Parametric Instabilities in a Magnetized Plasma

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Abstract
This paper makes a study of stimulated Raman scattering and stimulated Brillouin scattering of an incident electromagnetic pump wave in a magnetized plasma. The background magnetic field is taken to be parallel to the pump electric field. The growth rates of the two stimulated-scattering instabilities are found to be reduced in the presence of the background magnetic field.

1. Introduction
The parametric interaction of an intense coherent electromagnetic wave with collective modes in a plasma was investigated by Bornatici et al. (1969), Liu and Rosenbluth (1972), Drake et al. (1974), among others, in an attempt to provide an interpretation of observed phenomena in laser-produced plasmas and plasma-heating experiments. One of the consequences of the interaction of an incident (pump) electromagnetic wave with a plasma is the parametric excitation of two plasma waves. If the latter are both purely electrostatic, they are eventually absorbed in the plasma and this decay process then leads to enhanced (or anomalous) absorption of the incident electromagnetic wave. If one of the excited plasma waves is electromagnetic, it can escape from the plasma and show up as enhanced (or stimulated) scattering of the incident electromagnetic wave. This process can be of two types according to whether the other excited plasma wave is a Langmuir wave (stimulated Raman scattering) or an ion-acoustic wave (stimulated Brillouin scattering).

Experiments of Stamper et al. (1971) showed that intense spontaneously generated magnetic fields are present in laser-produced plasmas. These magnetic fields are usually strong enough to modify the spectrum of electrostatic modes in the plasma but not strong enough to influence the characteristics of propagation of the incident and scattered electromagnetic modes. The purpose of this paper is to study the stimulated Raman scattering and stimulated Brillouin scattering of an incident electromagnetic wave in a magnetized plasma.

2. A Prototype for Parametric Instabilities
As a prototype for the parametric processes discussed in this paper, consider a system acted on by an oscillatory pump (of large magnitude) of the form

\[ Z(t) = 2Z_0 \cos \omega_0 t, \] (1)
where \( Z_0 \) is taken to be a constant if one restricts consideration to initial stages of the ensuing instabilities in the system so that any depletion of the pump is then negligible. The pump induces a coupling between two natural modes of oscillation \( X(t) \) and \( Y(t) \) (with characteristic frequencies \( \omega_1 \) and \( \omega_2 \) respectively) say, in the form (Nishikawa 1968)

\[
\frac{d^2}{dt^2} + \omega_1^2 X(t) = \lambda Y(t) Z(t),
\]
\[
\frac{d^2}{dt^2} + \omega_2^2 Y(t) = \mu X(t) Z(t),
\]

where \( \lambda \) and \( \mu \) are coupling constants which are assumed to be such that \( \lambda \mu \) is real and positive.

Upon Fourier transforming according to

\[
Q(t) = \int_{-\infty}^{\infty} Q(\omega) \exp(-i\omega t) \, d\omega,
\]
equations (2) and (3) give

\[
(\omega^2 - \omega_1^2) X(\omega) = -\lambda Z_0 \{ Y(\omega - \omega_0) + Y(\omega + \omega_0) \},
\]
\[
(\omega^2 - \omega_2^2) Y(\omega) = -\mu Z_0 \{ X(\omega - \omega_0) + X(\omega + \omega_0) \}.
\]

Consider a resonant situation with

\[
\omega_0 \approx \omega_1 + \omega_2
\]

and retain in equations (5) and (6) only the terms \( X(\omega) \) and \( Y(\omega - \omega_0) \), so that the dispersion relation follows

\[
(\omega - \omega_1)(\omega - \omega_0 + \omega_2) + (\lambda \mu Z_0^2/4\omega_1 \omega_2) = 0.
\]

By putting

\[
\omega = \Omega + i\gamma,
\]
equation (8) gives us

\[
\gamma^2 \{ 1 + (\Delta^2/4\gamma^2) \} = \lambda \mu Z_0^2/4\omega_1 \omega_2,
\]

where

\[
\Delta \equiv \omega_0 - \omega_1 - \omega_2.
\]

The maximum growth rate occurs at perfect match (\( \Delta = 0 \)), and is given by

\[
\gamma_{\text{max}} = (\lambda \mu Z_0^2/4\omega_1 \omega_2)^{1/2}.
\]

3. Stimulated Raman Scattering

Consider a homogeneous plasma with a uniform background magnetic field \( B_0 \). A large amplitude plane-polarized electromagnetic pump wave

\[
E_i = 2E_i \cos(k_i x - \omega_i t),
\]

with \( E_i \) parallel to \( B_0 \), is incident on the plasma. The equilibrium state is comprised of electrons oscillating with velocity
\[ v_i = (2eE_i/m\omega_0) \sin(k_i \cdot x - \omega_i t), \]  

(13)

\( m \) being the mass of the electron, in the incident electric field \( E_i \), with the ions remaining stationary and making up a neutralizing background. Let us perturb this equilibrium and study the time development of these perturbations using the linearized fluid equations and Maxwell’s equations. In the stimulated Raman scattering process in a magnetized plasma, where only the electrons participate, the pump wave \((\omega_i, k_i)\) decays into an electromagnetic wave \((\omega_{i'}, k_{i'})\) and a modified Langmuir wave \((\omega, k_i)\) with the constraints

\[ \omega_i = \omega_{i'} + \omega_1, \quad k_i = k_{i'} + k_1. \]  

