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Cosmic Ray Anisotropy above 10^{15} eV

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Abstract

An examination is made of published data on cosmic ray anisotropy at energies above about 10^{15} eV. Both amplitude and phase results are examined in an attempt to assess the confidence which can be placed in the observations as a whole. It is found that whilst many published results individually may suggest quite high confidence levels of real measured anisotropy, the data taken as a whole are less convincing. Some internal consistency in the phase results suggests that a real effect may have been measured but, again, this is not at a high confidence level.

1. Introduction

The observed arrival directions of cosmic rays with energies above 10^{15} eV are remarkably isotropic. Experiments to study this isotropy at such energies require large collecting areas and long observation times, and individual experiments have rarely produced results which are of great statistical significance. However, in the literature there is now a considerable body of data on cosmic ray isotropy and the aim here is to put these data together to see how well the total data set fits our statistical ideas concerning the experimental results. I am aware that, in the past, a number of apparently significant results have not proved to be repeatable and a motive for this study was to develop some background for judging the significance of new results as they become available. I also wish here to identify any areas of the field which are in particular need of further work at the present time.

2. Experiments

Many studies of the deviation of the cosmic ray flux from isotropy (measurements of the anisotropy) have been made over the past 35 years. The early experiments were largely based on monitoring the counting rate of cosmic ray detectors operated in coincidence, with directional information being based on collimation due to shower absorption in the atmosphere (typical angular resolution $\sim\pm25°$). More recent experiments have tended to be based on the measurement of the arrival directions of individual cosmic ray showers, and the angular resolution is then improved by an order of magnitude. This improvement in angular resolution has probably hardly affected the usefulness of the data for overall anisotropy purposes, since the data

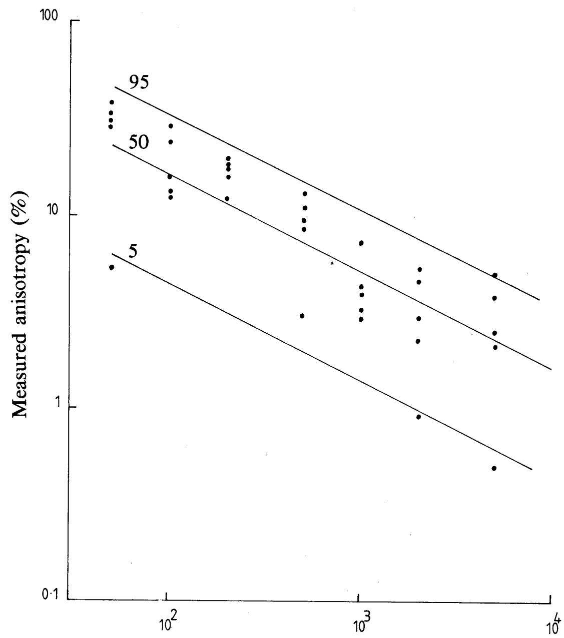

Fig. 1. Relationship between the derived percentage anisotropy and the number of events observed for events arriving randomly. The results of five trials are shown for each abscissa value and the expected 5%, 50% and 95% limit lines are indicated, derived on the basis of a random walk.

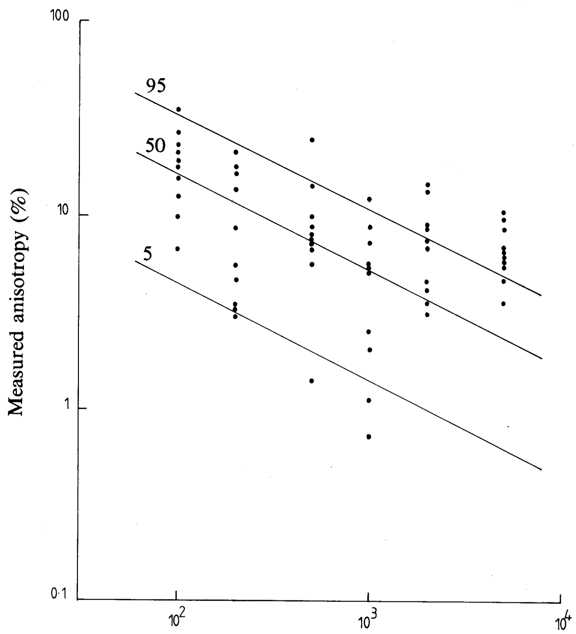

Fig. 2. Relationship between the derived percentage anisotropy and the number of events observed. The events were random except that 5% of the events were selected with phases randomly chosen only from 90° to 270°. Ten trials are shown for each case.

