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The Discrepancy in the Fermi Matrix Elements of the Isospin-forbidden $4^+ \rightarrow 4^+ \ \beta^-$ Decay of ^{46}Sc

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Abstract

A large number of measurements have been made on the γ-polarisation asymmetry coefficient $\tilde{A}$ for $4^+ \xrightarrow{\beta^-} 4^+ \xrightarrow{\gamma} 2^+$ of the $^{46}\mathrm{Sc} \rightarrow {}^{46}\mathrm{Ti}$ decay. Presently, there are two adopted values of $\tilde{A}$ yielding the Fermi nuclear matrix elements $|M_\mathrm{F}| = (0\cdot06^{+0\cdot26}_{-0\cdot06})\times10^{-3}$ and $(1\cdot4\pm0\cdot3)\times10^{-3}$, the latter being the more probable. Using a one-body spheroidal Coulomb potential with Nilsson wavefunctions, the theoretical value of M_F is found to be $0\cdot91\times10^{-3}$ in good agreement with the experimental value of $(1\cdot4\pm0\cdot3)\times10^{-3}$.

1. Introduction

Since 1957, close to 30 experimental reports (Raman *et al.* 1975) have been made on the measurement of the γ-polarisation asymmetry coefficient $\tilde{A}$ for $4^+ \xrightarrow{\beta^-} 4^+ \xrightarrow{\gamma} 2^+$ of the $^{46}\mathrm{Sc} \rightarrow {}^{46}\mathrm{Ti}$ decay, with a view to obtaining the Fermi nuclear matrix element M_F for the decay. The measured values of the asymmetry coefficient vary from 60×10^{-3} to 330×10^{-3}, with more recent measurements yielding much higher accuracies. Presently, the two adopted values are

$$\tilde{A} = (84\cdot3\pm3\cdot0)\times10^{-3}, \quad \text{yielding } |M_\mathrm{F}| = (0\cdot06^{+0\cdot26}_{-0\cdot06})\times10^{-3}; \tag{1}$$

$$\tilde{A} = (100\pm3)\times10^{-3}, \quad \text{yielding } |M_\mathrm{F}| = (1\cdot4\pm0\cdot3)\times10^{-3}. \tag{2}$$

The former is from a fairly accurate experiment by Pingot (1969), while the latter comes from the weighted average of the Daniel (1966) and Behrens (1967) experiments, values which are consistent with each other but not with Pingot's, and which had accuracies similar to Pingot's value. We believe this weighted average value to be the correct one.

The theoretical calculation of M_F for this decay (Bertsch and Wildenthal 1973), which yields a value five times larger than equation (2), uses isospin mixing obtained on the basis of the observed $A = 42$ spectra. This discrepancy is unsatisfactory and so in the present work we will calculate the M_F value for the $^{46}\mathrm{Sc} \rightarrow {}^{46}\mathrm{Ti}$ decay by using the Nilsson model. As this is an isospin-forbidden decay, the value of M_F arises from the isospin mixing of the daughter nucleus which has appreciable permanent axial deformation (Rebel and Habs 1973). In our previous work on deformed nuclei

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the Nilsson model yielded values of M_F in good agreement with experiment, both for large (Yap and Saw 1985) and small (Yap and Saw 1984) values.

2. Calculations and Results

The partial level diagram for the β^- decay of ^{46}Sc to ^{46}Ti is shown in Fig. 1, where $|P\rangle$, $|A\rangle$ and $|T_<\rangle$ are the parent state, the analogue state and the antianalogue state respectively. The deformed nuclei ^{46}Sc and ^{46}Ti have the rotational bands $K = 4$ and 0 respectively. By the K-selection rule for beta decay of $\Delta K \leqslant 1$, the beta matrix elements with $\Delta K = 4$ vanish, and thus the experimentally observed decay is due to the mixture of other K bands to the $K = 4$ ground state of ^{46}Sc and to the $K = 0$ excited state of ^{46}Ti. If we assume prolate deformation (Rebel and Habs 1973), the initial state is

$$\begin{aligned}|\mathrm{i}\rangle = {} & |J=4,\ M,\ K=4,\ T=2,\ T_z=-2\rangle \\ & +\bar{a}_0|J=4,\ M,\ K=0,\ T=2,\ T_z=-2\rangle \\ & +\bar{a}_1|J=4,\ M,\ K=1,\ T=2,\ T_z=-2\rangle \\ & +\bar{a}_0\,\bar{\alpha}_0|J=4,\ M,\ K=0,\ T=3,\ T_z=-2\rangle \\ & +\bar{a}_1\,\bar{\alpha}_1|J=4,\ M,\ K=1,\ T=3,\ T_z=-2\rangle \\ & +\bar{\alpha}_4|J=4,\ M,\ K=4,\ T=3,\ T_z=-2\rangle \\ & +\ldots,\end{aligned} \tag{3}$$

