Sea Level Measurements of Properties of Cosmic Ray Showers with Sizes Ranging from $2 \times 10^5$ to $10^7$ Particles

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Abstract

An analysis has been made of air shower data obtained over a 13 month period at Buckland Park. Showers with sizes of about $10^6$ particles were found to have an attenuation length of $185 \pm 5 \text{ g cm}^{-2}$, absorption lengths of about $100 \text{ g cm}^{-2}$ and a shower size spectrum which progressively steepens between sizes of about $5 \times 10^5$ and $10^7$ particles.

Introduction

Cosmic rays with an energy of $\sim 10^{16}$ eV are of particular importance since in this energy region there occurs the clearest structure in the energy spectrum. This 'knee' in the spectrum (Hillas 1974 and Olejniczak et al. 1977) is associated with a significant increase in the index of the power law to which the spectrum is usually fitted. The origin of the knee is not known and possible explanations have ranged from particle interactions in pulsar cosmic ray sources (Barrowes 1971 and Karakula et al. 1974), through enhanced leakage of cosmic rays from our galaxy (Hillas 1979) to the onset of new forms of particle physics (Clay et al. 1981b). In order that the various theoretical models can be tested as well as possible, we require to determine the properties of the observed cosmic rays at these energies as well as possible. We would like to know the primary particle energy spectrum and composition but these parameters cannot at present be determined directly and must be deduced from measurements of the air showers produced when the primary particles interact with the atmosphere. It is important therefore to determine with as great precision as possible the properties of such air showers. Conventional air shower arrays employ ground based particle detectors and we wish to discuss measurements made with one such array. The topics discussed below have been examined many times before but the operation of the array has recently been improved to provide significantly better data than have been available in the past; we wish to present here our present best estimates of some basic air shower properties as measured at sea level.

Buckland Park Air Shower Array

The Buckland Park extensive air shower array, operated by the University of Adelaide, has been described in detail elsewhere (Crouch et al. 1981). Briefly, it consists of twelve 1-m$^2$ plastic scintillators spaced with a total enclosed area of $\sim 3 \times 10^4$ m$^2$. Triggering levels are set to produce a minimum detectable sea-level shower size of $\sim 10^5$ particles which results in a mean time between events of $\sim 7$ min.
Shower directions are determined by fast timing over an inner square array with 30 m sides resulting in an estimated directional error of \( \sim 3 \sec \theta \) degrees (where \( \theta \) is the angle between the shower arrival direction and the zenith). Shower size analysis is performed using density measurements from all 12 sites. The analysis procedures have been improved recently by using the MINUIT computer package (James and Roos 1975) to fit showers to an Nishimura–Kamata–Greisen (NKG) particle lateral distribution function (Greisen 1960) with a variable age parameter \( S \) (Cocconi 1961). In the past, a fixed lateral distribution function had been used (Gerhardy \textit{et al}. 1981). Whilst this proved to be a good approximation to the NKG function and also provided well fitted analyses, recent results indicating that \( S \) (and hence the lateral distribution function) varies more rapidly with shower size than had been expected (Clay \textit{et al}. 1981\textit{a}) led us to re-analyse the basic properties of air showers detected by the array. This re-analysis is presented below.

**Shower Size Spectra**

The basic data produced by the Buckland Park array for each shower include the arrival direction of the shower, its impact point on the ground, the lateral distribution age parameter \( S \) and the shower size. In order to derive spectra using these data, we first group the data in zenith angle bins of width 4° (in this analysis, we use data out to zenith angles of 44°). For each zenith angle group we then derive a shower size spectrum. This is accomplished by placing the observed shower sizes in bins with widths progressively increasing by a factor of \( \sqrt{2} \) (with an arbitrary starting width of \( 10^4 \) particles at \( 10^4 \) particles). All showers which have an inadequate fit to the observed densities (with a reduced chi-squared > 5) are rejected.

The Buckland Park array was designed to have collecting areas for showers of given energies which were quite well defined (Crouch \textit{et al}. 1981). We are able therefore to select an area of the array which, for a given shower size, collects showers with close to 100% efficiency. The use of only those showers whose energies and core locations would have been such that the showers had 100% triggering efficiency removes major problems of calculating collecting areas since it is only necessary to define a physical area of the array within which showers of a given size are accepted for analysis. This area is taken to be circular in our analysis with a radius of 20 m for showers with sizes of \( \sim 4 \cdot 5 \times 10^5 \) particles up to 60 m for showers with sizes of \( \sim 2 \cdot 7 \times 10^6 \) particles. The total effective running time of the array (which nominally runs continuously) is derived by summing the total running time and making allowance for events rejected through poor analyses (high values of chi-squared).