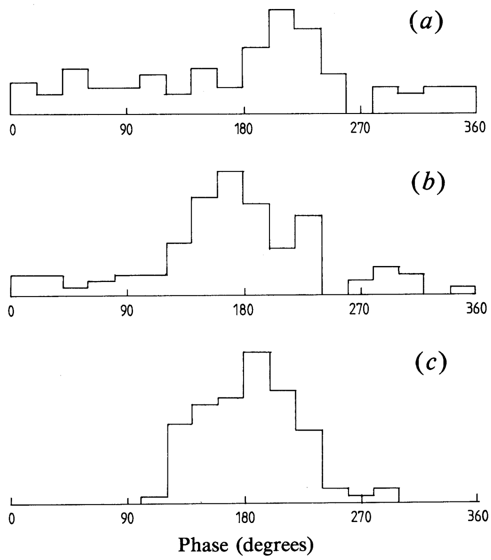

Fig. 3. Distribution of observed phases for the anisotropic sampled distributions as used in Fig. 2. One hundred trials were used in each case: (*a*) 100 events per trial, (*b*) 500 events per trial and (*c*) 2000 events per trial.

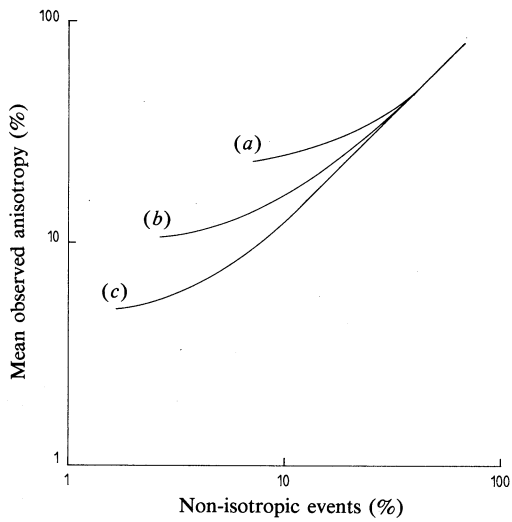

Fig. 4. 'Measured' anisotropies in experiments when a certain percentage of events is selected randomly only from the limited 90°–270° region: (*a*) 50 events per trial, (*b*) 200 events per trial and (*c*) 1000 events per trial.

set of an individual experiment has usually been analysed to give only the first and second harmonics of any deviation from isotropy. Since the time of the earliest experiments, allowance has been made for atmospheric pressure and temperature effects, often through the examination of Fourier sidebands due to the inclusion of spurious solar-time components (see e.g. Farley and Storey 1954).

Compilations of results of anisotropy experiments have been made by Sakakibara (1965) and by Linsley and Watson (1977). In addition, in the present paper, I have included data obtained at Adelaide (Gerhardy and Clay 1983), Akeno (Kifune *et al.* 1986), Haverah Park (Eames *et al.* 1985), Sydney (Horton *et al.* 1983), Utah (Baltrusaitis *et al.* 1985) and Yakutsk (Krasilnikov *et al.* 1983; Efimov *et al.* 1983).

3. Theory for Interpreting the Experiments

The theoretical ideas used in interpreting results from anisotropy experiments are well known and are based on ideas introduced by Lord Rayleigh (1880), which have been discussed in particular by Chapman and Bartels (1940) and, more recently, by Linsley (1975*a*, 1975*b*). The work presented by Linsley deals especially with the analysis of cosmic ray data.

If a data set is analysed in terms of the right ascension of each event, such that event vectors of equal magnitude are combined wih their phase corresponding to each event right ascension, a resultant fractional harmonic amplitude and phase may be obtained. If the events are random in phase then the properties of the resultant will be those given by Rayleigh's formulae (see e.g. Linsley 1975*b*).

