Aust. J. Phys., 1988, 41, 587–96

# An Investigation of r.f. Travelling Wave Current Drive Using the $\langle \tilde{J} \times \tilde{B} \rangle$ Model

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*Abstract*

Previous experimental investigations into the use of travelling r.f. waves to drive steady toroidal currents in a toroidal plasma have shown that $I_t$, the amount of current driven, is strongly dependent on the ratio of the static toroidal magnetic field $B_z$, to the strength of the r.f. magnetic field $B_\omega$. This dependence is characterised by an initial increase and subsequent decrease of $I_t$ when $B_t/B_\omega$ increases. It is shown that this observed behaviour is entirely consistent with the behaviour predicted by the $\langle \tilde{J} \times \tilde{B} \rangle$ current drive model. Results from numerical computations using the $\langle \tilde{J} \times \tilde{B} \rangle$ model show good quantitative agreement with the published experimental results.

## 1. Introduction

The method of driving steady currents in a toroidal plasma by means of travelling r.f. waves has been the subject of several experimental and theoretical studies. Most early experiments (Thonemann *et al.* 1952; Borzunov *et al.* 1964; Fukada *et al.* 1976) were performed using $m = 0$ travelling waves. More recently, attention has been focused on current drive using $m = 1$ and 2 r.f. coil configurations (Hotta *et al.* 1985; Dutch and McCarthy 1986; Kikunaga *et al.* 1982). Closely related to these experiments are the rotamak experiments (Jones 1986) which demonstrate the possibility of driving steady currents using a rotating magnetic field.

One feature common to most of these experiments is that the wave frequency and the electron–ion collision frequency lie between the ion and electron gyro frequencies. The r.f. wave then acts mainly on the electron fluid and the interaction between the wave and the plasma ions is relatively unimportant. At a very early stage (Thonemann *et al.* 1952; Borzunov *et al.* 1964), it was recognised that under these circumstances the driving force on the electron fluid originates from the non-linear Hall term $\boldsymbol{J} \times \boldsymbol{B}$ in the generalised Ohm's law.

The $\langle \tilde{J} \times \tilde{B} \rangle$ theory was developed further by Jones and Hugrass (1981) and Hugrass (1985), to provide a theoretical framework for the rotamak experiments. In most rotamak configurations the oscillating magnetic field $\tilde{\boldsymbol{B}}$ is perpendicular to the steady magnetic field $\boldsymbol{B}$. The $\langle \tilde{J} \times \tilde{B} \rangle$ current drive theory was never fully developed to describe the current drive mechanism for a tokamak in which $\tilde{\boldsymbol{B}} . \boldsymbol{B} \neq 0$. Consequently, when the experimental results for $m = 0$ current drive in a tokamak showed a strong dependence of the driven current on the strength of the applied

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toroidal field (Hirano *et al.* 1971; Fukada *et al.* 1976; Fukada 1978), it was believed that the observed results could not be explained in terms of the $\langle \tilde{J} \times \tilde{B} \rangle$ current drive mechanism. In particular, the substantial reduction in the amount of driven current when the toroidal field became very strong was explained using an electron trapping model (Fukada *et al.* 1976).

The hollow current density profiles observed by Fukada (1978) were investigated theoretically by Hugrass (1981) using the $\langle \tilde{J} \times \tilde{B} \rangle$ current drive model which included ion mobility as well as an applied toroidal field. However, in this work no attempt was made to explain the observed dependence of the driven current on the strength of the toroidal field.

This paper shows, by means of a numerical simulation, that the $\langle \tilde{J} \times \tilde{B} \rangle$ current drive mechanism alone can provide not only a qualitative description, but a good quantitative description of the observed dependence of the driven current on the applied toroidal field. This study is confined to the $m = 0$ case, since it is the only case for which sufficient experimental data are available for a meaningful comparison with the theory.

## 2. Basic Equations

The model is of an infinite cylindrical plasma of radius $a$ in which the ions are assumed to be immobile. The equations describing the $\langle \tilde{J} \times \tilde{B} \rangle$ current drive mechanism are Maxwell's equations and the generalised Ohm's law:

$$E = \eta J + \frac{1}{ne} J \times B. \tag{1}$$

The resistivity $\eta$ is assumed to be uniform but, to allow a better description of radial profiles, the electron number density $n$ is a specified function of the radial coordinate $n = \bar{n}\rho(r)$.

