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Isospin Mixing in 4^- States of ^{12}C*

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Abstract

Fits are made to measured $^{12}C(\pi^{\pm},\pi^{\pm})^{12}C$ cross sections in terms of an isospin-mixed doublet of 4^- ^{12}C levels near 19·5 MeV, using the *R*-matrix theory of nuclear reactions. Parameter values are restricted by information from other reactions and from bound-state shell model calculations.

1. Introduction

The $^{12}C(\pi^{\pm},\pi^{\pm})^{12}C$ cross sections measured by Morris *et al.* (1979) indicated the existence of a strongly isospin-mixed doublet near 19·5 MeV excitation in ^{12}C. This is believed to have $J^{\pi} = 4^-$ (Siciliano and Weiss 1980). Halderson *et al.* (1981) used the recoil corrected continuum shell model to calculate the 4^- contributions $\sigma_{\pi\pm}$ to the cross sections. They assumed that the ^{12}C ground state has the closed $1p_{3/2}$ shell configuration, that the ^{11}B and ^{11}C ground states are pure $1p_{3/2}^{-1}$ states and that the 4^- states of ^{12}C are the stretched states of the 1p–1h configuration $1p_{3/2}^{-1}d_{5/2}$. With a realistic effective nucleon–nucleon interaction, they were able to obtain cross sections giving a difference $\sigma_{\pi+}-\sigma_{\pi-}$ consistent with that observed by Morris *et al.*

Halderson *et al.* (1981) stressed the unbound nature of the 4^- states. It is possible, however, to make use of bound-state shell model calculations in conjunction with the *R*-matrix theory of nuclear reactions (Lane and Thomas 1958) in order to describe these states, as has been done for other cases of isospin mixing in unbound states (Barker 1977; Barker and Ferdous 1978); this is the approach followed here. The cross sections are expressed in terms of parameters (eigenenergies, reduced width amplitudes, feeding amplitudes), values of which may be obtained from the shell model calculations for comparison with those required to fit the data. One disadvantage is that the latter parameter values are dependent on the choice of channel radius. Another is that this method is less unified than that of Halderson *et al.*; however, it has the advantage of greater flexibility and simplicity, and it is not necessarily less accurate. In particular, in the present case involving the 4^- states of ^{12}C, we do not assume unreasonably simple configurations for the $A = 11$ and 12 ground states and the 4^- states of ^{12}C.

The measurements of the $^{12}C(\pi^{\pm},\pi^{\pm})^{12}C$ cross sections by Morris *et al.* (1979) had an energy resolution of about 500 keV. More recent measurements by the

* Dedicated to the memory of Professor S. T. Butler who died on 15 May 1982.

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Los Alamos group (C. L. Morris, personal communication) with a resolution of about 150 keV are shown in Fig. 1*. Fits to these data are made in Section 3, and information on the 4^- levels from other reactions is considered in Section 4.

2. Cross Sections for $^{12}C(\pi^\pm, \pi^\pm)^{12}C(4^-)$

We write the cross sections for populating the 4^- levels in $^{12}C(\pi^\pm, \pi^\pm)^{12}C$ as (Barker 1967; Barker and Ferdous 1978)

$$\sigma_x = \sum_c \sigma_{xc} \qquad (x = \pi^\pm), \tag{1}$$

with

$$\sigma_{xc} = kP_c |\sum_{\lambda\mu} \gamma_{\lambda c} g_{\mu x} A_{\lambda\mu}|^2, \tag{2}$$

where $A_{\lambda\mu}$ are elements of a matrix in level space, defined by its inverse:

$$(\mathbf{A}^{-1})_{\lambda\mu} = (E_\lambda - E)\delta_{\lambda\mu} - \sum_c L_c^0 \gamma_{\lambda c} \gamma_{\mu c}; \qquad L_c^0 = S_c - B_c + \mathrm{i} P_c. \tag{3}$$

Here E_λ is the eigenenergy of the 4^- level λ, $\gamma_{\lambda c}$ its reduced width amplitude for the decay channel c and $g_{\lambda\pi\pm}$ its feeding amplitude for the reaction $^{12}C(\pi^\pm, \pi^\pm)^{12}C$, while $S_c(E)$ and $P_c(E)$ are the shift factor and penetration factor, evaluated at the channel radius a_c, and B_c is the boundary condition parameter (Lane and Thomas 1958). Also, k is a constant if a weak dependence on the energy of the emitted $\pi^\pm$ is neglected.