The probability of obtaining a fractional amplitude greater than r by chance for n events is given by

$$P(>r) = \exp(-r^2 n/4).$$

Clearly, a useful parameter to describe these distributions is k_0 $(= r^2 n/4)$ so that

$$P(>k_0) = \exp(-k_0).$$

In cosmic ray work one expects an underlying anisotropy such that vectors in a certain distribution of directions are preferred. A major aim in this field is to determine the magnitude and direction of the underlying anisotropy through measurements of the resulting vector $\boldsymbol{r}$, which is given by

$$\boldsymbol{r} = \boldsymbol{s} + \boldsymbol{x},$$

such that $\boldsymbol{s}$ is the underlying anisotropy vector and $\boldsymbol{x}$ represents (ideally) the Rayleigh fluctuations.

Linsley has discussed the distribution $P_{s,\theta}$ which describes the probability that the population from which the data are drawn have s in $\mathrm{d}s$ and θ in $\mathrm{d}\theta$, when the data give values of amplitude r and phase ψ. The parameter k_0, which is a function of the ratio of the observed resulting amplitude to a mean random walk amplitude, determines the form of the appropriate distribution. Rarely, if ever, do cosmic ray results present us with the case $k_0 \gg 1$, corresponding to a very significant resulting amplitude. Linsley has considered cases over a broad range of k_0 (1/16 to 64) and it is interesting to see that, whilst for $k_0 = 1$ the 68·3% confidence limit of $|\theta|$ is less than 60° and only reaches 90° for $k_0 \sim 1/4$, the *95%* confidence limits of $|\theta|$ are still greater than 90° when k_0 is as high as 2, a situation hardly ever achieved in cosmic ray work.

4. Some Results of Simulations

Random walk results are simple to simulate and some of the above analytical results can be presented in ways which can allow a straightforward comparison with experimental data. Fig. 1 shows some results for random walk vectors to give an indication of the form of the data. The results of five 'experiments' or 'trials' are shown for each value of n (the number of events per experiment). There is a clear tendency for the measured anisotropy amplitude to decrease with $n^{1/2}$ as expected, and the distributions are in agreement with the expected limits.

If a number of non-isotropic events are added to the observed data set, the effect of an underlying anisotropy on the observations may be seen. Fig. 2 shows results obtained for anisotropy amplitudes when a randomly chosen 5% of the events had a phase chosen randomly over only a limited range (90° to 270°), while Fig. 3 shows the corresponding direction vectors that result.

An alternative view is shown in Fig. 4 which displays the relationship between the mean *observed* anisotropy and the *actual* percentage of non-isotropic events. The exact curves obtained depend on the number of events used in a given experiment and the phase distribution of the non-isotropic events, but generally there is an apparent threshold below which any anisotropy is masked and above which there is a progressive observable effect due to the anisotropic signal. As can easily be derived from Fig. 4, the threshold in this case is at a value of $k_0 \sim 0 \cdot 4$.

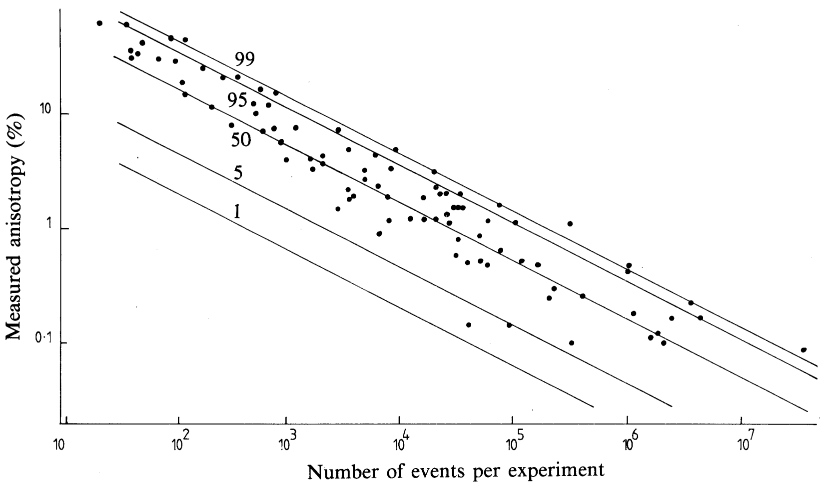

Fig. 5. Anisotropy amplitudes reported in the literature as a function of the number of events in the data set (energies are 10^{15} eV).