A total of 13 months data was used in this analysis (January–June 1980 and January–July 1981) and, of the \( \sim 7 \times 10^4 \) events which triggered the array, a total of 12795 events was retained. These events were used to construct differential size spectra and then the corresponding integral spectra at zenith angles spaced by 4°. The e-folding depth of the integral rate of showers above a given shower size as a function of absorber depth (Bourdeau \textit{et al}. 1980) is known as the absorption length of the showers. This depth is found by taking cuts at constant size through the integral size spectra, and Fig. 1 shows three such relationships, for sizes of \( 3 \times 10^5 \), \( 10^6 \) and \( 3 \times 10^6 \) particles. The integral spectrum was also used to derive the dependence of shower size on atmospheric depth at a constant integral intensity by making a cut through the spectrum at an integral rate of \( 10^{-6} \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1} \). This dependence is also expected to approximate to an exponential form with the e-folding depth.
Properties of Cosmic Ray Showers

Shower size

10^{-7} 3 \times 10^5 \text{ particles}

\text{Integral rate} \left( \text{m}^{-2} \text{s}^{-1} \text{sr}^{-1} \right)

10^{-10}

10^{-9}

10^{-8}

10^{-7}

10^{-6}

Shower size

3 \times 10^6 \text{ particles}

10^6

3 \times 10^6

Depth of atmospheric absorber (g cm\(^{-2}\))

Fig. 1. The relationship between integral rate and depth of atmospheric absorber (g cm\(^{-2}\) greater than at vertical incidence at sea level) for showers of sea-level size 3 \times 10^5, 10^6 and 3 \times 10^6 particles.

Fig. 2. The relationship between shower size and depth of atmospheric absorber (g cm\(^{-2}\) greater than at vertical incidence at sea level) at an integral rate of 10^{-8} \text{ m}^{-2} \text{s}^{-1} \text{sr}^{-1}.
known as the attenuation length (Ashton et al. 1975). Fig. 2 shows this relationship. We find that the attenuation length does not vary measurably over all the integral rates covered by our data and we can therefore use it to combine our data by deriving an equivalent vertical size from the observed size at a given intensity for all zenith angle ranges in our integral spectra. Hence an integral spectrum for 'vertical showers' can be derived which has good statistical accuracy. This spectrum was obtained through taking cuts at constant integral intensity and deriving the best value of the vertical shower size for that intensity. The result of this procedure is shown in Fig. 3. For comparison purposes, a power law index may be derived from the (assumed) power law differential spectrum above the position of the spectral knee (derived from the integral spectrum). This index was found to be $2.90 \pm 0.05$.

![Graph](image.png)

**Fig. 3.** The sea-level shower size spectrum for 'vertical' showers. The crossed error bars are Buckland Park data. The single error lines indicate the spread in a compilation by Hillas (1974) of such spectra.

**Discussion of Results**

The lateral distribution function of cosmic ray showers varies rapidly with shower size in the size range discussed in this paper. A parameter which characterizes this function is the lateral distribution age parameter $S$. The dependence found in this analysis of $S$ on shower size for showers with zenith angles less than or equal to $12^\circ$ is consistent with results found by us previously (Clay et al. 1981a) and also agrees with data from other experiments (Hara et al. 1979a). Between sizes of $4 \times 10^5$ and $4 \times 10^6$ particles, we find a mean rate of change of $S$ with shower size of $0.22 \pm 0.02$ per decade of size. The size of a shower which is found from a shower analysis depends on the lateral distribution which is chosen, and we would therefore expect improved shower size spectra to be obtained using our variable $S$ analysis procedure since a change of $0.22$ over the range of shower sizes of interest is quite large.

The absorption length of showers near the knee of the spectrum is of interest since Bourdeau et al. (1980) have made detailed calculations which relate this length to properties of the interaction mechanisms in the cascade. Fig. 1 shows the data obtained on the rate reduction of coincidences with increasing absorber. Absorption lengths of $102 \pm 3$, $99 \pm 3$ and $95 \pm 4 \text{g cm}^{-2}$ are found at shower sizes of $3 \times 10^5$, $10^6$ and $3 \times 10^6$ particles respectively. These results are in good agreement with
values previously obtained by ourselves and, as discussed in a previous paper (Clay and Gerhardy 1981b), put stringent limits on fits to the Bourdeau et al. calculations since most of the theoretical results predict values which are appreciably higher.

Fig. 2 shows data on the attenuation of showers in the atmosphere. The best fit of an exponential to the data gives an attenuation length of $185 \pm 5\text{ g cm}^{-2}$ at an integral rate of $10^{-8}\text{ m}^{-2}\text{s}^{-1}\text{sr}^{-1}$. The shower sizes measured at given depths (Clay and Gerhardy 1981a) and integral rates agree within $\pm10\%$ with our previously reported results and the attenuation curves in general would be barely statistically different.