In the present case, we include contributions from two 4^- levels, with $\lambda = \mathrm{a}, \mathrm{b}$ ($E_\mathrm{a} < E_\mathrm{b}$). The sum in equation (3) is over all channels c, both open and closed. We neglect contributions from closed channels. For energies near 19·5 MeV excitation in ^{12}C, only α-particle, proton and neutron channels are open. Decay of a 4^- level by f-wave α-particle emission to the 2^+ first excited state of ^8Be is allowed with an available energy of about 9 MeV; this channel is labelled $c = \alpha$. The α channel to the 4^+ excited state of ^8Be at 11·4 MeV is neglected because the available energy is low (~0·7 MeV). The proton channel to the $\frac{1}{2}^-$ first excited state of ^{11}B requires g-wave protons and is neglected. Two channels remain, the $^{11}B(g.s.)+p$ and the $^{11}C(g.s.)+n$ channels, which we label $c = \mathrm{p}$ and n respectively. Because of the large channel energy in the channel α, we assume that P_α and S_α are constants over the energy range of interest and choose $B_\alpha = S_\alpha$. For simplicity we choose $B_c = S_c(19\cdot5 \text{ MeV})$ for $c = \mathrm{p}$ and n, without loss of generality (Barker 1972). We also take $a_\mathrm{n} = a_\mathrm{p}$ and denote this by a_c.

In the two-state isospin-mixing model for nearly degenerate states, the states a and b are orthogonal mixtures of $T = 0$ and $T = 1$, 4^- states (Barker 1966, 1978):

$$\Psi_\mathrm{a} = \alpha\Psi_0 + \beta\Psi_1, \qquad \Psi_\mathrm{b} = \beta\Psi_0 - \alpha\Psi_1 \qquad (\alpha^2 + \beta^2 = 1). \tag{4a, b}$$

Then, we have

$$\begin{aligned} \gamma_{\mathrm{ap}} &= 2^{-\frac{1}{2}}(\alpha\gamma_0 + \beta\gamma_1), & \gamma_{\mathrm{bp}} &= 2^{-\frac{1}{2}}(\beta\gamma_0 - \alpha\gamma_1), \\ \gamma_{\mathrm{an}} &= 2^{-\frac{1}{2}}(-\alpha\gamma_0 + \beta\gamma_1), & \gamma_{\mathrm{bn}} &= 2^{-\frac{1}{2}}(-\beta\gamma_0 - \alpha\gamma_1), \\ \gamma_{\mathrm{a}\alpha} &= \alpha\gamma_{0\alpha}, & \gamma_{\mathrm{b}\alpha} &= \beta\gamma_{0\alpha}, \end{aligned} \tag{5}$$

* I am indebted to Professor B. H. J. McKellar for making these data available to me.

where γ_T ($T = 0, 1$) is the nucleon reduced width amplitude for the pure T state (excluding the isospin Clebsch–Gordan coefficient), and $\gamma_{1\alpha}$ is zero. Similarly

$$g_{a\pi\pm} = \alpha g_0 \pm \beta g_1\,, \qquad g_{b\pi\pm} = \beta g_0 \mp \alpha g_1\,, \tag{6a, b}$$

where we have used $g_{0\pi\pm} = g_0$ and $g_{1\pi\pm} = \pm g_1$.

Since we do not seek to calculate the absolute magnitudes of $\sigma_{\pi\pm}$, we require only the values of the ratio $g_0/g_1 = \zeta$ say. Similarly, we put $\gamma_0/\gamma_1 = \eta$. Then, apart from an overall normalization, the parameters occurring in the formula (1) for $\sigma_{\pi\pm}$ are a_c, E_a, E_b, γ_1^2, η, ζ, $P_\alpha \gamma_{0\alpha}^2$ and β [with $\alpha = (1-\beta^2)^{\frac{1}{2}}$].

Before considering in detail the calculated values of $\sigma_{\pi\pm}$, we note that a consequence of the form of the formulae (1)–(6) is an approximate relation that is independent of the value of any of these parameters. For this purpose we make approximations in the formulae by neglecting the off-diagonal terms of $\mathbf{A}^{-1}$ in equation (3) and by neglecting any overlap of the contributions of the two levels in (2). Then for $E \approx E_\lambda$ we get

$$\sigma_{\pi\pm}(E) \approx k \sum_c P_c \gamma_{\lambda c}^2 g_{\lambda\pi\pm}^2 \bigg/ \left\{ \left(E_\lambda - E - \sum_c \{S_c(E) - B_c\}\gamma_{\lambda c}^2 \right)^2 + \left(\sum_c P_c \gamma_{\lambda c}^2 \right)^2 \right\}.$$