(14)

One obtains for the Langmuir wave

\[ (\partial n_1/\partial t) + N_0 \nabla \cdot v_1 = 0, \]  

(15)

\[ \frac{\partial v_1}{\partial t} + \frac{3KT_e}{mN_0} \nabla n_1 + \frac{e}{m} E_i + \frac{e}{mc} v_1 \cross B_0 \]  

\[ = -(v_{i'\cdot} \nabla v_{i'} + v_{i'} \cdot \nabla v_i) - (e/mc)(v_{i'} \cross B_i + v_{i'} \cross B_0), \]  

(16)

where \( n_1 \) is the perturbation in the number density, \( v_1 \) the velocity, \( T_e \) the temperature of the electrons, \( N_0 \) the number density in the unperturbed state, and note from

\[ \nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t} \]  

(17)

that

\[ (e/mc)B_i = \nabla \times v_1 \quad \text{and} \quad (e/mc)B_{i'} = \nabla \times v_{i'}. \]  

(18)

By using (18), equation (16) becomes

\[ \frac{\partial v_1}{\partial t} + \frac{3KT_e}{mN_0} \nabla n_1 + \frac{e}{m} (E_i + c^{-1} v_1 \cross B_0) = -\nabla(v_{i'\cdot} v_{i'}). \]  

(19)

Taking the divergence of equation (19), using

\[ \nabla \cdot E_i = -4\pi e n_1 \]  

(20)

and equation (15) and considering propagation perpendicular to \( B_0 \) (i.e. \( k_i \cdot B_0 = 0 \)), one obtains

\[ (\partial^2/\partial t^2 + \omega_0^2)n_1 = N_0 \nabla^2(v_{i'\cdot} v_{i'}), \]  

(21)

where

\[ \omega_0^2 = \omega_1^2 + \omega_{ce}^2, \quad \omega_i^2 = \omega_{pe}^2 + 3k_i^2 (KT_e/m), \]  

\[ \omega_{ce} = eB_0/mc, \quad \omega_{pe}^2 = 4\pi N_0 e^2/m. \]

Next, from

\[ \nabla^2 E_{i'} - \frac{1}{c^2} \frac{\partial^2 E_{i'}}{\partial t^2} = -\frac{4\pi e}{c^2} \frac{\partial}{\partial t} (n_1 v_{i'}), \]  

(22)
one obtains for the scattered electromagnetic wave

$$\left( \frac{\partial^2}{\partial t^2} + \omega_i^2 \right) E_i' = 4\pi e \frac{\partial}{\partial t}(n_i v_i),$$  \hspace{1cm} (23)

where

$$\omega_i^2 = \omega_{pe}^2 + k_i^2 c^2,$$

and we have used the fact that the magnetic field $B_0$ is not strong enough to influence the characteristics of propagation of the incident and the scattered electromagnetic waves.

Noting that

$$\nabla^2(v_t \cdot v_i') = -\frac{e^2 k_t^2}{m^2 \omega_t \omega_i'} E_t \cdot E_i', \quad \frac{\partial}{\partial t}(n_i v_i) = -\frac{e\omega_t}{m\omega_i} n_i E_i,$$

equations (21) and (23) become

$$\left( \frac{\partial^2}{\partial t^2} + \omega_i^2 \right) n_i = \lambda R E_i E_i',$$ \hspace{1cm} (25)

$$\left( \frac{\partial^2}{\partial t^2} + \omega_i^2 \right) E_i' = \mu R E_i n_i,$$ \hspace{1cm} (26)

where

$$\lambda_R = -\frac{\omega_{pe}^2}{\omega_t \omega_i'} \frac{k_i^2}{4\pi m} (\hat{e}_i \cdot \hat{e}_i'), \quad \mu_R = -\frac{\omega_t}{\omega_i} \frac{\omega_{pe}^2}{N_0} (\hat{e}_i \cdot \hat{e}_i');$$

$\hat{e}_i$ and $\hat{e}_i'$ are the directions of polarization of $E_i$ and $E_i'$.

Equations (25) and (26) are of the same form as (2) and (3), so that the maximum growth rate for stimulated Raman scattering is given from equation (11) as

$$\gamma_R = \frac{\omega_{pe}^2 k_i |\hat{e}_i \cdot \hat{e}_i'| E_i}{4\omega_t \omega_i' \pi N_0 m (\omega_i^2 + \omega_{pe}^2)^{1/2}}.$$ \hspace{1cm} (27)

Observe that the growth rate is reduced in the presence of a background magnetic field. In the absence of the latter, (27) reduces to the one deduced by Bornatici et al. (1969).