### Table 1. Sea level shower 'vertical' size for a number of integral intensities

<table>
<thead>
<tr>
<th>Column</th>
<th>Description</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Integral size (m$^{-2}$s$^{-1}$sr$^{-1}$)</td>
<td>10$^{-7}$</td>
<td>6.44 ± 0.10</td>
</tr>
<tr>
<td>2</td>
<td>Buckland Park size (10$^6$ particles)</td>
<td>10$^{-7}$</td>
<td>7.3 ± 1.1</td>
</tr>
<tr>
<td>3</td>
<td>Akeno size (10$^6$ particles)</td>
<td>10$^{-7}$</td>
<td>7.0 ± 0.1</td>
</tr>
<tr>
<td>4</td>
<td>Buckland Park size (Akeno size, 10$^6$ particles)</td>
<td>10$^{-7}$</td>
<td>7.5 ± 1.5</td>
</tr>
<tr>
<td>5</td>
<td>Primary energy (eV)</td>
<td>10$^{-7}$</td>
<td>7.5 x 10$^{15}$</td>
</tr>
</tbody>
</table>

The size spectrum of air showers is a critical factor in determining the energy spectrum of the primary cosmic rays at energies above $\sim 10^{14}\text{ eV}$. This spectrum is particularly important near the knee and we have therefore attempted to obtain a precise set of measurements on the size spectrum. Data available so far are in fair agreement but discrepancies seem to arise in absolute calibration by $\sim 20\%$ in shower size. With the use of the MINUIT fitting procedure with a variable $S$ we expect our shower sizes to be as correct as possible (after including corrections in the calibration procedure discussed by Crouch et al. 1981). Fig. 3 shows our best estimate of the size spectrum together with an indication of the spread of the data available from other work (Hillas 1974). It is clear that our data agree with other results and have good statistical accuracy. The Buckland Park array is the only array of its type in the Southern Hemisphere and measurements of this kind on the size spectrum may well become useful for comparison with Northern Hemisphere data if models for the knee involve mechanisms which are directionally dependent. The radius of gyration of particles of this energy is becoming comparable with large scale structure in the Galaxy and it is not obvious that the cosmic ray sources or propagation paths available to us should have the same statistical distribution as those available to Northern Hemisphere observatories. In any case, the precise shape of the knee is important even now in fitting models of cosmic ray origin and propagation. For instance Hillas (1979) has fitted models of cosmic ray composition and rigidity spectra to the knee and has been able to conclude that a consistent fit cannot be found for a model which has the knee being related to trajectories of the primary particles in a magnetic field.
We have attempted to compare our results with the Northern Hemisphere data compiled by the Akeno group (Hara et al. 1979b). In order that our data be quite consistent with theirs, we must make two corrections to our shower sizes. Firstly, our calibration factor for determining the amplitude corresponding to a single particle through the detector is 11% higher than that used at Akeno. It is our practice to assume that the mode of the pulse height distribution for all particles passing through our detectors equals the mean response of the detectors to vertical particles (Crouch et al. 1981). The Akeno group found a difference of 11% between the two and made an appropriate allowance in their derived shower sizes. Secondly, the Akeno group used a modified NKG structure function which, as we have discussed previously (Gerhardy et al. 1981), caused them to estimate shower sizes higher than ours by about 20%. Table 1 shows the result of adjusting our shower sizes to allow for these two effects. Northern Hemisphere (Akeno compilation) data and Southern Hemisphere data can thus be compared. It can readily be seen that the agreement in sea-level size for a given integral rate is good and, allowing for calibration uncertainties, it would seem that one can say that at a fixed shower size, the mean Northern and Southern Hemisphere intensities cannot differ by more than \( \sim 10\% \) for showers in the sea-level size range from \( \sim 7 \times 10^5 \) to \( \sim 7 \times 10^6 \) particles. In terms of the statistics of the experiments, the agreement is probably better than \( \sim 4\% \).

Table 1 also shows the primary particle energies for some integral intensities as derived by Protheroe (1978). It is clear that for our showers, a reasonable approximation to the relationship between shower size and primary energy is

\[ \text{primary energy (eV)} \approx \text{shower sea-level size} \times 10^{10}. \]

We note that, if our error bars are realistic, the integral size spectrum is not fitted by a pure power law anywhere between \( 3 \times 10^5 \) and \( 10^7 \) particles but is progressively steepening with size all the way to \( 10^7 \) particles where the value of the power law index may be as high as 2.4. This progressive change in index rather reduces the sharpness sometimes reported for the knee.

Conclusions

We have re-analysed data from 13 months recording of air showers at Buckland Park and found that there is a rapid change in shower age with size over our size range of interest (\( 3 \times 10^5 \)–\( 10^7 \) particles). When allowance is made for this change in the lateral distribution function, an attenuation length of \( 185 \pm 5 \, \text{g cm}^{-2} \) and absorption lengths of \( \sim 100 \, \text{g cm}^{-2} \) are found, and a size spectrum results which, whilst being consistent with previous work, is rather more convex than most previously reported results.

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References


Properties of Cosmic Ray Showers


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