The first term in the large braces vanishes for $E \approx E_\lambda$, since $B_c = S_c(19{\cdot}5\text{ MeV}) \approx S_c(E_\lambda)$, and therefore

$$\sigma_{\pi\pm}(E_\lambda) \approx k g_{\lambda\pi\pm}^2 \bigg/ \left(\sum_c P_c \gamma_{\lambda c}^2 \right) = 2k g_{\lambda\pi\pm}^2 / \Gamma_\lambda\,,$$

where $\Gamma_\lambda = 2\sum_c P_c \gamma_{\lambda c}^2$ is the formal width of the level λ. From equations (6) we have

$$g_{a\pi+}^2 - g_{a\pi-}^2 = 4\alpha\beta g_0 g_1\,, \qquad g_{b\pi+}^2 - g_{b\pi-}^2 = -4\alpha\beta g_0 g_1\,,$$

and therefore

$$\Gamma_b[\sigma_{\pi+}(E_b) - \sigma_{\pi-}(E_b)] / \Gamma_a[\sigma_{\pi+}(E_a) - \sigma_{\pi-}(E_a)] \approx -1\,. \tag{7}$$

This is the approximate relation mentioned above. In comparing with experimental results, the values of $\sigma_{\pi+}(E_\lambda) - \sigma_{\pi-}(E_\lambda)$ may be taken as the extreme (positive and negative) values of $\sigma_{\pi+} - \sigma_{\pi-}$. Values of Γ_λ are not directly obtainable from experiment, but the FWHM $\Gamma_{\frac{1}{2}\lambda}$ of the measured peak λ is approximately equal to the observed width Γ_λ°, defined by (Lane and Thomas 1958):

$$\Gamma_\lambda^\circ = \Gamma_\lambda \bigg/ \left(1 + \sum_c (\mathrm{d}S_c/\mathrm{d}E)\gamma_{\lambda c}^2 \right). \tag{8}$$

For the parameter values of interest here, the denominator in (8) has more or less the same value for $\lambda = \text{a}$ and $\lambda = \text{b}$, so that one expects

$$R \equiv \Gamma_{\frac{1}{2}b}[\sigma_{\pi+} - \sigma_{\pi-}]_b / \Gamma_{\frac{1}{2}a}[\sigma_{\pi+} - \sigma_{\pi-}]_a \approx -1\,. \tag{9}$$

This relation is embodied in the formulae (6) of Morris *et al.* (1979), which were based on similar approximations (note that their $\sigma_A^{\pi\pm}$ is the value of the cross section integrated over the resonance A). Also, the calculated cross sections of Halderson *et al.* (1981), shown in their Figs 2 and 3, approximately satisfy the relation (9). The quantity R is useful for comparison between calculation and experiment, since $\Gamma_{\frac{1}{2}\lambda}[\sigma_{\pi^+}-\sigma_{\pi^-}]_\lambda$ is proportional to the area of the peak λ, and the area does not depend sensitively on the energy resolution.

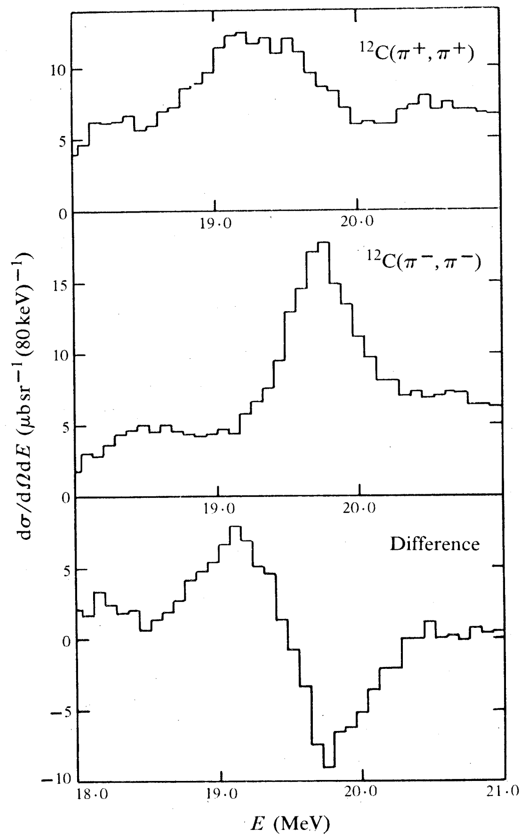

Fig. 1. Measured $^{12}C(\pi^{\pm},\pi^{\pm})^{12}C$ cross sections and their difference as functions of ^{12}C excitation energy E, for $E_\pi = 180$ MeV and $\theta = 75°$ (from C. L. Morris, personal communication).

