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Coronal Hole Dynamics

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Abstract

The dynamics of high speed streams of solar plasma emanating from a coronal hole is investigated by use of a two-fluid model with polytropic equations of state. Steady outflow is considered along a flow tube which has a radial orientation with respect to the Sun, and a cross-sectional area proportional to r^s where r is the heliocentric radius and s is a divergence parameter ($\geqslant 2$). All the flow variables are assumed to be functions of r only. The equations of continuity, momentum and state may be used to obtain a single, nonlinear, ordinary differential equation for the outflow velocity, and the problem reduces to the numerical solution of three pairs of simultaneous algebraic equations. It is found that the velocity profiles are generally highly dependent on the divergence parameter s, as well as the polytropic indices. Numerical results are given for a variety of cases most relevant to the solar corona. As s increases from 2, the value appropriate to purely spherically symmetric expansion, the outflow velocity *increases* throughout the range from the coronal base out to infinity, over a certain parameter range. Although the terminal outflow speed for $s > 2$ may be far in excess of the purely spherically symmetric value, we find that high speed streams emanating from coronal holes cannot be accounted for by geometrical effects alone. The results may have important applications in the general theory of stellar winds.

1. Introduction

A coronal hole is a region of the solar corona in which the density and temperature are abnormally low, and is characterized by diverging magnetic field lines of single polarity. Coronal holes were first recognized by Burton (1968) and Tousey *et al.* (1968), and are now generally considered to be the source of high speed streams in the solar wind and possibly the major source of solar wind plasma (Munro and Withbroe 1972; Kopp and Holzer 1976; Hundhausen 1977; Zirker 1977; Munro and Jackson 1977). Coronal hole models have been devised with varying degrees of sophistication (see Pneuman 1980, and references therein). In common with most previous authors, we shall assume an idealized, hypothetical magnetic field configuration, and examine coronal expansion in a specified non-radial flow tube. We shall extend the well known model of Parker (1963) to obtain the new result that, as the divergence of a flow tube increases, the expansion velocity of the fluid *increases* throughout the flow, under certain parameter restrictions. The application of this model to coronal holes is new. Although the model predicts that high speed streams emanating from coronal holes cannot be accounted for by purely geometric effects, the results may have important implications in stellar wind theory.

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2. Assumptions

(1) The coronal gas is a fully ionized, electrically neutral hydrogen plasma, i.e. an electron–proton gas in which the electron number density is equal to the proton number density.

(2) The electron and proton gases behave as fluids, each with a bulk radial streaming speed u, and respective macroscopic temperatures T_e and T_p.

(3) The electron and proton gases are ideal, and are described by polytropic equations of state in which the respective polytropic indices are α_e and α_p $(1 \leqslant \alpha_e \leqslant \alpha_p \leqslant \frac{5}{3})$.

(4) Coronal outflow is steady and takes place in flow tubes oriented radially to the Sun, but with a cross-sectional area proportional to r^s, where r is the heliocentric radius and s $(\geqslant 2)$ is a parameter which measures the divergence of the flow tube; purely spherically symmetric flow corresponds to $s = 2$.

(5) All dependent variables are functions of r only.

(6) The effects of solar rotation are negligible.

3. Equations of Model

For the equations of the model we have: the equation of continuity

$$\rho u r^s = \rho_0 u_0 r_0^s; \tag{1}$$

the equation of motion

$$\rho u \frac{\mathrm{d}u}{\mathrm{d}r} = -\frac{\mathrm{d}}{\mathrm{d}r}(P_e + P_p) - \frac{\rho G M}{r^2}; \tag{2}$$

the ideal gas laws

$$P_e = (k_B \rho/2m)T_e, \qquad P_p = (k_B \rho/2m)T_p, \tag{3a, b}$$

$$P_{e0} = (k_B \rho_0/2m)T_{e0}, \qquad P_{p0} = (k_B \rho_0/2m)T_{p0}; \tag{3c, d}$$

and the polytropic equations of state

$$P_e = P_{e0}(\rho/\rho_0)^{\alpha_e}, \qquad P_p = P_{p0}(\rho/\rho_0)^{\alpha_p}. \tag{4a, b}$$

In equation (1) r_0 is the coronal base radius. Throughout this paper the zero subscript is used to denote a quantity evaluated at $r = r_0$. The remaining quantities in equations (1)–(4) that have not been defined hitherto are as follows: ρ is the total mass density of the gas; P_e and P_p are the respective electron and proton gas pressures; m is the mean particle mass; k_B is the Boltzmann constant; M is the solar mass; and G is the constant of gravitation.

