An Absolute Measurement of Cherenkov Emission by Relativistic Muons in Pure Water

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Abstract
Relativistic cosmic ray muons passing through triple distilled water have been used to examine the absolute number of Cherenkov photons emitted between 250 and 650 nm. It is shown that almost all of the theoretically predicted photon number can be experimentally accounted for, provided that the apparatus is specifically designed to minimize losses due to reflection, absorption and coupling. It is noted that significant corrections need to be made for internal photomultiplier tube effects.

1. Introduction
The Falkiner Nuclear Department of the University of Sydney is currently conducting a feasibility study into the possibility of performing a solar neutrino experiment. The experiment envisaged is to detect scattered electrons in a large mass of distilled water from the Cherenkov radiation they emit. The advantages of this technique include some directionality and insensitivity to low energy background \(\alpha\)-particles and protons. As the maximum energy of the electrons will be a little less than 14 MeV, only a small number of photons is emitted (of the order of 1000), and it is important to know the exact number when designing the detector and assessing the feasibility of the experiment. Because the detection of solar neutrinos will be at best a difficult and long term procedure, errors in the number of Cherenkov photons available could tilt the balance to make it impracticable to achieve significance within a reasonable period of time.

Previous attempts in Sydney to measure an absolute photon number have been unable to account for more than about 25% of the theoretically predicted value. As a consequence of this, it was decided to set up a controlled experiment in which as many sources of photon loss as possible were minimized or removed. These include reflection loss, loss at coupling surfaces, indeterminate pathlength and absorption. By so doing, we hoped to determine whether the above discrepancies were real or the sum total of many effects, each of which might seem unimportant, but when acting in concert might significantly diminish the observed number of photons.
2. Apparatus

The detector consisted of a black polythene cylindrical container with a Philips 56 DVP photomultiplier tube protruding into it from below, as shown in Fig. 1. This tube had the highest gain and cathode efficiency of a sample of eight tubes measured in a separate experiment. The container was filled with triple distilled water in direct contact with the tube cathode. In order to detect as many photons as possible, it is necessary to maximize the number reaching the cathode; to achieve this, a small cylinder of the same diameter as the photomultiplier (5 cm) and of height 4·5 cm was placed over the tube. This cylinder was constructed from a highly reflective foil (Scotchtint sun control film type YS-91 supplied by the 3M Company), because many of the photons necessarily undergo some reflection due to the wide opening angle of the Cherenkov photons (41·3°). This foil cylinder was sealed with a black plastic cap to ensure that no extraneous light could reach the photomultiplier cathode.

The photons were produced by cosmic ray muons traversing the water. Four trigger scintillators (also shown in Fig. 1) ensured a very collimated beam, hence allowing an accurate knowledge of the muon path, which can be assumed to be almost vertical (with maximum deviation 3·5° and average deviation 1·2°). Because of this stringent control of the muon zenith angles, all photons incident upon the glass envelope of the tube will have zenith angles of 41·3±3·5°. These photons will impinge upon the cathode material with angles of incidence of 35·8±2·9° all lower than the critical angle (42·1°). As a consequence of this, we feel that any photon loss due to internal reflection will be negligible.

The output from the photomultiplier passed through a 450 ns delay (to allow the trigger pulse to arrive first at the multi-channel analyser, a Davidson type 4106A). The analyser was gated by the four trigger scintillators providing a 5 μs, 6 V pulse. A block diagram of the apparatus is shown in Fig. 2.

3. Method

Because of the need to define the angle of the incoming muons precisely, the aperture of the trigger system was necessarily small. This resulted in a slow trigger rate of approximately one count every ten minutes, and many runs were required to build up the necessary statistics. Each run usually lasted 24 hours, and was calibrated before and after to check for possible drifts. This calibration was accomplished by inspection of the noise spectrum, and the spectrum from a 137Cs source strapped to the back of a small sodium iodide crystal and placed directly onto the centre of the tube cathode. The response of the crystal to the 0·662 MeV γ rays provided a convenient standard reference light source. The positions of these calibration spectra were readjusted if necessary, so that all the various runs could be combined; the combined data from all the runs are shown in Fig. 3.

