The Discrepancy in the Fermi Matrix Elements of the Isospin-forbidden $4^+ \rightarrow 4^+ \beta^- \rightarrow 2^+$ Decay of $^{46}\text{Sc}$

E. L. Saw and C. T. Yap

Department of Physics, National University of Singapore, Kent Ridge, Singapore 0511.

Abstract

A large number of measurements have been made on the $\gamma$-polarisation asymmetry coefficient $\tilde{A}$ for $4^+ \beta^- \rightarrow 4^+ \gamma \rightarrow 2^+$ of the $^{46}\text{Sc} \rightarrow ^{46}\text{Ti}$ decay. Presently, there are two adopted values of $\tilde{A}$ yielding the Fermi nuclear matrix element $|M_F|$ of $^{46}\text{Sc} \rightarrow ^{46}\text{Ti}$ decay, with a view to obtaining the Fermi nuclear matrix element $M_F$ for the decay. The measured values of the asymmetry coefficient vary from $60 \times 10^{-3}$ to $330 \times 10^{-3}$, with more recent measurements yielding much higher accuracies. Presently, the two adopted values are

$$\tilde{A} = (84.3\pm3.0) \times 10^{-3},$$
$$|M_F| = (0.06^{+0.26}_{-0.06}) \times 10^{-3}; \quad (1)$$
$$\tilde{A} = (100\pm3) \times 10^{-3},$$
$$|M_F| = (1.4 \pm 0.3) \times 10^{-3}. \quad (2)$$

The former is from a fairly accurate experiment by Pingot (1969), while the latter comes from the weighted average of the Daniel (1966) and Behrens (1967) experiments, values which are consistent with each other but not with Pingot's, and which had accuracies similar to Pingot's value. We believe this weighted average value to be the correct one.

Theoretical calculation of $M_F$ for this decay (Bertsch and Wildenthal 1973), which yields a value five times larger than equation (2), uses isospin mixing obtained on the basis of the observed $A = 42$ spectra. This discrepancy is unsatisfactory and so in the present work we will calculate the $M_F$ value for the $^{46}\text{Sc} \rightarrow ^{46}\text{Ti}$ decay by using the Nilsson model. As this is an isospin-forbidden decay, the value of $M_F$ arises from the isospin mixing of the daughter nucleus which has appreciable permanent axial deformation (Rebel and Habs 1973). In our previous work on deformed nuclei
the Nilsson model yielded values of $M_F$ in good agreement with experiment, both
for large (Yap and Saw 1985) and small (Yap and Saw 1984) values.

2. Calculations and Results

The partial level diagram for the $\beta^-$ decay of $^{46}\text{Sc}$ to $^{46}\text{Ti}$ is shown in Fig. 1, where
$|P\rangle$, $|A\rangle$ and $|T\rangle$ are the parent state, the analogue state and the antianalogue state respectively. The deformed nuclei $^{46}\text{Sc}$ and $^{46}\text{Ti}$ have the rotational bands $K = 4$
and 0 respectively. By the $K$-selection rule for beta decay of $\Delta K < 1$, the beta matrix
elements with $\Delta K = 4$ vanish, and thus the experimentally observed decay is due to
the mixture of other $K$ bands to the $K = 4$ ground state of $^{46}\text{Sc}$ and to the $K = 0$
excited state of $^{46}\text{Ti}$. If we assume prolate deformation (Rebel and Habs 1973), the
initial state is

$$ |i\rangle = |J=4, M, K=4, T=2, T_z=-2\rangle $$
$$ + \tilde{a}_0 |J=4, M, K=0, T=2, T_z=-2\rangle $$
$$ + \tilde{a}_1 |J=4, M, K=1, T=2, T_z=-2\rangle $$
$$ + \tilde{a}_0 \tilde{a}_0 |J=4, M, K=0, T=3, T_z=-2\rangle $$
$$ + \tilde{a}_1 \tilde{a}_1 |J=4, M, K=1, T=3, T_z=-2\rangle $$
$$ + \tilde{a}_4 |J=4, M, K=4, T=3, T_z=-2\rangle $$
$$ + ... , $$

(3)

and the final state is

$$ |f\rangle = |J=4, M, K=0, T=1, T_z=-1\rangle $$
$$ + a_3 |J=4, M, K=3, T=1, T_z=-1\rangle $$
$$ + a_4 |J=4, M, K=4, T=1, T_z=-1\rangle $$
$$ + a_0 |J=4, M, K=0, T=2, T_z=-1\rangle $$
$$ + a_3 a_3 |J=4, M, K=3, T=2, T_z=-1\rangle $$
$$ + a_4 a_4 |J=4, M, K=4, T=2, T_z=-1\rangle $$
$$ + ... , $$

(4)

where $\tilde{a}_0$ and $\tilde{a}_1$ are the admixture amplitudes of $K=0$ and 1 in the initial state
respectively, and $a_3$ and $a_4$ are the admixture amplitudes of $K=3$ and 4 in the final
states respectively. The Fermi matrix element is

$$ M_F = \langle f | T^+ | i \rangle = 2(\alpha_0 \tilde{a}_0 + \alpha_4 a_4) , $$

(5)

where the isospin impurity amplitudes $\alpha_0$ and $\alpha_4$ are given by
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\[ \alpha_0 = -\frac{\langle J=4, M, K=0, T=1, T_z=-1 \mid \delta C \mid J=4, M, K=0, T=2, T_z=-1 \rangle}{\Delta E}, \]

and where \( \Delta E \) is the separation energy and \( \delta C \) the Coulomb potential.