If we adopt the nondimensional variables

$$\psi = mu^2/2k_B T_{e0}, \qquad \xi = r/r_0,$$

then from equations (1)–(4) we may derive the equation

$$\frac{\xi^2}{\psi}\frac{\mathrm{d}\psi}{\mathrm{d}\xi}\left\{\psi - \tfrac{1}{4}\alpha_e\left(\frac{\psi_0}{\psi}\right)^{\frac{1}{2}(\alpha_e-1)}\xi^{-s(\alpha_e-1)} - \tfrac{1}{4}\alpha_p\gamma\left(\frac{\psi_0}{\psi}\right)^{\frac{1}{2}(\alpha_p-1)}\xi^{-s(\alpha_p-1)}\right\}$$

$$= \tfrac{1}{2}\alpha_e s\left(\frac{\psi_0}{\psi}\right)^{\frac{1}{2}(\alpha_e-1)}\xi^{1-s(\alpha_e-1)} + \tfrac{1}{2}\alpha_p s\gamma\left(\frac{\psi_0}{\psi}\right)^{\frac{1}{2}(\alpha_p-1)}\xi^{1-s(\alpha_p-1)} - \beta, \tag{5}$$

where we have introduced the nondimensional constants

$$\beta = mGM/k_B T_{e0} r_0, \qquad \gamma = T_{p0}/T_{e0}.$$

Since all the flow variables in the model are expressible in terms of ψ only, we have thus reduced the problem to the solution of the nonlinear ordinary differential equation (5) for ψ.

We may note that by setting $\alpha_e = \alpha_p = \alpha$ and $\gamma = 1$ we obtain $T_e = T_p$, and equation (5) then describes a one-fluid model (Parker 1963).

4. Analysis

Equation (5) may be integrated to give the following (Bernoulli) energy integrals:

$$\psi + \tfrac{1}{2}\left(\frac{\alpha_e}{\alpha_e-1}\right)\left(\frac{\psi_0}{\psi}\right)^{\frac{1}{2}(\alpha_e-1)} \xi^{-s(\alpha_e-1)} + \tfrac{1}{2}\gamma\left(\frac{\alpha_p}{\alpha_p-1}\right)\left(\frac{\psi_0}{\psi}\right)^{\frac{1}{2}(\alpha_p-1)} \xi^{-s(\alpha_p-1)} - \frac{\beta}{\xi}$$

$$= \psi_0 + \tfrac{1}{2}\left(\frac{\alpha_e}{\alpha_e-1}\right) + \tfrac{1}{2}\gamma\left(\frac{\alpha_p}{\alpha_p-1}\right) - \beta, \qquad 1 < \alpha_e \leqslant \alpha_p \leqslant \tfrac{5}{3}; \tag{6}$$

$$\psi + \tfrac{1}{2}\log\left\{\left(\frac{\psi_0}{\psi}\right)^{\frac{1}{2}} \xi^{-s}\right\} + \tfrac{1}{2}\gamma\left(\frac{\alpha_p}{\alpha_p-1}\right)\left(\frac{\psi_0}{\psi}\right)^{\frac{1}{2}(\alpha_p-1)} \xi^{-s(\alpha_p-1)} - \frac{\beta}{\xi}$$

$$= \psi_0 + \tfrac{1}{2}\gamma\left(\frac{\alpha_p}{\alpha_p-1}\right) - \beta, \qquad 1 = \alpha_e < \alpha_p \leqslant \tfrac{5}{3}; \tag{7}$$

$$\psi + \tfrac{1}{2}(1+\gamma)\log\left\{\left(\frac{\psi_0}{\psi}\right)^{\frac{1}{2}} \xi^{-s}\right\} - \frac{\beta}{\xi} = \psi_0 - \beta, \qquad \alpha_e = \alpha_p = 1. \tag{8}$$