The relative width \( w \) (full-width-at-half-maximum divided by the peak position) allows an estimate of the number of photoelectrons emitted from the cathode. Provided that the sources of fluctuation are dominated by the Poisson process of photoelectron emission from the cathode, we can use the usual approximation for the photoelectron number

\[
N_{pe} \approx (2·36/w)^2. \tag{1}
\]
Fig. 1. Schematic diagram of the experimental arrangement.

Fig. 2. Block diagram of the electronics used in the experiment.
When this formula is applied to the peak in Fig. 3, we obtain an uncorrected estimate of 90 ± 30 photoelectrons. This is likely to be an underestimate, however, because of other possible effects contributing to the width of the distribution. For example, a slow non-systematic drift in the long runs might not be obvious in the calibration, yet still add somewhat to the width of the distribution. In principle, the zenith angle of the muons should also increase the width; however, calculations have shown that the tight trigger configuration makes this effect negligible.

![Fig. 3. Combined pulse height spectra from all the muon runs. The dashed line represents the equivalent number of photoelectrons as a function of peak channel number for the diode runs (see text).](image)

In order to confirm the above estimate, distributions from a light emitting diode were measured, and their width (and hence the equivalent photoelectron number) calculated as a function of peak position, for various diode drives. These runs were short (about five minutes) and hence not susceptible to slow non-systematic drifts. The diode results are also shown in Fig. 3 and suggest that the muon run data really represent something like 120 ± 12 photoelectrons. Even though the two values agree within errors, we believe that this latter figure is the more accurate, and use it in our subsequent discussion.

It should be re-emphasized that the approximation (1) is only valid when the cathode emission fluctuations are dominant. As the photoelectron number increases, there comes a point after which gain fluctuations in the tube are the most important factor. To observe this effect with our tube, we used the light emitting diode distributions to examine the number of photoelectrons inferred from the width as a function of the equivalent channel number (taking into account the necessary attenuation to bring the distribution into the same region of the analyser). These results are presented in Fig. 4 showing that the dependence up to about 1000 photoelectrons is linear. After this, the width is dominated by gain fluctuations and the approximation (1) is no longer applicable. However, because our estimates from the muon runs are around 100 photoelectrons, we feel that application of the formula will give meaningful results.
4. Corrections

Even though the experimental arrangement has been selected with a view to minimizing photon losses, there must be some correction made for the various inefficiencies inherent in the design system. We now consider these in turn:

**Losses due to Reflection from Foil Walls**

We calculated the average number of wall collisions for photons originating from the muon trajectories, by assuming the muons to be vertical and to uniformly irradiate the top surface of the foil cylinder. The mean value turns out to be 0.4.

We also made measurements of the absolute reflectance of the foil. The experiment used a $^{90}$Sr $\beta$-particle emitter (i.e. Cherenkov emission) and a green light emitting diode. Foil was used to line a black tank not dissimilar to the apparatus described before, except that it could be made longer, with central baffles to ensure multiple reflections. Assuming no attenuation in the water, and that the loss of light due to loss of solid angle has an inverse square dependence, an absolute reflectance of $92 \pm 5\%$ was obtained for the Cherenkov light and $94 \pm 4\%$ for the diode. It appears therefore that the reflectance of the foil is fairly constant over the whole wavelength range from green to ultraviolet. As our actual runs involved Cherenkov emission, we assumed a reflectance of 92%.

Combining this reflectance with the average number of collisions gives a correction factor of 0.97.

**Attenuation in Water**

Fig. 5 shows the attenuation coefficient of light in pure water as measured by various researchers (Hulbert 1945; Querry *et al.* 1978; Tam and Patel 1979; Smith...
and Baker 1981). These values imply attenuation lengths of 20 m or more in the wavelength region of interest. It is obvious from such values that the small distances travelled in the present experiment will result in virtually no absorption of photons by the triple distilled water. We have incorporated this attenuation into our calculation and verified that the resulting correction is significantly less than 1%, and hence can be ignored.