3. Comparison of Calculated and Experimental Values of $\sigma_{\pi^{\pm}}$

Extraction of reliable values for the 4^- contributions σ_{π^+} and σ_{π^-} from the experimental π^+ and π^- cross sections shown in Fig. 1 is hampered by the energy-dependent background. Nevertheless it is obvious that the lower member of the doublet contributes little to the π^- cross section. The most quantitative data come from the difference in the π^+ and π^- cross sections, which should be a reasonably accurate representation of $\sigma_{\pi^+}-\sigma_{\pi^-}$, apart from the effects of the energy resolution of about 150 keV.* The peak energies $E_{pa} \approx 19\cdot12$ MeV and $E_{pb} \approx 19\cdot75$ MeV and the ratio of the peak areas $R \approx -0\cdot8$ should be fairly insensitive to the energy resolution. More sensitive are the widths of the peaks $\Gamma_{\frac{1}{2}a} \approx 0\cdot55$ MeV and $\Gamma_{\frac{1}{2}b} \approx 0\cdot4$ MeV. We see to what extent the calculations can reproduce these values while retaining consistency with the qualitative features of the π^+ and π^- cross sections and with

* The nonzero value of the difference shown in Fig. 1 for $E \lesssim 18\cdot4$ MeV may be attributed to the 2^- level at $18\cdot32$ MeV, which is known to be the lower member of an isospin-mixed doublet (Moore *et al.* 1982).

information from other reactions, particularly ^{11}B(^{3}He, d)^{12}C (see Section 4). An immediate comment is that the experimental value of R is in rough agreement with the expectation given by equation (9).

Various considerations are used to obtain starting values of the parameters. We use the conventional value of the channel radius $a_c = 1 \cdot 45(11^{1/3}+1)$ fm $= 4 \cdot 67$ fm. The values of the level energies E_λ are initially taken equal to the peak energies $E_{p\lambda}$. A value of γ_1^2 may be obtained by fitting the observed width Γ° of the 4$^-$ state in ^{12}B at 4·52 MeV. This state is the analogue of the 4$^-$ $T = 1$ state of ^{12}C. Measured values of Γ° are 130±20 keV (Bockelman *et al.* 1951), 50 keV (Jaffe *et al.* 1960), 100±15 keV (Middleton and Pullen 1964), 77 keV (derived from the $\gamma_{\lambda c}$ and a_c values of Lane *et al.* 1970) and 86±20 keV (Ajzenberg-Selove *et al.* 1978). As an average value we take $\Gamma^\circ = 100$ keV. From $\Gamma^\circ = 2P\gamma_1^2/\{1+(\mathrm{d}S/\mathrm{d}E)\gamma_1^2\}$, we get γ_1^2 as a function of a_c. For $a_c = 4 \cdot 67$ fm, we have $\gamma_1^2 = 0 \cdot 65$ MeV. If the $A = 11$ ground states are assumed to have a simple $1\mathrm{p}_{3/2}^{-1}$ structure and the 4$^-$ $T = 0$ and $T = 1$ states a $1\mathrm{p}_{3/2}^{-1}\,1\mathrm{d}_{5/2}$ structure, then $\gamma_0 = \gamma_1$. If (3, 3) dominance is also assumed, then $g_0 = 2g_1$. Thus we take $\eta = 1$ and $\zeta = 2$ as starting values. Initially we take $P_\alpha \gamma_{0\alpha}^2 = 0$. Using their own ^{12}C($\pi^\pm, \pi^\pm$) data and formulae that neglect overlap between the two resonances, Morris *et al.* (1979) estimated $\beta \geqslant 0 \cdot 32$. Halderson *et al.* (1981) fitted the same data and obtained $\beta = 0 \cdot 45$, and we start with this value.

