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# A General Physical Model for RMF Current Drive

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*Abstract*

Rotating magnetic fields (RMF) have been used successfuly to drive steady currents in plasmas in many experiments. Some recent experimental and theoretical results did not seem to agree with the standard model based on assuming that the electrons are 'tied' to the lines of the RMF. A more general model based on the concept of flux preserving motion is developed in this paper. It appears that this model provides a unified approach for predicting the qualitative features of RMF current drive under a variety of conditions.

## 1. Introduction

Current drive by means of a rotating magnetic field (RMF) was suggested and demonstrated experimentally by Blevin and Thonemann (1962). It was later developed and applied to the rotamak device by Jones and other workers [see Jones (1984) and references therein, and also Hugrass (1984), Durance and Jones (1986), Bertram (1987, 1988*a*, 1988*b*), Brotherton-Ratcliffe and Storer (1988), Collins *et al.* (1988), Donnelly *et al.* (1987), Dutch *et al.* (1986), Dutch and McCarthy (1986, 1987), Durance *et al.* (1987), Jones *et al.* (1987), Kirolous *et al.* (1988)]. The technique is based on the concept that the electrons follow the motion of an RMF of suitable magnitude and frequency while the ions do not. The differential motion of the electrons and ions produces a steady current which can be utilised to confine the plasma.

The motion of plasmas (or conducting fluids in general) is often described in terms of the simple statement that the lines of the magnetic field are 'frozen' to the plasma (Alfvén 1950). In making such a statement it is implied that the field lines may move and that they have a definite velocity. The velocity of the field lines may be defined by comparing the magnetic field at a certain instant $t_0$ to that at $t_0+\mathrm{d}t$ where $\mathrm{d}t$ is infinitesimally small. Consider a point $r_0$ on a certain field line at $t_0$. If we can determine the location of this point $r_0+\mathrm{d}r$ at $t_0+\mathrm{d}t$, the velocity of the field lines at $r_0$, $t_0$ would be

$$v_0 = \underset{\mathrm{d}t\to 0}{\mathrm{limit}} \frac{\mathrm{d}r}{\mathrm{d}t}.$$

However, the information content of Maxwell's equations, plus the conventional definition of the field lines, is not sufficient to label each individual field line. In other

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words, there is no unique prescription for determining which field line at $t_0+\mathrm{d}t$ corresponds to a certain field line at $t_0$, and hence $\mathrm{d}\boldsymbol{r}$ and $\boldsymbol{v}_0$ are not uniquely defined (Newcomb 1958).

The majority of known physical phenomena occurring in plasmas can be described satisfactorily in terms of the fluid model where the plasma is considered as two (or more) interpenetrating fluids, and some effects can be explained using a single fluid model. Often the individual fluids (the electron fluid and the ion fluids) and the equivalent single fluid move at different velocities yet all of them are assumed to be glued to the field lines. This apparent contradiction is due to the fact that the velocity of the field lines is not a well defined physical quantity as mentioned above. The plasma motion can be described more accurately in terms of the flux preserving motion (defined in Section 2). Provided that certain conditions are satisfied, the motion of the electron fluid, the ion fluid and/or the single fluid is approximately flux preserving. For a given electromagnetic field, the flux preserving motion is not unique (Newcomb 1958). A number of fluids may move at different velocities and all these different motions are flux preserving. However, it is possible to have flux preserving motion only for fields with constant magnetic helicity. In this paper we will only consider situations where the magnetic helicity is at least approximately conserved.