![Fig. 5. Attenuation coefficient of light in pure water as a function of wavelength for the four experiments indicated.](image)

**Effective Cathode Loss**

The effective cathode diameter is less than the external tube diameter of 5 cm. This results in an outer annulus where photons will not be detected. We have compared runs using smaller cylinders of diameters 3.5 and 4.0 cm with the original run of 5.0 cm. These results indicate an effective cathode diameter of just under 4.8 cm (consistent with the manufacturers claim that the diameter is greater than 4.2 cm). As a consequence of this, only 92% of the incident photons can be assumed to be detected by the 5 cm cylinder.

**Tube Focussing Loss**

Not all the photoelectrons emitted from the cathode will be collected by the first dynode and contribute to the multiplication process. The figure supplied by the
manufacturers for this focussing efficiency is about 70–80% with the lower value applying when the cathode is uniformly illuminated. As our experimental situation implies almost uniform illumination for each muon, we take the figure of 70% as another correction factor to be considered.

**Dynode Multiplication Fluctuations**

The photomultiplier is run with a 14 stage dynode chain with a multiplication of about three per stage (our measured average figure for this tube at its operating voltage of 2200 V). This multiplication at each stage is expected to obey Poisson statistics increasing the fluctuation of the resulting output pulse at the anode. Therefore, we ran a Monte Carlo program starting with 100 photoelectrons, and incorporating Poisson fluctuations up to the third dynode, and found that the resulting anode distribution indicates only 70±5 photoelectrons because of the additional width from the dynode fluctuations. We thus have a correction factor of 0.7 to add into our calculations.

When allowance is made for all of the above corrections we find our best estimate for the experimental number of photoelectrons to be 274±27.

![Fig. 6. Quantum efficiency ξ(λ) of the DVP cathode material (solid curve) and the refractive index n(λ) of pure water (dashed curve) as a function of wavelength.](image)

5. Theoretical Prediction

Now that we have an experimental value for the number of photoelectrons, we must calculate the expected number. To do this we use the theory of Cherenkov radiation to calculate the number of photons emitted by a muon in the 4.5 cm pathlength. The cathode efficiency of the tube (an S11 response) is then incorporated to give the expected photoelectron number. Thus, the expression for the number of photoelectrons resulting from the group of photons with wavelength between λ₁ and λ₂ emitted in a pathlength L is given by

\[
N_{pe} = 2\pi x L \int_{\lambda_1}^{\lambda_2} \left( 1 - \frac{1}{\beta^2 n^2(\lambda)} \right) \xi(\lambda) \frac{d\lambda}{\lambda^2},
\]
where \( n(\lambda) \) is the refractive index, \( \xi(\lambda) \) is the conversion efficiency of the S11 cathode material, \( \alpha \) is the fine structure constant, and \( \beta = v/c \) is the velocity of the particle (assumed to be unity for muons). The dependences of \( n(\lambda) \) and \( \xi(\lambda) \) on wavelength are shown in Fig. 6. Using equation (2), we have calculated the photoelectron number from 250 to 650 nm and find the result of \( 290 \pm 10 \) photoelectrons.

6. Conclusions

We have estimated the number of photoelectrons emitted by relativistic muons traversing 4.5 cm of pure water. A simple carefully designed system has been used to minimize uncertainties in the calculation of the photon number and the causes of loss of these photons in their subsequent detection. Our corrected experimental figure of \( 274 \pm 27 \) agrees within errors with the expected number of \( 290 \pm 10 \).

Consequently, we feel that almost all of the theoretically predicted number of photoelectrons can be experimentally achieved, albeit with difficulty, provided that

(a) the tubes are immersed directly in distilled water thus dispensing with the need for coupling windows with their associated losses;
(b) the pathlengths travelled by the photons are small compared with their attenuation lengths;
(c) the number of reflections is kept small; and
(d) great care is taken to ensure that the focusing of the tube is optimized.

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