and the final state is

$$\begin{aligned}|\mathrm{f}\rangle = {} & |J=4,\ M,\ K=0,\ T=1,\ T_z=-1\rangle \\ & +a_3|J=4,\ M,\ K=3,\ T=1,\ T_z=-1\rangle \\ & +a_4|J=4,\ M,\ K=4,\ T=1,\ T_z=-1\rangle \\ & +\alpha_0|J=4,\ M,\ K=0,\ T=2,\ T_z=-1\rangle \\ & +\alpha_3 a_3|J=4,\ M,\ K=3,\ T=2,\ T_z=-1\rangle \\ & +\alpha_4\,a_4|J=4,\ M,\ K=4,\ T=2,\ T_z=-1\rangle \\ & +\ldots,\end{aligned} \tag{4}$$

where $\bar{a}_0$ and $\bar{a}_1$ are the admixture amplitudes of $K=0$ and 1 in the initial state respectively, and a_3 and a_4 are the admixture amplitudes of $K=3$ and 4 in the final states respectively. The Fermi matrix element is

$$M_{\mathrm{F}} = \langle \mathrm{f}|\,T^+\,|\mathrm{i}\rangle = 2(\alpha_0\,\bar{a}_0 + \alpha_4\,a_4), \tag{5}$$

where the isospin impurity amplitudes α_0 and α_4 are given by

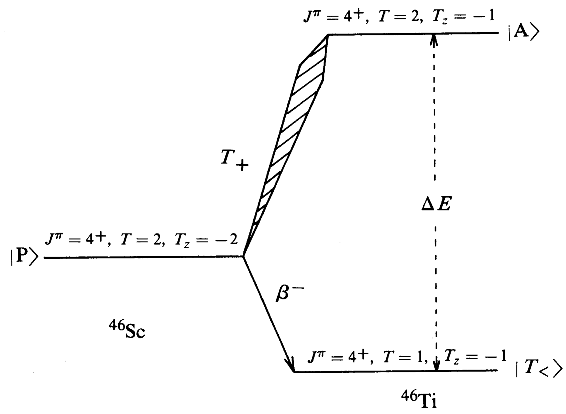

Fig. 1. Partial level diagram for the β^- decay of ^{46}Sc to ^{46}Ti.

$$\alpha_0 = -\frac{\langle J{=}4,\ M,\ K{=}0,\ T{=}1,\ T_z{=}-1|\,V_{\rm C}\,|J{=}4,\ M,\ K{=}0,\ T{=}2,\ T_z{=}-1\rangle}{\Delta E}, \tag{6}$$

$$\alpha_4 = -\frac{\langle J{=}4,\ M,\ K{=}4,\ T{=}1,\ T_z{=}-1|\,V_{\rm C}\,|J{=}4,\ M,\ K{=}4,\ T{=}2,\ T_z{=}-1\rangle}{\Delta E}, \tag{7}$$

and where ΔE is the separation energy and $V_{\rm C}$ the Coulomb potential.

The β^- decay of ^{46}Sc is of a mixed Fermi and Gamow–Teller (GT) type. The GT matrix element is calculated from the relation

$$M_{\rm GT}^2 = \frac{1}{2J+1}\sum_{\mu M_{\rm i} M_{\rm f}} |\langle J,\ M_{\rm f},\ K_{\rm f},\ T_{\rm f},\ T_{z{\rm f}}|\,D_{GT}(\mu)\,|J,\ M_{\rm i},\ K_{\rm i},\ T_{\rm i},\ T_{z{\rm i}}\rangle|^2. \tag{8}$$

When the operator $D_{\rm GT}\,(\mu)$ is transformed into the body-fixed coordinate system, we obtain

$$\begin{aligned} M_{\rm GT}^2 = |\ & \bar{a}_1 \sqrt{\tfrac{1}{2}}\,\langle \chi_0\, \chi^{T=1}_{T_z=-1}|\,D'_{\rm GT}(-1)\,|\chi_1\, \chi^{T=2}_{T_z=-2}\rangle \\ & + a_3 \sqrt{\tfrac{1}{5}}\,\langle \chi_3\, \chi^{T=1}_{T_z=-1}|\,D'_{\rm GT}(-1)\,|\chi_4\, \chi^{T=2}_{T_z=-2}\rangle \\ & + a_4\, 2\sqrt{\tfrac{1}{5}}\,\langle \chi_4\, \chi^{T=1}_{T_z=-1}|\,D'_{\rm GT}(0)\,|\chi_4\, \chi^{T=2}_{T_z=-2}\rangle|^2 . \end{aligned} \tag{9}$$