Since we expect the coronal gas to expand from the coronal base into the negligible pressure of interplanetary space via a subsonic–supersonic transition, then we shall require that the solution ψ passes through a critical point (ψ_c, ξ_c) (itself a sonic point) of equation (5), and that the branch ψ is chosen such that the pressure $P \to 0$ as $\xi \to \infty$. From equation (5) we find that

$$\psi_c - \tfrac{1}{4}\alpha_e\left(\frac{\psi_0}{\psi_c}\right)^{\frac{1}{2}(\alpha_e-1)} \xi_c^{-s(\alpha_e-1)} - \tfrac{1}{4}\alpha_p\gamma\left(\frac{\psi_0}{\psi_c}\right)^{\frac{1}{2}(\alpha_p-1)} \xi_c^{-s(\alpha_p-1)} = 0, \tag{9}$$

$$-\beta + \tfrac{1}{2}\alpha_e s\left(\frac{\psi_0}{\psi_c}\right)^{\frac{1}{2}(\alpha_e-1)} \xi_c^{1-s(\alpha_e-1)} + \tfrac{1}{2}\alpha_p s\gamma\left(\frac{\psi_0}{\psi_c}\right)^{\frac{1}{2}(\alpha_p-1)} \xi_c^{1-s(\alpha_p-1)} = 0. \tag{10}$$

Equations (9) and (10) may be expressed in the form

$$2s\xi_c\psi_c = \beta, \tag{11}$$

$$\psi_c^{\frac{1}{2}\{2s+1-\alpha_e(2s-1)\}} - \tfrac{1}{4}\alpha_e\psi_0^{\frac{1}{2}(\alpha_e-1)}\left(\frac{2s}{\beta}\right)^{s(\alpha_e-1)}$$

$$-\tfrac{1}{4}\alpha_p\gamma\psi_0^{\frac{1}{2}(\alpha_p-1)}\left(\frac{2s}{\beta}\right)^{s(\alpha_p-1)}\psi_c^{\frac{1}{2}(2s-1)(\alpha_p-\alpha_e)} = 0. \tag{12}$$

Moreover, the critical point (ψ_c, ξ_c) must be a saddle-point (or X-type singular point). The conditions that the point (ψ_c, ξ_c) both lies on the integral curves (6)–(8) and is also a saddle-point lead to the further results:

$$\begin{aligned}
&\tfrac{1}{2}\left(\frac{\alpha_e}{\alpha_e-1}\right)\psi_0^{\frac{1}{2}(\alpha_e-1)}\left(\frac{2s}{\beta}\right)^{s(\alpha_e-1)}\psi_c^{\frac{1}{2}(2s-1)(\alpha_e-1)} \\
&+\tfrac{1}{2}\gamma\left(\frac{\alpha_p}{\alpha_p-1}\right)\psi_0^{\frac{1}{2}(\alpha_p-1)}\left(\frac{2s}{\beta}\right)^{s(\alpha_p-1)}\psi_c^{\frac{1}{2}(2s-1)(\alpha_p-1)}-(2s-1)\psi_c \\
&= \psi_0+\tfrac{1}{2}\left(\frac{\alpha_e}{\alpha_e-1}\right)+\tfrac{1}{2}\gamma\left(\frac{\alpha_p}{\alpha_p-1}\right)-\beta, \qquad 1<\alpha_e<\alpha_p\leqslant\frac{2s+1}{2s-1}, \quad s\geqslant 2 \\
&\qquad\qquad \text{or} \quad 1<\alpha_e=\alpha_p<\frac{2s+1}{2s-1}, \quad s\geqslant 2; \qquad (13)
\end{aligned}$$

$$\begin{aligned}
&\tfrac{1}{2}\log\left\{\psi_0^{\frac{1}{2}}\left(\frac{2s}{\beta}\right)^{s}\psi_c^{\frac{1}{2}(2s-1)}\right\}+\tfrac{1}{2}\gamma\left(\frac{\alpha_p}{\alpha_p-1}\right)\psi_0^{\frac{1}{2}(\alpha_p-1)}\left(\frac{2s}{\beta}\right)^{s(\alpha_p-1)}\psi_c^{\frac{1}{2}(2s-1)(\alpha_p-1)}-(2s-1)\psi_c \\
&= \psi_0+\tfrac{1}{2}\gamma\left(\frac{\alpha_p}{\alpha_p-1}\right)-\beta, \qquad 1=\alpha_e<\alpha_p\leqslant\frac{2s+1}{2s-1}, \quad s\geqslant 2; \qquad (14)
\end{aligned}$$

$$\begin{aligned}
&\tfrac{1}{2}(1+\gamma)\log\left\{\psi_0^{\frac{1}{2}}\left(\frac{2s}{\beta}\right)^{s}\psi_c^{\frac{1}{2}(2s-1)}\right\}-(2s-1)\psi_c \\
&= \psi_0-\beta, \qquad \alpha_e=\alpha_p=1, \quad s\geqslant 2. \qquad (15)
\end{aligned}$$