The boundary conditions are chosen to correspond to a steady applied magnetic field $B = (0, 0, B_z^{\mathrm{v}})$ and an oscillating field

$$\tilde{B} = (-B_\omega^0\, I_1(kr)\cos(kz - \omega t),\, 0,\, B_\omega^0\, I_0(kr)\sin(kz - \omega t)). \tag{2}$$

Each physical variable $Q(r, z, t)$ is assumed to be of the form

$$Q = Q(r) + \mathrm{Re}\{ q(r)\, e^{i(m\theta + kz - \omega t)} \}, \tag{3}$$

where, in accordance with previous practice (Hugrass 1985; Bertram 1987), we ignore the effects of second and higher harmonics. The justification for this assumption will be discussed later (Section 5). Considering only $m = 0$, the components of the oscillating magnetic field in the plasma can be expressed in terms of two complex scalar functions $\psi$ and $g$:

$$b_r = -i\kappa B_\omega^0\, \psi, \qquad b_\theta = B_\omega^0\, g, \qquad b_z = (B_\omega^0/x)\frac{d}{dx}(x\psi), \tag{4}$$

where $x = r/a$ and $\kappa = ka$.

Using the method outlined in Bertram (1987), Maxwell's equations and Ohm's law yield a system of coupled differential equations for $\psi$, $g$, $B_\theta$ and $B_z$:

$$\Delta\psi + 2\mathrm{i}\,\lambda^2\psi = \mathrm{i}(\gamma/\rho)(B_z\, g - \mu_0\, J_z\,\psi)\,, \tag{5}$$

$$\Delta g + 2\mathrm{i}\,\lambda^2 g = \mathrm{i}(\gamma/\rho)\left\{ B_z\,\Delta\psi + x\psi\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mu_0\, J_\theta}{x}\right) + \frac{2}{x}\, B_\theta\, g + \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x}(B_\theta\, g - \mu_0\, J_\theta\,\psi)\right\}, \tag{6}$$

$$\frac{\mathrm{d}B_\theta}{\mathrm{d}x} = \mu_0\, J_z - \frac{1}{x}\, B_\theta\,, \qquad \frac{\mathrm{d}B_z}{\mathrm{d}x} = -\mu_0\, J_\theta\,, \tag{7, 8}$$

$$\mu J_\theta = (\gamma/2\rho x^2)\,\mathrm{Im}\,\frac{\mathrm{d}}{\mathrm{d}x}(x^2\psi^* g)\,, \tag{9}$$

$$\mu_0\, J_z = (\gamma/2\rho)\,\mathrm{Im}(\psi^*\Delta\psi)\,, \tag{10}$$

where $\lambda^2 = \mu_0\,\omega a^2/2\eta$, $\gamma = \kappa B^0_\omega/e\bar{n}\eta$, $\rho$ is the normalised electron density, and the operator $\Delta$ is defined by

$$\Delta\psi = \frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} + \frac{1}{x}\frac{\mathrm{d}\psi}{\mathrm{d}x} - \left(\kappa^2 + \frac{1}{x^2}\right)\psi\,. \tag{11}$$

The boundary conditions at $x = 0$ are obtained from the requirements of continuity, i.e. $\psi = g = B_\theta = 0$ at $x = 0$. The conditions at $x = 1$ are determined by matching the internal and external fields at the plasma–vacuum interface; thus at $x = 1$,

$$K_1(\kappa)\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x}(x\psi) + \kappa K_0(\kappa)\psi = -\mathrm{i}/\kappa\,, \tag{12}$$

$$g = 0\,, \qquad B_z = B^{\mathrm{v}}_z\,.$$

When $B^{\mathrm{v}}_z = 0$, solution of these equations yields $g = B_z = J_\theta = 0$. The remaining equation is

$$\Delta\psi + 2\mathrm{i}\,\lambda^2\psi = -\mathrm{i}\,\gamma\mu_0\, J_z\,\psi/\rho\,. \tag{13}$$

Multiplying this by $\psi^*$ and using equation (9) to eliminate $\psi^*\Delta\psi$ results in the relation given by Thonemann *et al.* (1952) and by Borzunov *et al.* (1964):

$$\mu_0\, J_z = -\frac{\lambda^2\gamma\psi^*\psi/\rho^2}{1+\gamma^2\psi^*\psi/2\rho^2}\,. \tag{14}$$

In general, when $B^{\mathrm{v}}_z \neq 0$ this relation is not correct and the full set, equations (5)–(10), must be solved numerically.