It follows from the above discussion that the simple explanation of the RMF current drive technique (namely that the electrons are tied to the lines of the RMF while the ions are not) is not satisfactory. In this paper we develop an alternative physical model based on the concept of flux preserving motion. The motion of the electron fluid has to be flux preserving, while the motion of the ion fluid does not have to be provided that the magnitude and frequency of the field satisfy certain conditions. Note that the flux preserving velocity is not unique. For the special case of RMF current drive a rigid rotation at the same rotational frequency as the RMF is obviously flux preserving. However, other modes of flux preserving motion are also possible. Any of these modes is 'kinematically' possible, but the actual motion of the electron fluid would be approximately given by one or a certain combination of the flux preserving modes such that energy and momentum conservation, as well as the initial and boundary conditions, are satisfied. The need for this refined understanding of the RMF current drive arose only recently when the theory for RMF current drive in the presence of a strong azimuthal magnetic field (Bertram 1987), as well as the experimental results (Collins *et al.* 1988), seemed inconsistent with the standard theory developed for zero azimuthal steady field insofar as the application of a steady azimuthal field reduced the driven current, despite improved penetration of the RMF into the plasma.

The paper is organised as follows. Line and flux preserving motions are defined in Section 2. The concept of flux preserving motion is applied to the multi-fluid model for plasmas in Section 3, to RMF current drive in Section 4 and to double helix current drive in Section 5.

## 2. Line Preserving and Flux Preserving Velocity

The concepts of line preserving velocity and flux preserving velocity were introduced by Newcomb (1958) who defined them as follows. Consider the Eulerian description of fluid motion, where the fluid velocity $\boldsymbol{v}(\boldsymbol{r},t)$ is a function of the independent variables $\boldsymbol{r}$ and $t$ and is defined in a certain domain D in $\boldsymbol{r}$. The electromagnetic fields

$\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{B}(\boldsymbol{r},t)$ are also defined in the domain D. Let the curve $C_l$ coincide with a line of $\boldsymbol{B}$ at a certain instant $t_0$ and let each point $\boldsymbol{r}$ on $C_l$ move with velocity $\boldsymbol{v}(\boldsymbol{r},t)$ for $t \geqslant t_0$. The velocity field $\boldsymbol{v}(\boldsymbol{r},t)$ is line preserving if any such curve $C_l$ continues to be a line of $\boldsymbol{B}$. Now consider a closed curve $C_f$ moving with $\boldsymbol{v}$. The velocity field is flux preserving if the magnetic flux in $C_f$ is constant for any such curve. It can be shown (Newcomb 1958) that any flux preserving velocity is line preserving as well. The converse of this statement is not true as this can be demonstrated by considering a constant straight magnetic field, $\boldsymbol{B} = B_0(x, y)\hat{z}$. Any velocity field satisfying $\partial \boldsymbol{v}/\partial z = 0$ is line preserving. However, such velocity is flux preserving only if the condition $(\boldsymbol{v}.\nabla)\boldsymbol{B}+(\nabla.\boldsymbol{v})\boldsymbol{B} = 0$ is satisfied as well. It was shown by Newcomb (1958) that one can ascribe a flux preserving velocity to the magnetic field lines without creating any contradictions with the known physical laws. Line preserving motion is not relevant to the present discussion and will not be considered any more.

The rate of change of the magnetic flux through any surface S bounded by $C_f$ is

$$\frac{d\psi}{dt} = \int_S \frac{\partial \boldsymbol{B}}{\partial t}.ds + \oint_{C_f} \boldsymbol{B}.(\boldsymbol{v}\times d\boldsymbol{l})$$
$$= -\oint_{C_f} (\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}).d\boldsymbol{l}. \tag{1}$$

It follows that $\boldsymbol{v}$ is flux preserving provided that

$$\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B} = -\nabla\phi, \tag{2}$$

where $\phi$ is any single valued scalar function. It is not always possible to find a flux preserving velocity. Consider, for simplicity, the special case where the magnetic field lines close on themselves. Equation (2) can be satisfied, if and only if

$$\oint_{C_l} \boldsymbol{E}.d\boldsymbol{l} = 0, \tag{3}$$

where $C_l$ is any line of $\boldsymbol{B}$. It follows that one cannot define a flux preserving velocity for situations where the flux linking any line of $\boldsymbol{B}$ is not constant, i.e. when the magnetic helicity (Moffatt 1978) is not conserved. This conclusion is not surprising because in this case, any attempt to describe the evolution of the magnetic field in terms of motion of the lines of force would necessarily involve the absurd notion of lines crossing each other. In what follows we will only consider situations where the magnetic helicity is at least approximately conserved and it is possible to define a flux preserving velocity.