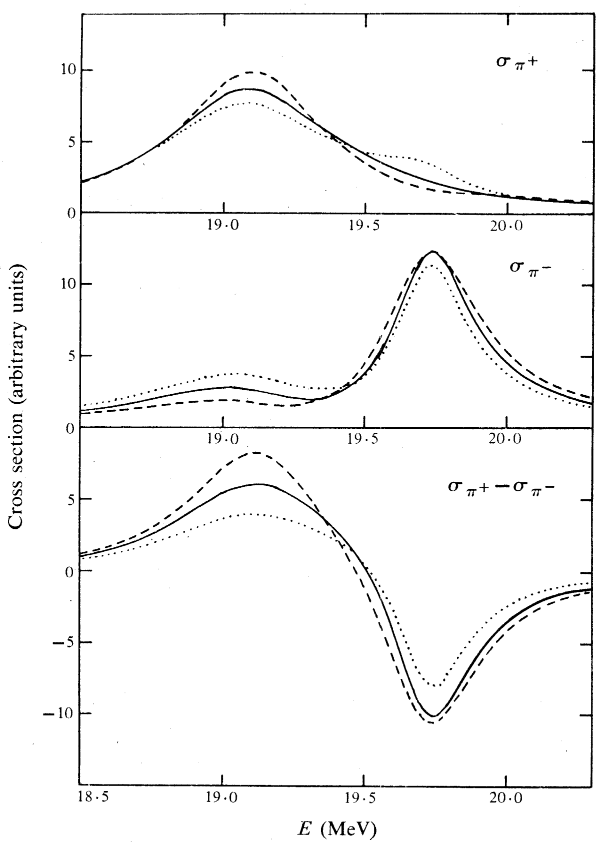

Fig. 2. Calculated cross sections $\sigma_{\pi\pm}$ for ^{12}C($\pi^\pm, \pi^\pm$)^{12}C(4$^-$) and their difference as functions of ^{12}C excitation energy E, for standard parameter values (solid curves) and for $\beta = 0 \cdot 3$ (dotted curves) and $\beta = 0 \cdot 6$ (dashed curves).

Some of these parameter values must be changed in order to fit certain features of the data. A reasonable overall fit is obtained with the values $a_c = 4\cdot67$ fm, $E_a = 19\cdot05$ MeV, $E_b = 19\cdot78$ MeV, $\gamma_1^2 = 0\cdot65$ MeV, $\eta = 0\cdot2$, $\zeta = 1\cdot5$, $P_\alpha \gamma_{0\alpha}^2 = 0\cdot4$ MeV and $\beta = 0\cdot45$, which we refer to as standard values. These give the values of $\sigma_{\pi^+}, \sigma_{\pi^-}$ and their difference shown in Fig. 2. The dependence of these cross sections on the value of β is also shown. The effects of changing the parameter values one at a time are indicated in Table 1. Equivalent fits and sets of parameters can be obtained for other values of a_c.

Table 1. Experimental and calculated values for $^{12}C(\pi^\pm, \pi^\pm)^{12}C(4^-)$

Parameter modified	Change in parameter	E_{pa} (MeV)	E_{pb} (MeV)	R	$\Gamma_{\frac{1}{2}a}$ (MeV)	$\Gamma_{\frac{1}{2}b}$ (MeV)
		Standard parameter set				
		19·12	19·75	−0·86	0·59	0·31
		Modified parameter sets				
E_a (MeV)	19·05 → 19·10	19·17	19·75	−0·88	0·58	0·31
E_b (MeV)	19·78 → 19·73	19·12	19·70	−0·84	0·59	0·30
γ_1^2 (MeV)	0·65 → 0·86	19·10	19·73	−0·87	0·59	0·34
η	0·2 → 0·4	19·12	19·74	−0·93	0·61	0·30
ζ	1·5 → 2·0	19·12	19·75	−0·87	0·60	0·31
$P_\alpha \gamma_{0\alpha}^2$ (MeV)	0·4 → 0·3	19·09	19·74	−0·89	0·49	0·29
β	0·45 → 0·6	19·11	19·74	−0·91	0·51	0·36
Experimental values		19·12	19·75	−0·8	0·55	0·4

It is seen from Table 1 that the value of R is insensitive to the parameter values and agrees reasonably with the expectation (9) and with the experimental value. Better agreement with the experimental peak widths could be obtained by decreasing η and/or increasing β from the standard values, but such changes would reduce the agreement with the $^{12}C(e, e')$ and $^{11}B(^3He, d)$ data (see Section 4). Most features of the cross sections are insensitive to the value of ζ, and the reason for the reduction of ζ from the starting value of 2 to the standard value of $1\cdot5$ is entirely to lower the σ_{π^-} peak near 19 MeV, as seems necessary from the data in Fig. 1.