### 3. Numerical Results

Equations (5)–(10) with the appropriate boundary conditions were solved as a two-point boundary value problem using a standard finite difference integration

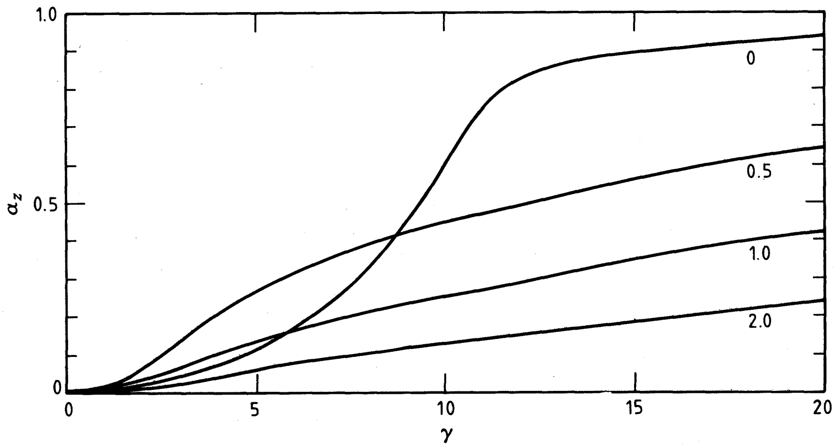


**Fig. 1.** Dependence of the normalised total driven axial current on the parameter $\gamma$ for $\lambda = 4$ and $B_z^0 = 0$, $0 \cdot 5$, $1 \cdot 0$ and $2 \cdot 0$.

program. Solutions of the equations depend on the three dimensionless variables $\lambda$, $\gamma$ and $B_z^0 = \kappa B_z^{\mathrm{v}}/\mu_0 \omega n e a^2$. The maximum current that can be driven is the synchronous current $J_{\mathrm{s}}$ when the entire electron fluid moves as a rigid body with the phase velocity of the r.f. wave:

$$J_{\mathrm{s}} = -2\rho\lambda^2/\mu_0 \gamma \,. \tag{15}$$

It is convenient to define the normalised current densities $J_z^0 = J_z/J_{\mathrm{s}}$ and $J_\theta^0 = J_\theta/J_{\mathrm{s}}$ and the normalised total current

$$\alpha_z = \int J_z \, x \, \mathrm{d}x \bigg/ \int J_{\mathrm{s}} \, x \, \mathrm{d}x \,. \tag{16}$$

The dependence of $\alpha_z$ on the parameter $\gamma$ is shown in Fig. 1. When $B_z^0 = 0$, this dependence is characteristic of $\langle \tilde{J} \times \tilde{B} \rangle$ current drive (Hugrass 1985), exhibiting a fairly well-defined threshold value of $\gamma$ above which the total driven current almost reaches the synchronous value. The effect of the applied axial field is also typical (Bertram 1987) with $\alpha_z$ decreasing above the threshold and increasing below the threshold.

As seen from Fig. 2*a*, under certain conditions the application of an axial magnetic field can result in a significant enhancement of the driven current. Particularly interesting are the results for $\gamma = 5 \cdot 0$ and $7 \cdot 5$, which are very similar to those obtained experimentally by Hirano *et al.* (1971). These authors suggested that the increase in driven current is due to the excitation of the whistler wave. This is at least partially correct as can be seen from Fig. 2*b*, which shows the degree of r.f. field penetration at the centre of the plasma as a function of $B_z^0$. There is evidence of a resonating wave mode, but the position of the maximum in the driven current does not coincide with the position of this resonance.