## 3. Plasma Motion

Most of the physical phenomena occurring in plasmas can be satisfactorily described in terms of a multi-fluid model where the plasma is treated as a number of interpenetrating fluids, an electron fluid and one or more ion fluids. The equation of motion for each fluid component is

$$m_\alpha n_\alpha \frac{\partial \boldsymbol{v}_\alpha}{\partial t} + m_\alpha n_\alpha \boldsymbol{v}_\alpha . \nabla \boldsymbol{v}_\alpha = e_\alpha n_\alpha (\boldsymbol{E} + \boldsymbol{v}_\alpha \times \boldsymbol{B})$$

$$-\nabla . P_\alpha - m_\alpha n_\alpha \sum_{\beta \neq \alpha} \nu_{\alpha\beta} (\boldsymbol{v}_\alpha - \boldsymbol{v}_\beta), \qquad (4)$$

where $m_\alpha$ and $e_\alpha$ are the mass and charge of particle species $\alpha$, $n_\alpha$ is the number density of this species, $\boldsymbol{v}_\alpha$ its macroscopic velocity, $\boldsymbol{P}_\alpha$ its kinetic stress tensor and $\nu_{\alpha\beta}$ the momentum transfer collision frequency between particle species $\alpha$ and $\beta$. Equation (4) can be rearranged in the form

$$\boldsymbol{E} + \boldsymbol{v}_\alpha \times \boldsymbol{B} = \boldsymbol{R}_\alpha , \qquad (5)$$

$$\boldsymbol{R}_\alpha = \boldsymbol{R}_{\alpha \mathrm{i}} + \boldsymbol{R}_{\alpha \mathrm{c}} + \boldsymbol{R}_{\alpha \mathrm{k}} , \qquad (6)$$

where $\boldsymbol{R}_\alpha$ consists of an inertial part

$$\boldsymbol{R}_{\alpha \mathrm{i}} = \frac{m_\alpha}{e_\alpha} \left( \frac{\partial \boldsymbol{v}_\alpha}{\partial t} + \boldsymbol{v}_\alpha . \nabla \boldsymbol{v}_\alpha \right), \qquad (7)$$

a collisional part

$$\boldsymbol{R}_{\alpha \mathrm{c}} = \frac{m_\alpha}{e_\alpha} \sum_{\beta \neq \alpha} \nu_{\alpha\beta} (\boldsymbol{v}_\alpha - \boldsymbol{v}_\beta), \qquad (8)$$

and a kinetic part

$$\boldsymbol{R}_{\alpha \mathrm{k}} = \frac{1}{e_\alpha n_\alpha} \nabla . P_\alpha . \qquad (9)$$

Comparing equations (2) and (5) it is seen that the motion of the fluid is flux preserving provided that

$$\nabla \times \boldsymbol{R}_\alpha = 0 . \qquad (10)$$

Non-flux-preserving motion can occur because of inertial, collisional or kinetic effects. The kinetic effects are best understood in the special case where the velocity distribution is isotropic when observed from the frame of reference moving with $\boldsymbol{v}_\alpha$. The kinetic stress tensor can be expressed, in this case, in terms of a scalar pressure, $p_\alpha = n_\alpha T_\alpha$, where $T_\alpha$ is the temperature (in energy units). It follows that

$$\mathscr{E}_{\mathrm{t}} = \nabla \times \boldsymbol{R}_{\alpha \mathrm{k}} = \frac{1}{e_\alpha n_\alpha} \nabla T_\alpha \times \nabla n_\alpha , \qquad (11)$$

and the fluid motion would not be flux preserving if the temperature and density gradients are not parallel. Flux variation is brought about by the thermal electromotive force $\mathscr{E}_{\mathrm{t}}$. This effect may be important in laser produced plasmas where large magnetic fields can be self-generated as a result of the grossly non-equilibrium distributions established by the intense laser heating. Transport processes normally maintain gas discharge plasmas very near to a state of local thermodynamic equilibrium, and the thermal electromotive force is therefore relatively unimportant and will not be considered in the rest of this paper.

