$$

since for $\lambda = 1$ this is merely a uniform translation of the centre of mass of the system, which is not an intrinsic excitation. Therefore any collective excitation components must be at least two step (e.g. $2^+ \to 3^-$); note, however, that single-particle excitation is not forbidden.

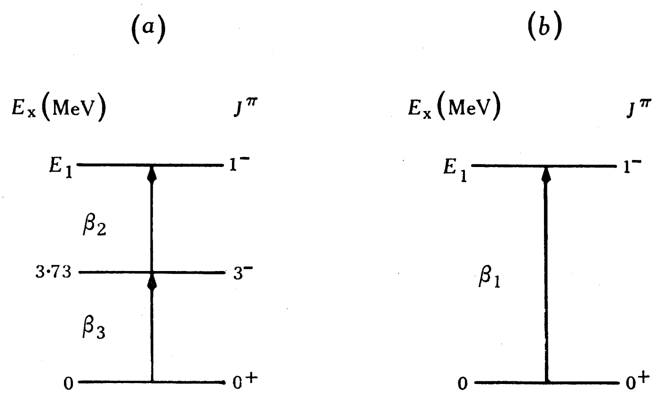
Deformation parameters $\beta_1^{5\cdot91}$ and $\beta_1^{6\cdot94}$ can be extracted for comparison with other analyses using the relation $\delta = \beta R$, by choosing an appropriate radius (Thompson and Eck 1977). With the choice $R = 1{\cdot}35(40^{1/3}+9^{1/3}) = 7{\cdot}43$ fm, we find

$$\beta_1^{5\cdot91} = 0{\cdot}016\pm0{\cdot}002, \qquad \beta_1^{6\cdot94} = 0{\cdot}049\pm0{\cdot}003.$$

It is of interest to compare the present deformation parameters with those of the more detailed coupled-channels analysis of α+^{40}Ca scattering by Cramer *et al.* (1973). They use the two-step octupole–quadrupole coupling scheme described above and obtain the deformation parameters tabulated in Fig. 2. Since the cross sections in a DWBA analysis scale as

$$\sigma_1 \propto (\beta_2\,\beta_3)^2,$$

the products $\beta_2\beta_3$ from the coupled-channels analysis should be compared with the β_1's of the present analysis (i.e. it is expected that $\beta_2\beta_3 \approx \beta_1$). These comparisons are given in Fig. 2. The agreement between the β_1 values, obtained from application of the simplified scaling formula to the inelastic cross sections for excitation of the 5·91 and 6·94 MeV states in ^{40}Ca by ^{9}Be scattering, and the products $\beta_2\beta_3$ from the $\alpha+^{40}$Ca analysis is surprisingly good, although this is probably somewhat fortuitous due to a wide range of choice of a suitable radius for extraction of the deformation parameters from the measured deformation lengths.



E_1 (MeV)	β_2	β_3	$\beta_2\beta_3$	β_1
5·91	0·050	0·25	0·0125	0·016
6·94	0·180	0·25	0·0450	0·049

Fig. 2. Comparison between the coupling schemes used and measured deformation parameters for inelastic scattering to the 5·91 and 6·94 MeV levels in ^{40}Ca from (*a*) the analysis by Cramer *et al.* (1973) of $\alpha+^{40}$Ca, and (*b*) the present analysis of ^{9}Be+^{40}Ca.

Conclusions

The simple scaling formula derived by Frahn for the inelastic scattering to spin 1 states in even–even nuclei has been shown to yield good agreement with the experimental cross sections for excitation of the 1^- states at 5·91 and 6·94 MeV in ^{40}Ca by ^{9}Be scattering. Furthermore, the scaling formula yields the correct energy dependence for the cross sections, at least for the limited energy range investigated here. The only free parameter is the deformation length δ_1. The deformation parameters derived from the present analysis compare well with those extracted from more elaborate analyses through the use of simple scaling arguments, although the theoretical interpretation of the measured δ_1 parameters is unclear.

Considering the simplicity of the present analysis and its apparent success, it is of interest to apply the more elaborate Frahn formalism to the scattering to states other than those with spin 1, as has been done recently by Frahn and Rehm (1978).

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