The radial distributions of the currents (Fig. 3), calculated using a uniform density profile ($\rho = 1$), show the initial improvement of the driven axial current within the

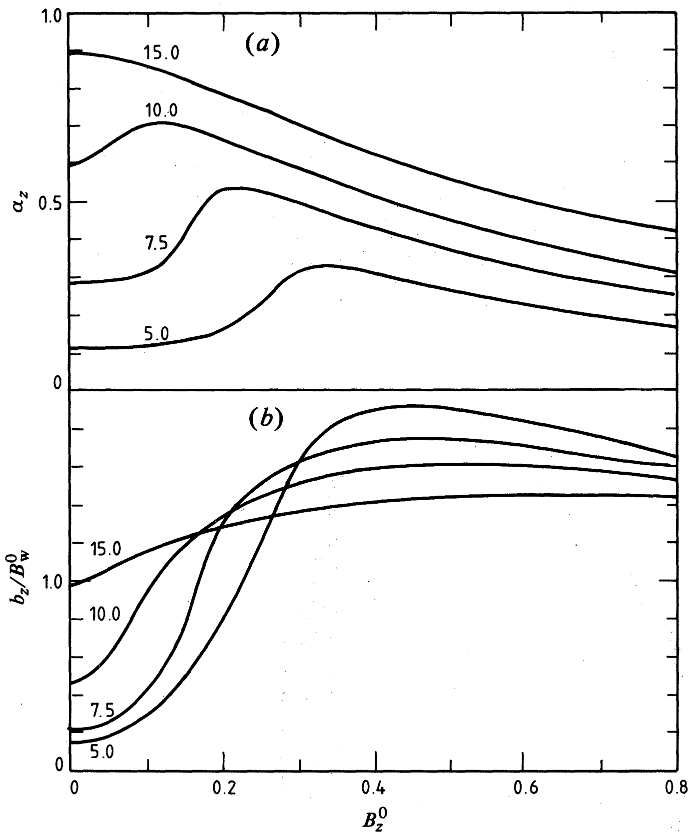


**Fig. 2.** Dependence of (*a*) the total driven axial current and (*b*) the strength of the r.f. field at $x = 0$ on the magnitude of the applied steady field $B_z^0$. Results are shown for $\lambda = 4$ and $\gamma = 5, 7{\cdot}5, 10$ and 15.

plasma when $B_z^0$ is increased. When $B_z^0$ becomes very large the current decreases and tends to a skin current where the only significant current driven is confined to the boundary region of the plasma. For intermediate values of $B_z^0$ there is also some azimuthal steady current. From the analysis of Klima (1973, 1974), it can be shown that the travelling r.f. wave cannot impart net angular momentum to the plasma. This gives rise to the condition

$$\int_0^1 \eta\rho x^2 J_\theta \, \mathrm{d}x = 0. \tag{17}$$

Our numerical results always satisfy this condition as is evident from equation (9) and the boundary conditions.

## 4. Comparison with Experiment

The validity of the $\langle \tilde{J} \times \tilde{B} \rangle$ current drive model is best demonstrated by a quantitative comparison with experimental results. Most of the recent available data for $m = 1$ and 2 current drive are not sufficiently detailed to allow such a comparison.

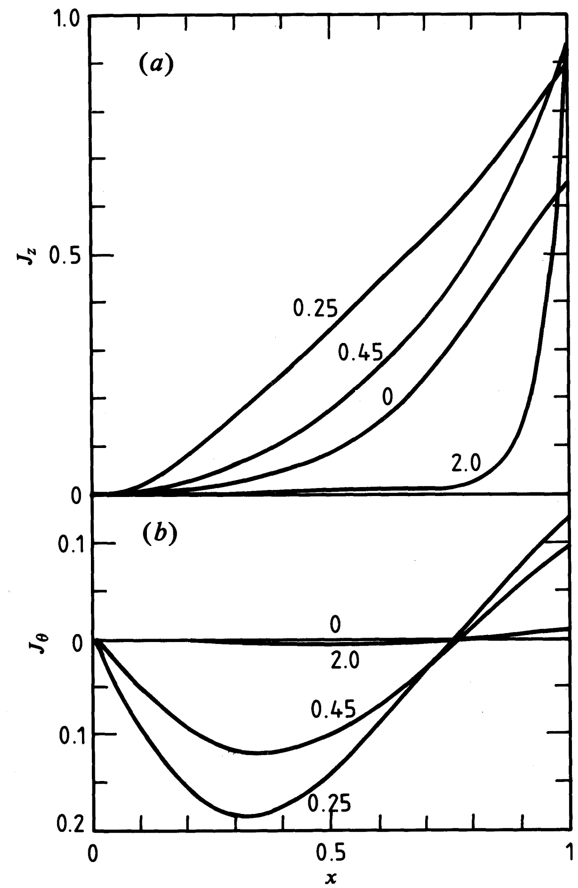


**Fig. 3.** Radial distributions of the driven (*a*) axial and (*b*) azimuthal current densities when $\lambda = 4$, $\gamma = 7\cdot5$ and $B_z^0 = 0$, $0\cdot25$, $0\cdot45$ and $2\cdot0$.