The collisional and inertial effects are more difficult to analyse because both $\boldsymbol{R}_{ac}$ and $\boldsymbol{R}_{ai}$ depend on the fluid velocity $\boldsymbol{v}_a$. Roughly speaking, the motion would be approximately flux preserving provided that both $R_{ac}$ and $R_{ai}$ are much smaller than the magnitude of either of the two terms on the left-hand side of equation (5), i.e. provided that

$$\frac{\nu_a\, m_a}{e_a\, B} \sim \frac{\nu_a}{\omega_{ca}} \ll 1 \qquad \text{and} \qquad \frac{\omega\, m_a}{e_a\, B} \sim \frac{\omega}{\omega_{ca}} \ll 1\,, \tag{12, 13}$$

where $\nu_a$ is the effective momentum transfer collision frequency for species $a$, $\omega$ is the characteristic frequency of the motion under consideration and $\omega_{ca}$ is the cyclotron frequency. It is important to emphasise the approximate and qualitative nature of the conditions (12) and (13). These two conditions were obtained by comparing the magnitudes of various terms in a vector equation. In particular, the collisional term and the $\boldsymbol{v}\times\boldsymbol{B}$ term can be very nearly at perpendicular directions and it is not strictly correct to compare their magnitudes. Also, one should compare the curl of these terms rather than their magnitudes. Flux preserving motion can occur in situations where the collisional and/or the inertial terms are not small but their curl is very small. Therefore, conditions (12) and (13) are sufficient but not necessary for approximate flux preservation.

As mentioned earlier, the flux preserving velocity is in general not unique. All flux preserving velocities are for the purpose of this discussion 'kinematically' possible, i.e. are consistent with the approximate 'constraint'

$$\nabla\times(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}) = 0\,.$$

A unique velocity can be determined only if we consider all the forces and the initial conditions in a particular problem. This will become clear when we consider the rotating magnetic field current drive in the following section.

## 4. Rotating Magnetic Field Current Drive

### *(a) General Remarks*

We consider the motion of the electron and ion fluids in a magnetic field consisting of a constant azimuthally symmetric magnetic field $\boldsymbol{B}_0$ and a rotating magnetic field $\boldsymbol{B}_{\mathrm{R}}$. The constant magnetic field can be written in the form

$$\boldsymbol{B}_0 = \frac{1}{r}\nabla\psi\times\hat{\boldsymbol{\theta}} + B_{\mathrm{T}}\,\hat{\boldsymbol{\theta}}\,, \tag{14}$$

where the poloidal flux $\psi$ and the toroidal field $B_{\mathrm{t}}$ are functions of $r$ and $z$ only, and $r$, $\theta$ and $z$ are the standard cylindrical coordinates. Note that for azimuthally symmetric fields the toroidal component is purely in the azimuthal ($\theta$) direction and hence the terms toroidal and azimuthal will be used interchangeably. The poloidal direction, in this case, lies in the $r$–$z$ plane.

The rotating magnetic field has the general form

$$\boldsymbol{B}_{\mathrm{R}} = \boldsymbol{B}_{\omega}\, \mathrm{e}^{\mathrm{i}(\omega t - m\theta)}, \tag{15}$$

where $\boldsymbol{B}_{\omega}$ is in general complex, $\omega$ is the angular frequency of the field, $m$ is the azimuthal mode number and we use the standard complex phasor notation. Note that the angular rotational frequency of this field is $\omega/m$. One can also associate a phase velocity with this field,

$$\boldsymbol{v}_{\mathrm{ph}} = (\omega/m) r\hat{\boldsymbol{\theta}}. \tag{16}$$