Using Nilsson (1955) wavefunctions with the experimental value of deformation $\beta = 0{\cdot}3$ (Stelson and Grodzins 1965) to calculate the matrix elements between the intrinsic states $|\chi_{K_{\rm i}}\, \chi^{T_{\rm i}}_{T_{z\rm i}}\rangle$ and $|\chi_{K_{\rm f}}\, \chi^{T_{\rm f}}_{T_{z\rm f}}\rangle$, it was found that the third term on the right-hand side of (9) is much larger than the first two, so that

$$\begin{aligned}M_{\mathrm{GT}}^{2} &= \tfrac{4}{5}\,a_{4}^{2}\left|\langle\chi_4\,\chi_{T_z=-1}^{T=1}\,|\,D'_{\mathrm{GT}}(0)\,|\,\chi_4\,\chi_{T_z=-2}^{T=2}\rangle\right|^{2}\\ &= \tfrac{4}{5}\,a_{4}^{2}\Big|\sqrt{\tfrac{3}{4}}\langle+\tfrac{5}{2}^{-}[31\dot{2}]\mathrm{p}\,|\,D'_{\mathrm{GT}}(0)\,|+\tfrac{5}{2}^{-}[312]\mathrm{n}\rangle\\ &\qquad -\sqrt{\tfrac{1}{12}}\langle-\tfrac{3}{2}^{-}[321]\mathrm{p}\,|\,D'_{\mathrm{GT}}(0)\,|-\tfrac{3}{2}^{-}[321]\mathrm{n}\rangle\\ &\qquad -\sqrt{\tfrac{1}{6}}\langle-\tfrac{1}{2}^{-}[321]\mathrm{p}\,|\,D'_{\mathrm{GT}}(0)\,|-\tfrac{1}{2}^{-}[321]\mathrm{n}\rangle\Big|^{2}, \qquad (10)\end{aligned}$$

which gives a value of $a_4 = |M_{\mathrm{GT}}|/1\cdot113$.

The value of M_{GT} can also be obtained from the well-known relation (Raman *et al.* 1975)

$$\begin{aligned}|M_{\mathrm{GT}}| &= \frac{G_{\mathrm{V}}}{G_{\mathrm{A}}}\left(\frac{2ft\ \text{(superallowed)}}{ft\ \text{(decay under study)}}\right)^{\frac{1}{2}}\frac{1}{(1+y^{2})^{\frac{1}{2}}}\\ &= \frac{1}{1\cdot19}\left(\frac{6222}{10^{6\cdot2}(1+y^{2})}\right)^{\frac{1}{2}}, \qquad (11)\end{aligned}$$

where $y = G_{\mathrm{V}}\,M_{\mathrm{f}}/G_{\mathrm{A}}\,M_{\mathrm{GT}}$ and is related to the experimental value of the asymmetry coefficient. Either of the adopted values of $\tilde{A}$ yields $|M_{\mathrm{GT}}| = 0\cdot053$, resulting in $a_4 = 0\cdot047$.

In the calculation of the isospin impurity amplitudes given by equations (6) and (7), the Coulomb potential V_{C} for the interaction is taken as (Damgard 1966)

$$V_{\mathrm{C}} = \frac{(Z-1)e^{2}}{R}\{\tfrac{3}{2}-\tfrac{1}{2}(r/R)^{2}\} + a(r/R)^{2}\,Y_{20}, \quad \text{for } r<R \qquad (12a)$$

$$= \frac{(Z-1)e^{2}}{r} + a(R/r)^{3}\,Y_{20}, \qquad \text{for } r>R, \qquad (12b)$$

where R is the radius of the nucleus and a is related to the Bohr deformation parameter β by

$$a = \tfrac{3}{5}\beta(Z-1)e^{2}/R.$$

Calculation yields a much larger value for the isospin impurity amplitude α_4 than α_0, which can then be neglected. The final theoretical value for the Fermi nuclear matrix element becomes

$$(M_{\mathrm{F}})_{\mathrm{th}} = 0\cdot91\times10^{-3},$$

which compares favourably with the experimental value of

$$(M_{\mathrm{F}})_{\mathrm{exp}} = (1\cdot4\pm0\cdot3)\times10^{-3}.$$

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