The more detailed results from recent rotamak experiments (Collins *et al.* 1988) are not suitable owing to the geometrical complications. More suitable for our purposes are the measurements by Fukada *et al.* (1976) which give the total driven current $I_z$ and the average electron density $\bar{n}$ as a function of the applied toroidal field $B_z^{\mathrm{v}}$. The basic parameters for this experiment, which was carried out in a toroidal vessel, are $a = 0\cdot04$ m, $k = 0\cdot47$, $\omega = 2\cdot26\times10^7$ rad/s and $B_\omega^0 = 2\times10^{-3}$ T. From these values it is possible to obtain the ratio

$$\lambda^2/\gamma = 3\cdot9\times10^{-18}\,\bar{n}\,. \tag{18}$$

The value of the resistivity is not known. However, the parameter $\lambda$ can also be expressed as $\lambda = a/\delta$ (Hugrass 1985), where $\delta$ is the classical skin depth. Fukada *et al.* (1976) gave the estimate $\delta \approx 0\cdot01$ m which corresponds to $\lambda \approx 4$.

Measurements of the density profile by Fukada (1978) under similar experimental conditions indicated that the electron density is fairly constant throughout the plasma and falls off sharply towards the edge. Accordingly, we parametrise $\rho$ in the form

$$\begin{aligned} \rho &= C, & 0 \leqslant x \leqslant x_0 \\ &= C\left\{1-\left(\frac{x-x_0}{1-x_0}\right)^2\right\}, & x_0 < x < 1. \end{aligned}$$

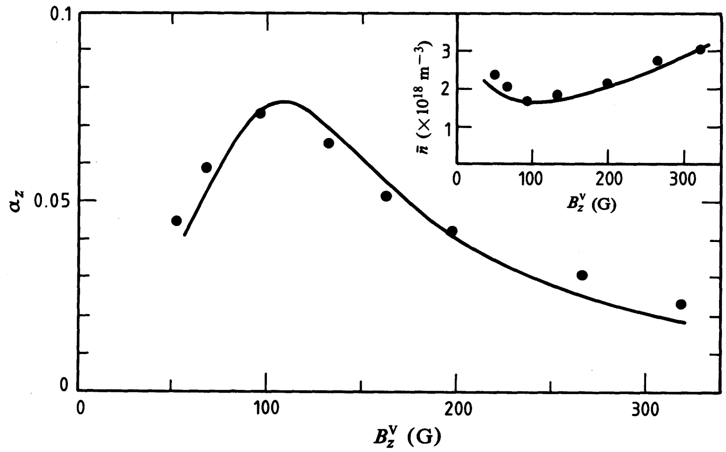


**Fig. 4.** Comparison between theoretical and experimental results. The dots represent the results of measurements by Fukada (1978). The theoretical results were obtained for $\lambda = 3 \cdot 5$ and $\gamma$ calculated from equation (18) with $\bar n$ dependent on $B_z^{\rm V}$ as indicated in the inset.

The results of the calculations with $\lambda = 3\cdot 5$ and $x_0 = 0\cdot 8$ (Fig. 4) show that, with realistic parameters, the $\langle \tilde{J} \times \tilde{B} \rangle$ theory provides a good quantitative description of the initial rise and subsequent decay of the driven current when the strength of the applied toroidal field is increased.

Theoretical comparison with measured radial profiles is more difficult owing to a number of experimental factors that are not included in the theoretical model. Toroidal effects are evident in the experimental results by their lack of symmetry. The use of discrete r.f. coils and their proximity to the plasma also make accurate modelling difficult. Nevertheless, the calculated distributions of the r.f. field strengths (Fig. 5) are in good qualitative agreement with the results of Fukada *et al.* (1976), showing the change from hollow to bell-shaped profiles when $B_z^{\rm V}$ is increased. The calculated current densities (Fig. 6) are similar to those measured by Fukada (1978). However, the measured profiles are not as sharply peaked as those calculated, perhaps because of diffusion of the current into the plasma by, for example, resistive tearing.