The electric field associated with the RMF is given by

$$\boldsymbol{E} = -\nabla\phi_1 - \boldsymbol{v}_{\mathrm{ph}}\times\boldsymbol{B}_{\mathrm{R}}, \tag{17}$$

where $\phi_1$ is an arbitrary scalar function. It is noted that

$$\nabla\times(\boldsymbol{v}_{\mathrm{ph}}\times\boldsymbol{B}_0) = 0 \tag{18}$$

for the given steady field. Hence, we have

$$\boldsymbol{E} + \boldsymbol{v}_{\mathrm{ph}}\times\boldsymbol{B} = -\nabla\phi, \tag{19}$$

where $\phi$ is an arbitrary scalar function. It follows that a rigid rotational motion at angular frequency $\omega/m$ is flux preserving for any field $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_{\mathrm{R}}$, where $\boldsymbol{B}_0$ and $\boldsymbol{B}_{\mathrm{R}}$ are given by (14) and (15). This result is self-evident and could have been stated without formal proof. There may be other flux preserving motions which are not as obvious. In the limit $B_{\omega} \ll B_0$, any motion corresponding to one of the low frequency cold plasma waves is approximately flux preserving (because it satisfies the condition $\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}_0 = 0$, where $\boldsymbol{v}$ is either the velocity of the electron fluid or the velocity of the equivalent single fluid). Such motion is oscillatory and does not produce a steady current.

All the flux preserving motions, namely rigid rotation at $\omega/m$ and the oscillatory wave motion, are kinematically possible. A combination of rigid rotation at angular speed other than $\omega/m$ and certain wave motions is also possible. The actual motion in any particular case is uniquely determined by the initial and boundary condition as well as other effects such as collisional effects and particle recycling.

### *(b) Rotational Motion in the Absence of the RMF*

We consider the rotational motion of the electron and ion fluids in a steady azimuthally symmetric magnetic field (equation 14). The rotational velocity of a certain fluid can be written as

$$\boldsymbol{v} = r\Omega\hat{\boldsymbol{\theta}}, \tag{20}$$

where $\Omega(r, z)$ is the angular velocity. The condition for flux preservation is obtained using (2), (14) and (20):

$$\nabla\Omega\times\nabla\psi = 0. \tag{21}$$

The rotational motion is flux preserving provided that $\nabla\Omega$ is parallel to $\nabla\psi$, i.e. the angular velocity is constant on the poloidal flux surfaces. This is the isorotation condition derived by Ferraro (1937). Note that the isorotation condition should be approximately satisfied by both the electron and ion fluids. Hence the toroidal current density should be approximately given by $J_T \sim ern[\Omega_i(\psi)-\Omega_e(\psi)]$. The actual motion deviates slightly from the ideal motion described above. A discussion of the general case is beyond the scope of this paper. We consider briefly the special case where the magnetic field is purely poloidal to the first approximation. This equilibrium is known as the field reversed configuration (FRC) (Tuszewski 1988). Both the electron and ion fluids in an FRC rotate toroidally. The actual motion deviates from ideal isorotation in a number of ways. Ambipolar diffusion, whereby both the electron and ion fluids move outwards (at the same velocity) across the flux surfaces, arises from momentum transfer collisions between the electron and ion fluids. The diffusion velocity in large systems is much smaller than the rotational velocity of the electron fluid. The motion associated with the ambipolar diffusion is obviously not flux preserving. In the absence of a toroidal magnetic field (and the associated poloidal current), the toroidal rotational velocity is determined by the balance of four forces: the force associated with the toroidal electric field arising from the decay of the poloidal flux, the $\boldsymbol{v}\times\boldsymbol{B}$ force caused by the ambipolar diffusion across the poloidal flux, the collisional friction force between the electron and ion fluids and the viscous force arising from velocity shear. The toroidal velocity does not necessarily satisfy the isorotation condition. If the condition is not satisfied, an antisymmetric (with respect to the equatorial plane) toroidal field is generated spontaneously. This field exerts toroidal torque and transfers angular momentum between different parts of the plasma. This can be seen by considering a horizontal plane $z = c$ dividing the configuration into two parts. The magnetic torque exerted by one part on the other is given by