From fitting the data it is possible to determine approximately the values of at least some of the parameters, including $E_b - E_a \approx 0\cdot73$ MeV and $\beta \approx 0\cdot45$. From these we can derive the energy difference of the unmixed $T = 0$ and $T = 1$ states and the isospin-mixing matrix element coupling them:

$$E_1 - E_0 = (\alpha^2 - \beta^2)(E_b - E_a) \approx 0\cdot43 \text{ MeV}, \tag{10a}$$

$$V_{01} = -\alpha\beta(E_b - E_a) \approx -0\cdot29 \text{ MeV}. \tag{10b}$$

Irrespective of the value of β, one has $|V_{01}| \lesssim 0\cdot36$ MeV. The values (10) do not depend sensitively on the choice of channel radius. For $a_c = 4\cdot0$ fm, for example, the cross sections for the standard parameter set are essentially reproduced by changing only E_b to $19\cdot80$ MeV and γ_1^2 to $1\cdot41$ MeV (which fits $\Gamma^\circ = 100$ keV for the 4^- state of ^{12}B), leading to $E_1 - E_0 \approx 0\cdot45$ MeV and $V_{01} \approx -0\cdot30$ MeV.

4. Experimental Data from Other Reactions

The first evidence for a 4^- $T = 1$ level of ^{12}C at 19·6 MeV came from inelastic scattering of high-energy electrons (Donnelly *et al.* 1968; Yamaguchi *et al.* 1971). The 4^- contribution to the ^{12}C(e, e′) cross section may be calculated from equations (1)–(5), with x = e, and with equations (6) replaced by $g_{\mathrm{ae}} = \beta g_{1\mathrm{e}}$ and $g_{\mathrm{be}} = -\alpha g_{1\mathrm{e}}$, corresponding to feeding of only the $T = 1$ parts of the states. The standard parameter values predict a single peak at about 19·7 MeV. Increasing β leads to a shoulder near 19·2 MeV, which does not appear to be present in the data at high momentum transfer.

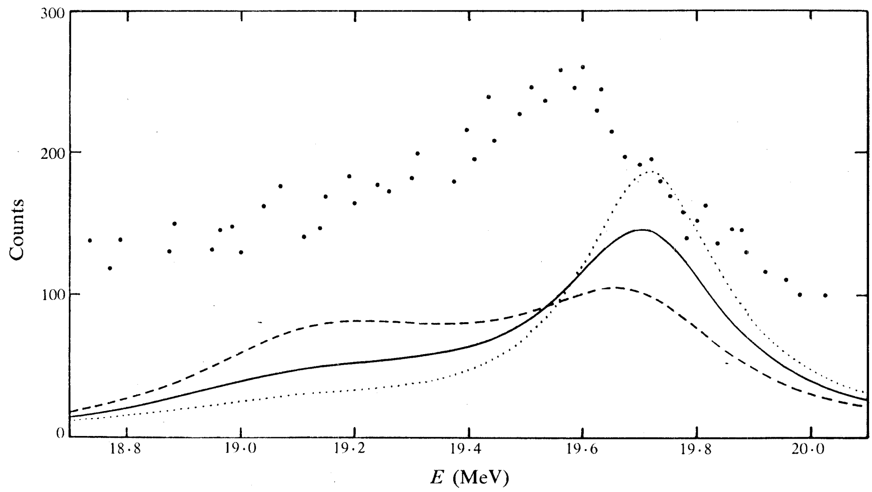

Fig. 3. Deuteron spectrum from ^{11}B(^{3}He, d)^{12}C as a function of ^{12}C excitation energy E. The experimental points are from Reynolds *et al.* (1971) and include a background under the 4^- contribution. The calculated 4^- contributions are for standard parameter values (solid curve) and for $\beta = 0·3$ (dotted curve) and $\beta = 0·6$ (dashed curve).

Peaks observed in ^{11}B(^{3}He, d)^{12}C at excitation energies of 18·27 and 19·56 MeV have been attributed to 4^- $T = 0$ and 4^- $T = 1$ levels respectively by Reynolds *et al.* (1971). In Fig. 3 we compare their deuteron spectrum in the region of the 19·56 MeV peak with that calculated for populating the 4^- levels that we have assumed, using standard parameter values in equations (1)–(5) with $g_{\mu x} \propto \gamma_{\mu \mathrm{p}}$. The dependence on β is also shown. We attribute the discrepancy between the experimental and calculated peak positions to an inconsistency in the energy calibrations of the ^{12}C($\pi^{\pm}, \pi^{\pm}$) and ^{11}B(^{3}He, d) measurements. Reynolds *et al.* attributed the plateau at about 19·2 MeV to a 2^- $T = 1$ level, however our 4^- contribution also shows such a plateau and suggests that $\beta \lesssim 0·5$. We comment later on the possible 4^- level at 18·27 MeV, in connection with shell model calculations.