5. Measured Anisotropies

(a) Amplitudes

I have compiled a set of data for comparison with the expected statistical distributions which, I believe, cover most of the independent anisotropy results available. I used the compilation of Linsley and Watson (1977) and the more recent results from Akeno, Adelaide, Yakutsk, Sydney, Haverah Park and Utah. Fig. 5 shows the available data above the claimed energies of 10^{15} eV in terms of the derived anisotropy amplitude and the total number of detected events.

Two points concerning Fig. 5 are worth noting. Firstly, over a range of experimental event numbers covering more than six orders of magnitude, when compared with the form of Fig. 2 there is no clear progressive deviation from the general form one would have expected if there was no anisotropy. Secondly, there is, nonetheless, a general tendency at all event numbers to have derived anisotropies greater than those expected on a random basis.

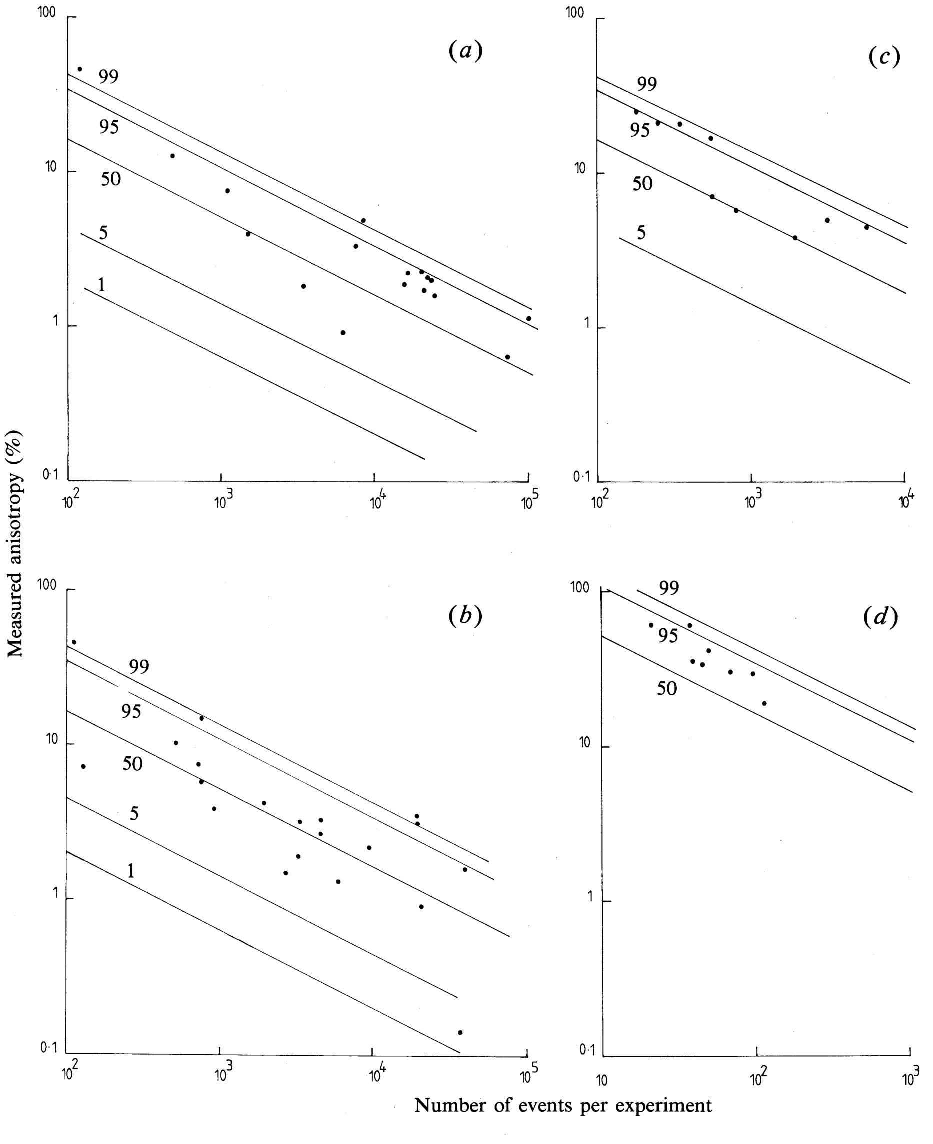

Fig. 6. Anisotropy amplitudes reported in the literature as a function of the number of events in the data set for limited ranges of energy: (*a*) 5×10^{15} to 5×10^{16} eV, (*b*) 5×10^{16} to 10^{18} eV, (*c*) 10^{18} to 10^{19} eV and (*d*) above 10^{19} eV.

Fig. 5 covers data for all energies from 10^{15} to 10^{20} eV. It is possible that the anisotropy may be energy dependent so that, when the experimental data are combined as in Fig. 5, there would still be no clear trend to deviate from 'random' amplitudes. Fig. 6 shows the data displayed in the four limited ranges of energy indicated. A break was made at 5×10^{15} eV in case there was some change above and below the knee of the energy spectrum. Only in the energy range between 5×10^{16} and 10^{18} eV (Fig. 6*b*) is there a distribution of amplitudes which is convincingly close to a random expectation. In every other case, there is a substantial deficit of experiments with integral probabilities below 50%.

In the range 5×10^{16} to 10^{18} eV, which most clearly approaches a random distribution, there are 20 experiments. Four of the results have amplitudes greater than the 95% probability level (where we expect only one on a random basis). Two of these have amplitudes which were measured with small numbers of events. If the results in these two cases truly measured the anisotropy, the results in the rest of the data set with much higher event numbers could not be compatible as members of the same parent population. In other words, the data set as a whole is not compatible with unbiased measurements of an anisotropy. Similar comments apply also to each other energy range and in these cases one could also note that there is no evidence of any flattening such as is found at the higher event levels in Fig. 2. For the moment, my conclusion is that there is no convincing internally consistent evidence, from the measured amplitudes, for any observed anisotropy, but I cannot explain the form of the observed distribution of amplitudes. It is not a simple Rayleigh form.