$$T_z = \int_0^R 2\pi r^2 \frac{B_z B_\theta}{\mu_0}\, \mathrm{d}r,$$

where $R$ is the radius of the configuration at $z = c$. It is clear this magnetic torque vanishes for a configuration with purely poloidal magnetic field ($B_T = B_\theta = 0$). For small values of the self-generated toroidal magnetic field, the $z$ component of the magnetic field is unperturbed (to first approximation) and hence the torque is approximately proportional to a properly weighted average of the self-generated toroidal field. The magnitude of the self-generated toroidal field continues to increase until the transfer of angular momentum is sufficient to establish approximate isorotation. Note that exact isorotation (and hence flux preservation) cannot be established because the self-generated toroidal field is maintained by the equivalent emf arising from the slight deviation from isorotation.

### (c) RMF Current Drive in Cylindrical Plasmas with Zero Toroidal Field

We first consider the special case where the constant field is purely axial. Referring to equation (14) we have $B_T = 0$ and $\phi$ is a function of $r$ only, and hence $(1/r)\nabla\psi\times\hat{\theta} = B_0(r)\hat{z}$. Assuming that both the plasma cylinder and the RMF coils are infinitely long, it follows that all quantities are independent of $z$. Two classes of low frequency waves can propagate in such plasmas: (1) waves with $k \neq 0$ ($k$ is the $z$ component of the wavevector) such as the whistler waves and (2) waves with $B_z \neq 0$

(compressional Alfvén waves). Neither of these two classes of low frequency waves is excited by the RMF in this particular geometry since $\boldsymbol{B}_{\mathrm{R}} \cdot \hat{z} = 0$ and $(\partial/\partial z)\boldsymbol{B}_{\mathrm{R}} = 0$. Hence rigid rotation at an angular velocity $\omega/m$ is the only flux preserving motion (rigid translation in the $z$ direction is also flux preserving, but this motion is not relevant for our purpose). For RMF current drive, the magnitude and frequency of the rotating field are chosen such that conditions (12) and (13) are satisfied for the electrons and are not satisfied for the ions, namely

$$\omega/\omega_{\mathrm{ce}} \ll 1, \quad \nu_{\mathrm{ei}}/\omega_{\mathrm{ce}} \ll 1, \quad \omega/\omega_{\mathrm{ci}} \gg 1, \qquad (22,23,24)$$

where $\omega_{\mathrm{ce}} = eB_{\omega}/m_{\mathrm{e}}$ and $\omega_{\mathrm{ci}} = eB_{\omega}/m_{\mathrm{i}}$. Note that we use the magnitude of the rotating field (and not the steady field nor the total field) to calculate the cyclotron frequencies $\omega_{\mathrm{ce}}$ and $\omega_{\mathrm{ci}}$, because the analysis of the equations describing RMF current drive for this particular geometry (Jones and Hugrass 1981) showed that the cyclotron frequencies in (22) and (23) should be calculated using the magnitude of the rotating field. However, this is not true in general as will be seen from our discussion of the RMF current drive in different geometries.