The \( \beta^- \) decay of \( ^{46}\text{Sc} \) is of a mixed Fermi and Gamow-Teller (GT) type. The GT matrix element is calculated from the relation

\[ M_{GT}^2 = \frac{1}{2J+1} \sum_{\mu M_t M_{t'}} |\langle J, M_\beta, K_\beta, T_\beta, T_{z\mu} \mid D_{GT}(\mu) \mid J, M_1, K_1, T_1, T_{z1} \rangle|^2. \]

When the operator \( D_{GT}(\mu) \) is transformed into the body-fixed coordinate system, we obtain

\[ M_{GT}^2 = |a_1 V_1^J \langle \chi_0 \chi_{T_1}^{T=1} \mid D_{GT}(1) \mid \chi_1 \chi_{T_1}^{T=2} \rangle|^2 + |a_3 V_3^J \langle \chi_3 \chi_{T_1}^{T=1} \mid D_{GT}(1) \mid \chi_4 \chi_{T_1}^{T=2} \rangle|^2 + |a_4 V_4^J \langle \chi_4 \chi_{T_1}^{T=1} \mid D_{GT}(0) \mid \chi_4 \chi_{T_1}^{T=2} \rangle|^2. \]

Using Nilsson (1955) wavefunctions with the experimental value of deformation \( \beta = 0.3 \) (Stelson and Grodzins 1965) to calculate the matrix elements between the intrinsic states \( |\chi_K \chi_{T^*}^{T'}\rangle \) and \( |\chi_K \chi_{T^*}^{T'}\rangle \), it was found that the third term on the right-hand side of (9) is much larger than the first two, so that

Fig. 1. Partial level diagram for the \( \beta^- \) decay of \( ^{46}\text{Sc} \) to \( ^{46}\text{Ti} \).
\[ M_{GT}^2 = \frac{4}{3} a_4^2 \left| \langle \chi_4 \chi_4^{T=-1} | D'_{GT}(0) | \chi_4 \chi_4^{T=-2} \rangle \right|^2 \]
\[ = \frac{4}{3} a_4^2 \left\{ \sqrt{\frac{3}{4}} \left[ + \frac{5}{2} - [312]p | D'_{GT}(0) | + \frac{5}{2} - [312]n > \right. \right. \]
\[ - \sqrt{\frac{1}{12}} \left\{ - \frac{3}{2} - [321]p | D'_{GT}(0) | - \frac{3}{2} - [321]n > \right. \left. \right. \]
\[ - \sqrt{\frac{1}{6}} \left\{ - \frac{1}{2} - [321]p | D'_{GT}(0) | - \frac{1}{2} - [321]n > \right\}^2 , \]
(10)

which gives a value of \( a_4 = |M_{GT}| / 1.13 \).

The value of \( M_{GT} \) can also be obtained from the well-known relation (Raman et al. 1975)

\[ |M_{GT}| = \frac{G_V}{G_A} \left( \frac{2f_t \text{ (superallowed)}}{f_t \text{ (decay under study)}} \right)^{\frac{1}{2}} \frac{1}{(1 + y^2)^{\frac{1}{2}}} \]
\[ = \frac{1}{1.19} \left( \frac{6222}{10^6 \cdot 2(1 + y^2)} \right)^{\frac{1}{2}}, \]
(11)

where \( y = G_V M_f / G_A M_{GT} \) and is related to the experimental value of the asymmetry coefficient. Either of the adopted values of \( \tilde{A} \) yields \( |M_{GT}| = 0.053 \), resulting in \( a_4 = 0.047 \).

In the calculation of the isospin impurity amplitudes given by equations (6) and (7), the Coulomb potential \( V_C \) for the interaction is taken as (Damgard 1966)

\[ V_C = \frac{(Z-1)e^2}{r} \left\{ \frac{3}{2} - \frac{1}{2}(r/R)^2 \right\} + a(r/R)^2 Y_{20}, \text{ for } r < R \]
(12a)
\[ = \frac{(Z-1)e^2}{r} + a(R/r)^3 Y_{20}, \text{ for } r > R, \]
(12b)

where \( R \) is the radius of the nucleus and \( a \) is related to the Bohr deformation parameter \( \beta \) by

\[ a = \frac{3}{2} \beta (Z-1)e^2 / R. \]

Calculation yields a much larger value for the isospin impurity amplitude \( a_4 \) than \( a_0 \), which can then be neglected. The final theoretical value for the Fermi nuclear matrix element becomes

\[ (M_F)_{\text{th}} = 0.91 \times 10^{-3}, \]

which compares favourably with the experimental value of

\[ (M_F)_{\text{exp}} = (1.4 \pm 0.3) \times 10^{-3}. \]
References


Manuscript received 7 February, accepted 6 March 1986