(b) Directions

With any measurement of an anisotropy amplitude, there is also a measured direction. One would like to know confidence limits for this measured direction. Linsley (1975*a*) and, in a sense, Sakakibara (1965) have examined this problem and, as discussed above, Linsley has given an expression for the distribution $P_{s,\theta}$.

At first sight it seems anomalous that one can define a limited angular uncertainty when k_0 is smaller (and indeed substantially smaller) than 1. Effectively, one is saying that a direction may be measured for a signal which is 'buried' in noise. However, the measured signal r is a sample from a distribution of all possible signals from all possible parent amplitudes, and the angular uncertainty derived by Linsley is limited even for small measured signals r because the parent amplitude can have a substantial magnitude and still give that small magnitude for r. As Linsley has stated: 'the population having been selected at random from an ensemble in which all possible values of s (magnitude and phase) are equally represented'. On the other hand, I feel that the previous discussion of signal amplitudes gives sufficient grounds for regarding the ensemble of underlying anisotropies as limited to only small values. Linsley's result should then be regarded as a lower limit to the angular uncertainty on any new datum in the previously studied ranges.

One can take an operational attitude and ask just how consistent are the observed phases. I do not have complete faith in the conventional confidence limits, both for the reasons explained immediately above and because I believe that the distribution of $|r|$, the observed amplitude, is not sufficiently well understood, which is a factor used to calculate the confidence limits. As a consequence, I have selected and examined certain experimental results which one could imagine may be closer to s than others. For these results I took from the data in Fig. 6 the experiments which contained event numbers within one order of magnitude of the number of events in the largest experimental data set of each energy range, and also used only those experiments which gave amplitudes above the 50% probability line. I did not just combine all the data in a given energy range, since I do not understand any effects which may cause the amplitude distribution to be other than expected, but merely plotted the raw experimental phase distributions. The results of this selection are shown in Fig. 7, while Fig. 8 shows a compilation from Fig. 7 of the ranges of the data.