Thus, the problem is finally reduced to the solution of the three pairs of simultaneous algebraic equations (12) and (13), (12) and (14), and (12) and (15). For the required critical solution corresponding to an appropriate solution pair (ψ_0, ψ_c) of equations (12) and (13), the speed of the coronal gas increases monotonically to approach a terminal value u_∞ given by

$$\psi_\infty = \frac{mu_\infty^2}{2k_B T_{e0}} = \psi_0+\tfrac{1}{2}\left(\frac{\alpha_e}{\alpha_e-1}\right)+\tfrac{1}{2}\gamma\left(\frac{\alpha_p}{\alpha_p-1}\right)-\beta. \qquad (16)$$

For similar critical solutions obtained from equations (12) and (14), and (12) and (15), the gas speed increases monotonically and without bound according to the asymptotic forms $\psi = \frac{1}{2}s\log\xi + \ldots$, and $\psi = s\log\xi + \ldots$, as $\xi \to \infty$, in the respective cases. In the special case $\alpha_e = \alpha_p = 1$, equation (12) yields the solution $\psi_c = \frac{1}{4}(1+\gamma)$, the corresponding value of ψ_0 then being given implicitly by equation (15). In general, the pairs of equations (12) and (13), and (12) and (14), do not possess explicit analytic solutions. However, in the special and important case $s = s^* = 2\beta/(\alpha_e+\gamma\alpha_p)$, *each* pair of equations (12) and (13), (12) and (14), and (12) and (15) has the solution $\psi_0 = \psi_c = \frac{1}{4}(\alpha_e+\gamma\alpha_p)$, corresponding to $\xi_c = 1$ from equation (11). Physically, this solution corresponds to the limiting case when the outflow of coronal gas is supersonic immediately upon leaving the coronal base. We shall make particular use of this solution in the following section to demonstrate how in the present model an increase in the flow divergence parameter s causes the critical (sonic) point in the flow to approach the coronal base, with a consequent increase in velocity throughout the flow and, in particular, at the orbit of the Earth.

Table 1. Coronal expansion parameters derived from equations (6)–(8) and (11)–(16)

The expansion velocities u_0 at the coronal base, u_E at the orbit of the Earth, and u_∞ at infinity, are expressed in $\mathrm{km\,s^{-1}}$. The upper and lower values in each entry correspond to $\beta = 5\cdot773$ ($T_0 = 2\times10^6$ K) and $\beta = 7\cdot697$ ($T_0 = 1\cdot5\times10^6$ K) respectively

α_e, α_p	$[u_0]_{s=2}$	$[u_E]_{s=2}$	$[u_\infty]_{s=2}$	$[\xi_c]_{s=2}$	s^*	$[u_0]_{s=s^*}$	$[u_E]_{s=s^*}$	$[u_\infty]_{s=s^*}$
1, 1	21·3	753	—	2·89	5·77	182	1354	—
	4·75	628	—	3·85	7·69	157	1341	—
1·01, 1·01	20·02	714	2508	2·99	5·72	183	1229	2514
	4·14	589	2150	4·06	7·62	158	1180	2155
1·1, 1·1	9·6	421	588	4·69	5·25	191	587	618
	0·29	267	404	9·90	6·99	165	425	437
1, 1·1	16·0	610	—	3·44	5·49	186	1017	—
	2·27	487	—	5·04	7·33	161	966	—
1, 1·2	11·1	539	—	4·11	5·25	191	915	—
	0·89	428	—	6·34	6·99	165	876	—
1, 1·3	7·18	508	—	4·78	5·02	195	861	—
	0·37	409	—	7·19	6·69	169	828	—
1, 1·4	4·86	493	—	5·26	4·81	199	822	—
	0·19	403	—	7·53	6·41	172	792	—
1, 1·5	3·45	488	—	5·53	4·62	203	790	—
	0·12	402	—	7·65	6·16	176	762	—
1, 1·6	2·61	486	—	5·66	4·44	207	763	—
	$8\cdot7\times10^{-2}$	402	—	7·68	5·92	179	736	—
1, $\frac{5}{3}$	2·21	486	—	5·71	4·33	210	747	—
	$7\cdot4\times10^{-2}$	402	—	7·69	5·77	182	719	—
1·1, 1·2	4·08	302	424	5·118	5·02	195	443	467
	$8\cdot1\times10^{-4}$	199	199	37·01	6·69	169	251	261