### 4. Stimulated Brillouin Scattering

In the stimulated Brillouin scattering process the pump wave ($\omega_i, k_i$) decays into an electromagnetic wave ($\omega_i', k_i'$) and an ion-acoustic wave ($\omega_s, k_s$), with the constraints

$$\omega_i = \omega_i' + \omega_s, \quad k_i = k_i' + k_s.$$ \hspace{1cm} (28a, b)

Here both electrons and ions participate in the motion of the ion-acoustic wave. One obtains for the electrons moving in this wave

$$\frac{Kn_e}{mN_0} \nabla n_e + \frac{e}{m} \left( E_s + \frac{1}{c} v_e \times B_0 \right)$$

$$= -\left( v_t \cdot \nabla v_t + v_i' \cdot \nabla v_i' \right) - \frac{e}{mc} (v_t \times B_t + v_i' \times B_i)$$

$$= -\nabla (v_t \cdot v_t),$$ \hspace{1cm} (29)
where we have ignored the electron inertia, and have assumed that the electrons respond isothermally to the ion-acoustic wave. One has for the ions moving in the wave

\[
(\frac{\partial n_{is}}{\partial t} + N_0 \nabla \cdot v_{is}) = 0 ,
\]

\[
\frac{\partial v_{is}}{\partial t} + \frac{3KT_i}{MN_0} \nabla n_{is} - \frac{e}{M} \left( E_s + \frac{1}{c} v_{is} \times B_0 \right) = 0 ,
\]

with

\[
\nabla \cdot E_s = 4\pi e (n_{is} - n_{es}) ,
\]

where \( M \) is the mass of an ion, and \( T_i \) the temperature of the ions.

Taking the divergence of equation (30), using equations (29) and (33), and considering propagation perpendicular to \( B_0 \) (i.e. \( k_s \cdot B_0 = 0 \)), one obtains

\[
n_{es} = \frac{1}{1 + k_s^2 \lambda_D^2 + \omega_{ce}^2/\omega_{pe}^2} \left( n_{is} + \frac{N_0}{\omega_{pe}} \nabla^2 (v_1 \cdot v_1) \right) ,
\]

where

\[
\lambda_D^2 = \frac{KT_e}{m \omega_{pe}^2} .
\]

Further, taking the divergence of equation (32) and using equations (31), (33) and (34), one obtains

\[
\left( \frac{\partial^2}{\partial t^2} + \omega_s^2 + \omega_s^2 \right) n_{is} = \frac{(m/M)N_0}{1 + k_s^2 \lambda_D^2 + \omega_{ce}^2/\omega_{pe}^2} \nabla^2 (v_1 \cdot v_1) ,
\]

where

\[
\omega_s^2 = \frac{k_s^2 C_s^2}{1 + k_s^2 \lambda_D^2 + \omega_{ce}^2/\omega_{pe}^2} + 3k_s^2 \frac{KT_i}{M} ,
\]

\[C_s = \frac{KT_e}{M} .\]

Using

\[
\nabla^2 E_{s'} - \frac{1}{c^2} \frac{\partial^2 E_{s'}}{\partial t^2} = - \frac{4\pi e}{c^2} \frac{\partial}{\partial t} (n_{es} v_t)
\]

and equation (34), one obtains for the scattered electromagnetic wave

\[
\left( \frac{\partial^2}{\partial t^2} + \omega_{t'}^2 \right) E_{s'} = \frac{4\pi e}{1 + k_s^2 \lambda_D^2 + \omega_{ce}^2/\omega_{pe}^2} \frac{\partial}{\partial t} (v_1 n_{is}) .
\]

From (24), equations (35) and (37) can be written as

\[
(\frac{\partial^2}{\partial t^2} + \omega_s^2 + \omega_s^2) n_{is} = \lambda_B E_s E_{s'} ,
\]

\[
(\frac{\partial^2}{\partial t^2} + \omega_t^2) E_{s'} = \mu_B E_s n_{is} ,
\]

where

\[
\lambda_B = - \frac{(\omega_{pe}^2/\omega_s \omega_{t'}) (k_s^2/4\pi M)}{1 + k_s^2 \lambda_D^2 + \omega_{ce}^2/\omega_{pe}^2} (\hat{e}_s \cdot \hat{e}_{s'}) ,
\]

\[
\mu_B = - \frac{(\omega_{s}/\omega_t)(\omega_{pe}^2/N_0)}{1 + k_s^2 \lambda_D^2 + \omega_{ce}^2/\omega_{pe}^2} (\hat{e}_s \cdot \hat{e}_{s'}) .
\]
Equations (38) and (39) are again of the same form as (2) and (3), so that the maximum growth rate for stimulated Brillouin scattering is given from equation (11) as
\[
\gamma_B = \frac{\{\omega_{ee}^2 k_s/\omega_l(4\pi MN_0)^{1/2}\} |\vec{e}_i \cdot \vec{e}_r| E_i}{2(1 + k_x^2 \lambda_B^2 + \omega_{ce}^2/\omega_{pe}^2)\{\omega_l(\omega_{ee}^2 + \omega_{ei}^2)^{1/2}\}^{1/2}}. \tag{40}
\]

Observe that the growth rate is again reduced in the presence of a background magnetic field. In the absence of the latter, equation (40) reduces to the one deduced by Liu and Rosenbluth (1972).

References


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