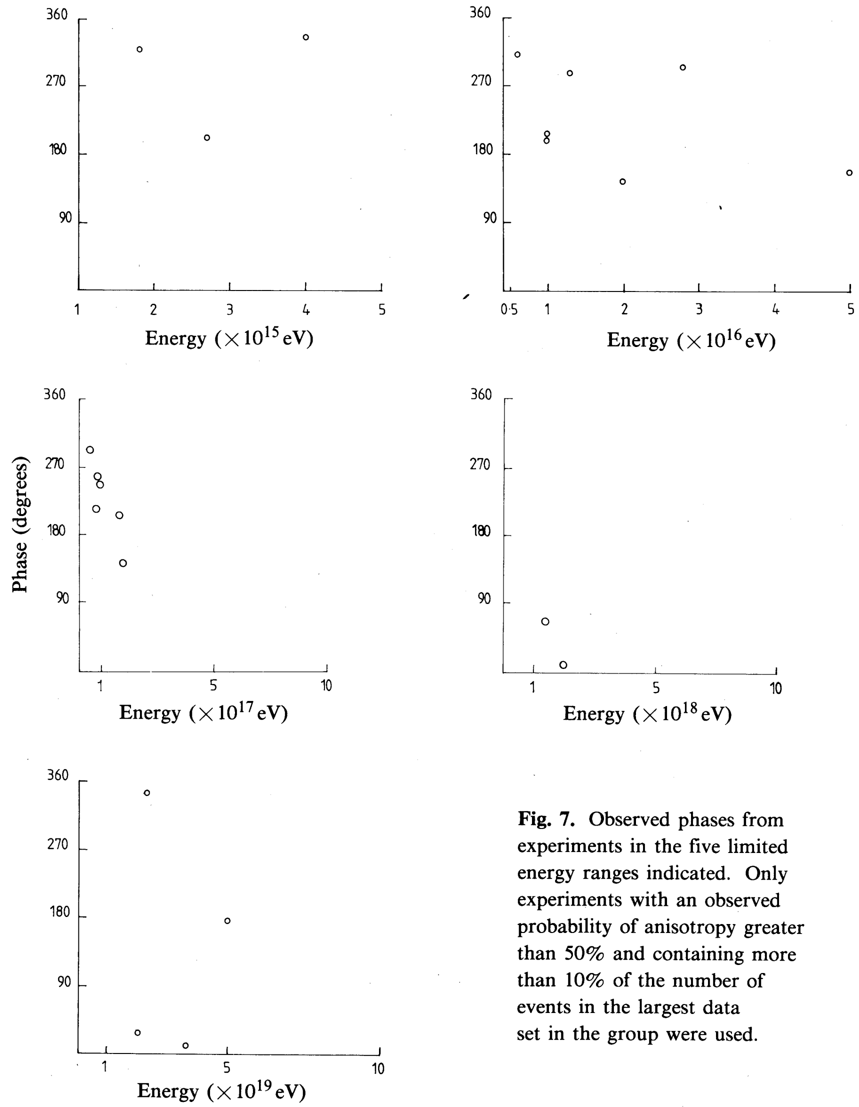

Fig. 7. Observed phases from experiments in the five limited energy ranges indicated. Only experiments with an observed probability of anisotropy greater than 50% and containing more than 10% of the number of events in the largest data set in the group were used.

The selection of experiments for Fig. 7 is such that experiments at energies low in any given energy range tend to be selected because the cosmic ray energy spectrum falls steeply. The particular experiments chosen then tend, of course, to depend on the limits of the selected energy ranges. It may be noteworthy that, in each case in Fig. 7, the results are limited in phase to a total spread of not more than 180°, and one might view Fig. 8 as indicating a progressive phase change with energy. Fig. 8 is (necessarily) similar to the standard form of the dependence often given in compilations of phase variations (see e.g. Watson 1984), but now with a different meaning for the error bars.