In ^{12}C(p, p′)^{12}C, Buenerd *et al.* (1977) have observed a peak at about 19·6 MeV with a width of about 0·5 MeV, but have assigned it 4^+ $T = 0$.

Ball and Cerny (1969) observed a broad level or group of levels at 19·58 MeV in the ^{12}C(^{3}He, ^{3}He′)^{12}C reaction. They also observed a broad level or group of levels at 4·24 MeV in ^{12}N from ^{12}C(^{3}He, t)^{12}N. By a comparison of excitation energies, relative intensities and angular distributions, they concluded that these two levels are analogues and therefore have $T = 1$. They also took them to be analogues of the ^{12}B level at 4·54 MeV, then thought to be 3^- but now known to be 4^- (Ajzenberg-Selove and Busch 1980). The 4·24 MeV level of ^{12}N has also been seen in other reactions and its width is given as $\sim$400 keV (Ajzenberg-Selove and Busch 1980). We can calculate the observed width of a 4^- level of ^{12}N at this energy, using the value of the reduced width obtained by fitting the observed width of the 4^- level of ^{12}B at 4·52 MeV (see Section 3), and obtain $0\cdot44 \pm 0\cdot05$ MeV for a channel radius of $4\cdot67 \mp 1$ fm. This is consistent with all these observed levels being 4^-.

5. Shell Model Calculations

Jäger and Kirchbach (1977) carried out shell model calculations for the negative parity $A = 12$ states, using as basis states all non-spurious $1\hbar\omega$ excitations with harmonic oscillator single particle wavefunctions. These are bound-state shell model calculations in the sense of those referred to by Halderson *et al.* (1981). With the non-central interaction of Milliner and Kurath, Jäger and Kirchbach predicted the lowest 4^- $T = 0$ state of ^{12}C at 14·63 MeV, followed by a second 4^- $T = 0$ state at 19·18 MeV and a 4^- $T = 1$ state at 19·31 MeV. It seems natural to identify the latter two states with those whose isospin mixing produces the 4^- levels observed near 19·5 MeV, the agreement with the value of $E_1 - E_0$ given in equation (10a) being reasonably good. It is therefore of interest to calculate the isospin-mixing matrix element of the Coulomb interaction between these two shell model states, and also to calculate their reduced widths for the ^{11}B(g.s.)+p channel.

Jäger and Kirchbach (1977) did not give values of spectroscopic factors. We were not able to repeat their full shell model calculation, since we could not eliminate spurious states nor could we include the $1s^4\,1p^7 2s$ or $1s^3 1p^9$ configurations with the code available here (a version of the Glasgow shell model code). These approximations are probably not very serious as far as the 4^- states are concerned; in particular, we note that the 4^- states of the simplest configuration $1s^4 1p^7_{3/2} 1d_{5/2}$ do not contain any spurious states. We find 4^- $T = 0$ states at 14·58 and 19·02 MeV and a 4^- $T = 1$ state at 19·30 MeV, in reasonable agreement with the energies given by Jäger and Kirchbach. The calculated spectroscopic factor of the 4^- $T = 1$ state for the $A = 11$(g.s.)+nucleon channel is $\mathscr{S}_1 = 0\cdot83$ (normalized so that $\mathscr{S}_1 = 1$ for the simple configurations $1p^{-1}_{3/2}$ for the $A = 11$ ground state and $1p^{-1}_{3/2}\,1d_{5/2}$ for the 4^- $T = 1$ state). This may be related to the reduced width γ_1^2 by

$$\gamma_1^2 = \mathscr{S}_1(\hbar^2/2M_c a_c)\left(u^2(a_c)\bigg/\int_0^{a_c} u^2(r)\,\mathrm{d}r\right), \tag{11}$$

where $u(r)$ is the radial wavefunction. Using Woods–Saxon wavefunctions, we obtain values of γ_1^2 about 30% greater than those obtained in Section 3 from fitting $\Gamma^\circ = 100$ keV for the 4·52 MeV state of ^{12}B; for example $\gamma_1^2 = 0\cdot86$ MeV for $a_c = 4\cdot67$ fm. The value of $\eta = \gamma_0/\gamma_1 \approx \mathscr{S}_0^{\frac{1}{2}}/\mathscr{S}_1^{\frac{1}{2}}$ obtained from the shell model calculation is $\eta = 0\cdot65$, which is less than the value $\eta = 1$ for the simplest configurations but greater than the standard value of 0·2.