The dependence of the total driven current and the average electron density on the filling pressure was measured by Fukada and Matsuura (1978) at a fixed value of the toroidal field $B_z^{\rm V} = 0\cdot 0128$ T. The values of the basic parameters in this experiment were the same as those of Fukada *et al.* (1976). The quoted electron frequency, $\nu_{\rm e} = (2 \sim 7)\times 10^7\ {\rm s}^{-1}$, corresponds to $\eta \approx \frac{1}{3}\times 10^{-4}\ \Omega\,{\rm m}$ independent of $\bar n$. The value of the parameter $\lambda$ is then $\lambda \approx 12$.

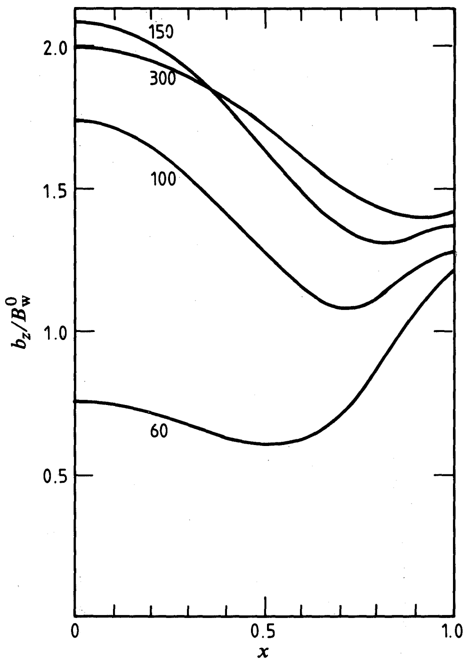


**Fig. 5.** Radial distributions of the normalised r.f. field strength at different values of the applied steady field $B_z^V = 60$, 100, 150 and 300 G.

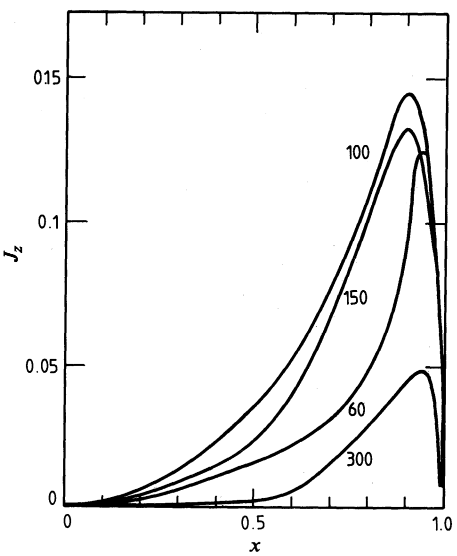


**Fig. 6.** Radial distributions of the driven axial current calculated at $B_z^V = 60$, 100, 150 and 300 G.

The measured data can be used to obtain $\alpha_z$ as a function of $\bar{n}$ as shown in Fig. 7. The theoretical results, calculated with $\lambda = 12$ and $x_0 = 0 \cdot 8$, again show that excellent quantitative agreement with experiment can be obtained from the $\langle \tilde{J} \times \tilde{B} \rangle$ current drive model with physically realistic values of the parameters.

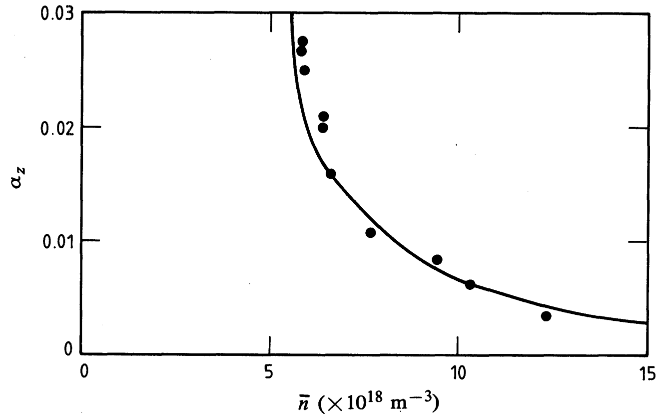


**Fig. 7.** Dependence of the total axial current on the electron number density. The dots represent values measured by Fukada and Matsuura (1978). The theoretical results were obtained using $\lambda = 12$ and $\gamma$ given by equation (18).