The trend in Fig. 8 bears comment since it illustrates the cyclic feature of such representations. With cyclic data, there is always freedom to choose the cycle to which a given phase relates, so that any given anisotropy phase result may have multiples of 360° added or subtracted. This is relevant when the variation of phase

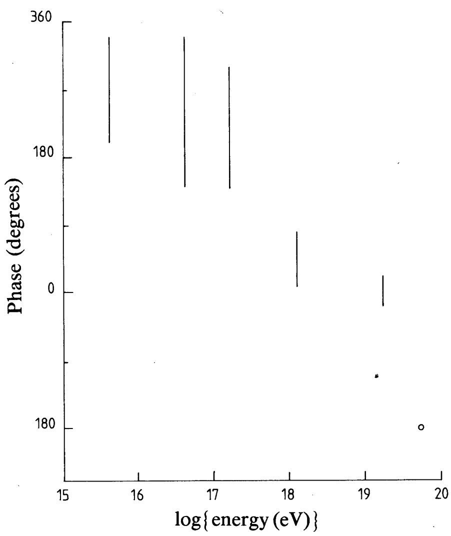

Fig. 8. Energy variation of the spread of phases in Fig. 7. The 'error' bars indicate the limits of spread in the data.

with energy is considered. If data in compilations of phase versus energy are moved by 360° above 10^{17} eV, a new plausible phase dependence similar to Fig. 8 results, and the standard form with a rapid change at 10^{17}–10^{18} eV is not unique. Indeed, the new phase dependence may have some appeal since no drastic change in phase variation is required at $\sim 10^{18}$ eV, an energy which is not associated with any particular structure in the energy spectrum. There are unfortunately few experimental points, perhaps only two, in the uncertain range in which, by convention, there is a rapid phase change.

6. Phase Agreement and Limited Data Sets

The freedom to move phase data by 360° suggests that one should be wary when comparing data from various experiments, particularly when one might suspect that the angular uncertainties may have been underestimated. One can take data which are generated randomly in phase and ask whether or not apparently genuine variations such as those in Fig. 8 or the standard compilations may be obtained, given realistic experimental uncertainties plus the freedom to move data points by multiples of 360° in phase.

I have taken randomly chosen phases and assigned them to typical experimental sets of combinations of primary energy and phase uncertainty (in this case those obtained by Kifune *et al.* 1986) to generate random data with appropriate phase uncertainties. These were then plotted in the usual way as in the compilation of Watson (1984) (his Fig. 3) to display phase against energy, and to see whether any phase trend appeared. As in the real experiments, freedom to change phase by 360° was assumed and employed in order to make the data into apparently smooth eye fits. Encouraging fits were produced in $\sim 80\%$ of the random data sets when viewed from an aesthetic point of view only. In other words, one cannot in general use an apparent phase consistency as a criterion for confirming that a data set contains 'true' anisotropies, since apparently convincing phase dependences can readily result from randomly chosen data sets. However, if one tries to make an eye fit of a smoothly varying function to the data, and then examines this fit in terms of a goodness of fit chi-squared parameter (ignoring possible phase uncertainties of 360°), it becomes clear that most of the apparently satisfactory fits are indeed poor. The smooth curve usually fits the data points with large errors rather well, but the data points with small phase uncertainties which are not so important in 'eyeball fits' almost invariably dominate the goodness of fit parameter, and large 'chi-squareds' are found in almost all cases for random phase data. The real data in compilations are appreciably better than most (80%) randomly generated data sets in terms of having a low 'chi-squared' for an eye-function fit.

7. Discussion

It seems from the above examination that there is no convincing evidence for any clear anisotropy amplitude in the presently available data. Where one might expect certain data sets to be most likely to show an anisotropy on the basis of results from smaller data sets at similar energies, it is *not* the case that this is so. The general distributions of observed amplitudes appear more like random results with some additional non-random effects, such as one might find if there were some residual systematic effects in some data sets not removed by techniques such as anti-sidereal analysis. However, there is phase consistency between results but this is not yet convincingly strong, perhaps only at the 80% level at the moment.

A further possibility is that inherent in the data are experimental effects which may make it unlikely that a significant anisotropy will be observed and agreed upon by a number of real arrays nominally operating at the same energy or, indeed, by a single real array between adjacent energy ranges.