Values of the isospin-mixing matrix element V_{01} may be calculated using the formulae and approximations given by Barker (1978). The surface contribution to V_{01} depends sensitively on the channel radius, being -202, -117 and -70 keV for

$a_c = 4$, 5 and 6 fm respectively. These values are about 55% of those for the simplest configurations. The dominant contribution comes from the $A = 11$ ground-state channel, due to the small products of spectroscopic amplitudes for the excited-state channels. We estimate the internal contribution to V_{01} in an approximate way, assuming it also to be 55% of the value for the simplest configurations, which is -171 keV (for the harmonic oscillator length parameter $b = 1 \cdot 67$ fm). Thus, agreement with the experimental value (10b) of V_{01} is obtained for $a_c \approx 4$ fm.

The shell model calculations of Jäger and Kirchbach (1977) predicted the lowest 4^- $T = 0$ state at 14·63 MeV with an $A = 11$ ground-state spectroscopic factor of 0·34, about the same as that of their second 4^- $T = 0$ state. Reynolds *et al.* (1971) made a 4^- $T = 0$ assignment to a peak that they observed at 18·27 MeV, but Jäger and Kirchbach considered the energy difference to be too great for this to be identified with the predicted 14·63 MeV state. A more attractive identification would appear to be that of the level observed at 13·35 MeV in ^{12}C with the predicted 4^- $T = 0$ state (cf. Fig. 4 of Jäger and Kirchbach). This 13·35 MeV level has been given the uncertain assignment $J^\pi = (2^-)$ (Ajzenberg-Selove and Busch 1980). Although this level has been seen in several reactions, this spin assignment is based entirely on one study of the ^{10}B(^{3}He, p$\alpha\alpha\alpha$) reaction by Waggoner *et al.* (1966). They suggested a J^π assignment of $J \geqslant 1$, unnatural parity, with some preference for 2^- on the basis of the large α-particle width. From all the observations the width is given as 375 ± 40 keV (Ajzenberg-Selove and Busch 1980); if this is due entirely to the f-wave α-particle channel to ^{8}Be first excited state, the required spectroscopic factor is about 2 (for channel radii $\gtrsim 5$ fm). Such a large value implies considerable α clustering, and α-particle cluster calculations do indeed predict a low-lying 4^- $T = 0$ level. Fujiwara *et al.* (1980) suggested that the 4^- level they predicted at about 12–13 MeV excitation should be identified with the observed 13·35 MeV level, and calculated its width as 270 keV, in reasonable agreement with the experimental value.

6. Discussion

The present description of the ^{12}C($\pi^\pm, \pi^\pm$)^{12}C(4^-) cross sections in terms of an isospin-mixed doublet of 4^- levels near 19·5 MeV in ^{12}C is based on the *R*-matrix theory of nuclear reactions and bound-state shell model calculations. With similar assumptions of simple configurations for the states involved, this treatment gives results similar to those obtained by Halderson *et al.* (1981) using the recoil corrected continuum shell model. Because they assumed these simple configurations, Halderson *et al.* attributed all the widths of the levels to neutron and proton channels, and consequently suggested that the fragmentation of the 4^- states in ^{12}C is relatively small. The description of the states given by the shell model calculations of Jäger and Kirchbach (1977) and that we find necessary in order to fit the data is, however, very different from these simple configurations. Jäger and Kirchbach calculated the spectroscopic factors of the 4^- $T = 0$ and 4^- $T = 1$ states near 19 MeV as $\mathscr{S}_0 = 0 \cdot 35$ and $\mathscr{S}_1 = 0 \cdot 83$. We fit the observed width of the 4^- state of ^{12}B with $\mathscr{S}_1 = 0 \cdot 63$ and find even smaller values of $\mathscr{S}_0$ ($\eta = 0 \cdot 2$ corresponds to $\mathscr{S}_0 = 0 \cdot 03$). Consequently, we attribute most of the widths of the 4^- levels to the α-particle channel. It may be noted that these α-particle widths of the order of several hundred keV are much larger than those of the 4^- levels of ^{16}O at about 19 MeV excitation, which each have total widths of $\lesssim 50$ keV (Kemper *et al.* 1983). A critical feature of our description would be the identification of the 4^- $T = 0$ state predicted by Jäger and Kirchbach to lie several MeV below the 4^- $T = 0$ state involved in the isospin-mixed doublet; the most likely candidate is the observed 13·35 MeV level.

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