## 5. Conclusions

Calculations by Hugrass (1985) have shown that, although the inclusion of higher harmonics can have significant effects on the numerical results, these effects are small when either the resistive term or the Hall term in Ohm's law (1) dominates. For the two experiments we have examined, the parameter ranges were $\lambda = 3\cdot5$, $\gamma = 1\cdot2$–$2\cdot5$ and $\lambda = 12$, $\gamma = 2\cdot5$–$7\cdot0$. The very low values of $\alpha_z$ ($<0\cdot1$) obtained with these parameters are characteristic of a current drive dominated by the resistive term, so that the effect of neglecting the higher harmonics is expected to be negligible.

It has been shown that most of the experimentally observed features of $m = 0$ r.f. current drive in the presence of a toroidal field can be explained in terms of the $\langle \tilde{J} \times \tilde{B} \rangle$ current drive model. Many of the effects observed in $m = 0$ current drive have also been observed with $m \neq 0$ coil configurations (Hotta *et al.* 1985; Collins *et al.* 1988). The principal reason for the recent interest in $m \neq 0$ configurations was the anticipation of improved current drive within the plasma by avoiding the hollow current profiles associated with $m = 0$. Although the geometric effect which requires $J_z = 0$ on axis when $m = 0$ does not occur when $m \neq 0$, Bertram (1987) has shown that a strong toroidal magnetic field inhibits current drive within the plasma regardless of the values of $m$. In general, when the ratio $B_z^{\mathrm{v}}/B_\omega^0$ is very large, the $\langle \tilde{J} \times \tilde{B} \rangle$ current drive mechanism, as well as the $\langle \tilde{U} \times \tilde{B} \rangle$ current drive associated with compressional Alfvén waves (Bellan 1985), will only drive skin currents. There are mechanisms by which these currents can diffuse into the plasma (Bellan 1985), but their usefulness for driving steady currents in a large tokamak remains to be demonstrated.

## References

Bellan, P. M. (1985). *Phys. Rev. Lett.* **54**, 1381.
Bertram, W. K. (1987). *J. Plasma Phys.* **37**, 423.
Borzunov, N. A., *et al.* (1964). *Sov. Phys. Dokl.* (Engl. Trans.) **8**, 914.
Collins, G. A., Durance, G., Hogg, G. R., Tendys, J., and Watterson, P. A. (1988). *Nucl. Fusion* (in press).

Dutch, M. J., and McCarthy, A. L. (1986). *Plasma Phys. Control. Fusion* **28**, 695.
Fukada, M. (1978). *J. Phys. Soc. Jpn* **45**, 283.
Fukada, M., and Matsuura, K. (1978). *J. Phys. Soc. Jpn* **44**, 1344.
Fukada, M., Matsuura, K., Hirano, K., Mohri, A., and Fukao, M. (1976). *J. Phys. Soc. Jpn* **41**, 1376.
Hirano, K., Matsuura, K., and Mohri, A. (1971). *Phys. Lett.* A, **36**, 215.
Hotta, E., Suzuki, M., Ohta, H., and Hayashi, I. (1985). *Jap. J. Appl. Phys.* **24**, 110.
Hugrass, W. N. (1981). *Nucl. Fusion* **21**, 1326.
Hugrass, W. N. (1985). *Aust. J. Phys.* **38**, 157.
Jones, I. R. (1986). *Plasma Phys. Control. Fusion* **10**, 115.
Jones, I. R., and Hugrass, W. N. (1981). *J. Plasma Phys.* **26**, 441.
Kikunaga, T., Hotta, E., and Hayashi, I. (1982). *Jap. J. Appl. Phys.* **21**, 1657.
Klima, R. (1973). *Plasma Phys.* **15**, 1031.
Klima, R. (1974). *Czech. J. Phys.* B **24**, 846.
Thonemann, P. C., Cowhig, W. T., and Davenport, P. A. (1952). *Nature* **169**, 34.

Manuscript received 5 November 1987, accepted 2 March 1988