(a) Anisotropy, Composition and Arrays

Charged cosmic ray primaries are expected to travel in paths which are controlled by galactic or intergalactic magnetic fields. The paths will then be rigidity dependent and, for a fixed primary energy, one expects different paths for whatever different elemental species there may be in the beam. Kifune *et al.* (1986) have attempted to exploit this effect by looking for an anisotropy of the subset of muon-rich showers in the beam. Their intention was to select a group of primaries which are expected to be deficient in lighter elements (see also Tkaczyk *et al.* 1985).

A well-known effect in air shower work (see e.g. Clay 1985) is that different arrays respond differently to different composition components in the primary beam at a given primary energy. Particular differences are expected between arrays sensitive to different air shower components (electrons, muons, Cerenkov light, etc.), although there will also be differences between relatively similar arrays at different altitudes or with different triggering criteria, or indeed between results for a given array at different observed energies. Thus, for a primary beam containing different elemental components, we might *a priori* expect different arrays to measure different anisotropies as they each observe their own compositional subset of showers at a given apparent energy. These differences are not likely to be trivial. If the beam contains both proton and iron primaries and two arrays select beams with say 50% iron or 80% iron at a given energy parameter (see e.g. Clay 1985), then one can examine Fig. 8 and note that it would not be surprising if components with rigidities differing by 26 times were up to 180° out of phase, and the observed results for the two arrays would be quite unlikely to be similar.

(b) Some General Considerations of Cosmic Ray Propagation

Roughly speaking, the radius of gyration for a proton will be ~ 1 pc at 10^{15} eV in a $1\ \mu G$ ($\equiv 10^{-10}$ T) magnetic field. Our galaxy is a spiral with a very small ratio of thickness to diameter, ~ 1 kpc to 25 kpc. As a result, if we assume that the dimensions of the galactic magnetic field are similar to (within an order of magnitude) the dimensions of the galaxy and that the observed fields of a few μG are typical, then the ratio of the radius of gyration to the smallest galactic dimension will be $\sim 10^{-4} E$, where E is measured in units of 10^{15} eV. This is likely to be an underestimate for considering cosmic ray propagation since the galactic magnetic field may well be lumpy with regions of significant size which have low magnetic field strengths (at the very least in inter-arm regions), and also since the scale of the galactic thickness may well be overestimated as 1 kpc, with matter in the galaxy on a scale more likely 100–300 pc. We can naively see that at 10^{18} eV, there can be little serious effect of the magnetic field on propagation in terms of containment in the galaxy and also that, as a result, if sources of cosmic rays at these energies are galactic, there must be large anisotropies associated with the grossly anisotropic distribution of the Milky Way in the sky. This point of view is illustrated graphically when results from cosmic ray proton propagation in proposed models of galactic magnetic fields are examined (e.g. Karakula *et al.* 1971). There is always a gross expected anisotropy at these energies, not a few per cent but approaching 100%. This is *not* observed.

It is true that one may consider particles with large electric charges, e.g. iron nuclei (see Tkaczyk *et al.* 1985), but then acceleration without breaking up the nuclei is not straightforward, and one must also explain the lack of protons. It may be necessary to again consider ultra-high energy particles coming from large distances, where it is the isotropy of the Universe which ensures directional isotropy. Conventional nuclei are not satisfactory as primaries for this purpose, due to attenuation by the microwave background, and we may again have to look at the possibility of the observed particles being photons. Recent discussions (see e.g. Watson 1985; Clay 1985; Protheroe 1986) on the structure of ultra-high energy gamma-ray showers emphasise that one cannot unequivocally eliminate this possibility on the basis of observations of shower structure, although some rethinking of the role of the muon component, for instance, would be necessary.

8. Concluding Remarks

The most remarkable fact emerging from studies of cosmic ray anisotropies is that, above 10^{15} eV, there is no convincing evidence that any real anisotropy has been observed, despite cosmic rays of these energies being affected relatively little by galactic magnetic fields and the gross local structure of the galaxy being very anisotropic. If the experimental situation is to be improved, then we must regard present anisotropy measurements as upper limits, and attempt experiments which will retain good long-term stability and either record very large data sets, at least two orders of magnitude greater than presently available at a given energy, or have very good compositional resolution. In the former case, a return to long-term counting experiments may well be appropriate with large, simple, arrays and with counting rates of ~ 1 Hz at say $\gtrsim 10^{16}$ eV.

Acknowledgment

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