5. Results and Discussion

Equations (6)–(8) and (11)–(15) were solved for a variety of conditions expected to prevail in the solar corona. The results are given in Table 1 in which the upper entries correspond to $\beta = 5\cdot773$ ($T_0 = 2\times10^6$ K) and the lower entries to $\beta = 7\cdot697$ ($T_0 = 1\cdot5\times10^6$ K). Values for the expansion velocity at the coronal base, the orbit of the Earth and infinity (u_0, u_E and u_∞ respectively) are given both in the purely spherically symmetric case ($s = 2$), and in the case in which the divergence parameter s takes on its special (maximum) value $s = s^* = 2\beta/(\alpha_e+\gamma\alpha_p)$. For each case of purely spherically symmetric flow the location of the critical (sonic) point is also given. It has been assumed that $\gamma = T_{p0}/T_{e0} = 1$, and that $r_0 = R_\odot$, the solar radius. The change in the divergence parameter from $s = 2$ to $s = s^*$ is designed to simulate a change from purely spherically symmetric coronal outflow to outflow from a coronal hole, and hence the results in Table 1 may be readily used to check the effectiveness of the present model. Observations reveal that the 'quiet' solar wind velocity u_E at the orbit of the Earth is about 450 $\mathrm{km\,s^{-1}}$, corresponding to a coronal base velocity $u_0 \leqslant 10\ \mathrm{km\,s^{-1}}$ and temperature $T_0 \leqslant 2\times 10^6$ K. High speed solar wind streams typically reach speeds greater than or equal to $600\ \mathrm{km\,s^{-1}}$ and, while corresponding coronal base conditions are not completely known (Munro and Jackson 1977; Crifo-Magnant and Picat 1980; Kovalenko and Molodykh 1980), it is expected that coronal expansion speeds reach at least $100\ \mathrm{km\,s^{-1}}$ within a distance of one solar radius of $r = R_\odot$, and that coronal base temperatures are lower than those in quiet regions. In all cases in Table 1 the values of u_0, u_E and u_∞ (u_∞ does not exist if $\alpha_\mathrm{e} = 1$) for $s = s^*$ (>2) exceed their corresponding values for $s = 2$. In particular, in all cases we find that $[u_0]_{s=s^*} \geqslant 157\ \mathrm{km\,s^{-1}}$. These two features support the argument that the dynamical configuration in the present model reflects some of the gross characteristics of coronal hole dynamics. It is clear from the results that an increase in the divergence parameter from $s = 2$ can lead to a great increase in solar wind speed at the Earth, in some cases more than a 100% increase while commonly more than 60%. The 'best-fit' cases, namely those cases that typically might represent observed high speed streams, are included in the cases $\alpha_\mathrm{e} = 1$ (isothermal electrons), and $1\cdot 1 \leqslant \alpha_\mathrm{p} \leqslant 1\cdot 3$. We observe that the two-fluid case with non-isothermal electrons $\alpha_\mathrm{e} = 1\cdot 1$, $\alpha_\mathrm{p} = 1\cdot 2$ for $s = 2$ and $\beta = 5\cdot 773$ produces a rather low value of u_E ($= 302\ \mathrm{km\,s^{-1}}$), and for $s = s^* = 5\cdot 02$ does not produce a realistic high speed stream. Of the one-fluid cases ($\alpha_\mathrm{e} = \alpha_\mathrm{p}$) the case $\beta = 5\cdot 773$, $\alpha_\mathrm{e} = \alpha_\mathrm{p} = 1\cdot 1$, $s = s^* = 5\cdot 25$ best simulates a high speed stream. Nevertheless, it is clear from the analysis in Section 4 and the results in Table 1 that it is not possible to account for high speed streams emanating from a coronal hole by an appropriate divergence of the flow tube. A more effective model of gas emanating from a coronal hole must incorporate a realistic energy equation, as opposed to the above polytropic equations of state. Unfortunately, at present neither theory nor observation is sufficiently advanced to permit a formulation of the appropriate energy equation. The results of the present paper may have important implications in stellar wind theory with regard to the calculation of the terminal velocity of a stellar wind. On the basis of the present calculations based on solar parameters, the terminal wind speed appropriate to a strongly diverging flow tube may be more than *twice* its value corresponding to purely spherically symmetric flow. Thus it is clear that the assumed flow-tube geometry in a general stellar wind model may be a crucial factor in the calculation of its terminal speed.

We note that Adams and Sturrock (1975) incorporated in their coronal hole model a one-fluid version of the above flow tube with divergence parameter s, but in their model the asymptotic (terminal) flow speed *decreases* as s increases, contrary to observation.

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