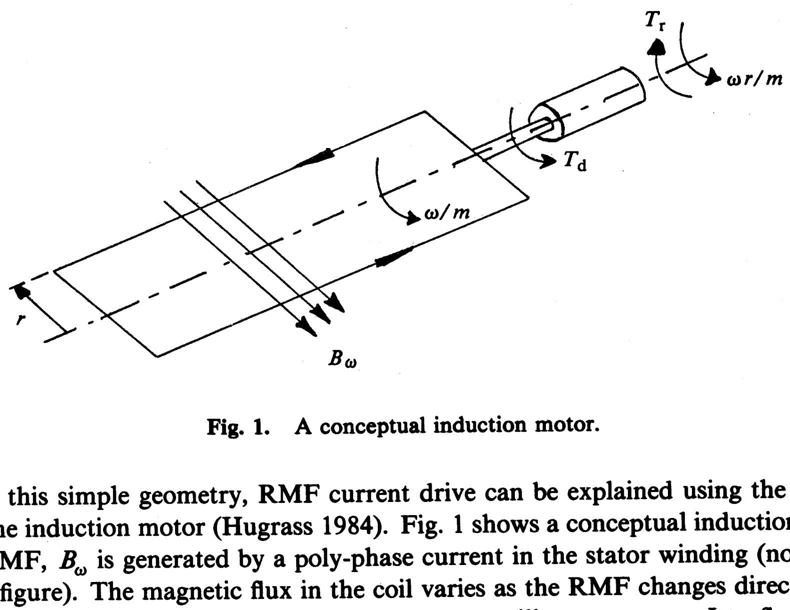


**Fig. 1.** A conceptual induction motor.

For this simple geometry, RMF current drive can be explained using the analogy with the induction motor (Hugrass 1984). Fig. 1 shows a conceptual induction motor. The RMF, $B_{\omega}$ is generated by a poly-phase current in the stator winding (not shown in the figure). The magnetic flux in the coil varies as the RMF changes direction and this induces an oscillatory emf which causes an oscillatory current $I$ to flow in the coil. The magnetic forces acting on the sides of the coil produce, on the average, a unidirectional torque $T_{\mathrm{d}}$ in the same direction as $\omega$. The coil rotates at an angular frequency $\omega_{\mathrm{r}}/m$, where $\omega_{\mathrm{r}} < \omega$, such that the driving torque equals the retarding torque $T_{\mathrm{r}}$ applied by the mechanical load. The analogy between the induction motor and the RMF current drive is described as follows (see Hugrass 1984 for details). The coils generating the RMF represent the stator windings of the induction motor and the electron fluid represents the rotor which has a rotational angular velocity $\omega_{\mathrm{r}}/m$. The slip $s$ is defined by the equation

$$s = (\omega - \omega_{\mathrm{r}})/\omega. \qquad (25)$$

In a frame of reference rotating with the electron fluid, the Doppler shifted frequency of the RMF is $\omega_d = \omega - \omega_r = s\omega$.

The induced emf is therefore proportional to the slip and oscillates at angular frequency $\omega_d$. Oscillatory current (at angular frequency $\omega_d$) flows in the axial direction in response to the emf. The current depends on the emf and the equivalent inductance and resistance of the rotor (electron fluid). This current known as 'screening current' tends to attenuate the RMF in the plasma according to the classical skin effect (with effective frequency $\omega_d$). Note, however, that the frequency of the screening current as observed in the laboratory frame is $\omega$. The presence of the screening current leads to a steady flow of energy from the source of the RMF to the electron fluid,

$$\langle P \rangle = \tfrac{1}{2} \int \mathrm{Re}\{ J_z^* E_z \} \, d\tau \,, \tag{26}$$

where $\langle P \rangle$ is the time averaged power, the asterisk denotes complex conjugate and the integration is carried out over the volume of the plasma. This power flow is accompanied by a steady flow of angular momentum (Klima 1974),

$$\langle T \rangle = (m/\omega) \langle P \rangle \,, \tag{27}$$

where $\langle T \rangle$ is the time averaged torque. It is this torque that maintains the rotation of the electron fluid against the frictional retarding torque caused by momentum transfer collisions with the ion fluid (the ion fluid resembles the mechanical load of a motor). The retarding torque is approximately proportional to $\omega_r$ (i.e. to $1-s$), while the driving torque is proportional to $B_\omega^2$ and depends on $s$ in a manner similar to the schematic shown in Fig. 2. The driving torque and the retarding torque are equal when the electron fluid rotates at $\omega_r/m$. For zero slip, the induced emf in the electron fluid vanishes and hence no screening current flows and the RMF completely

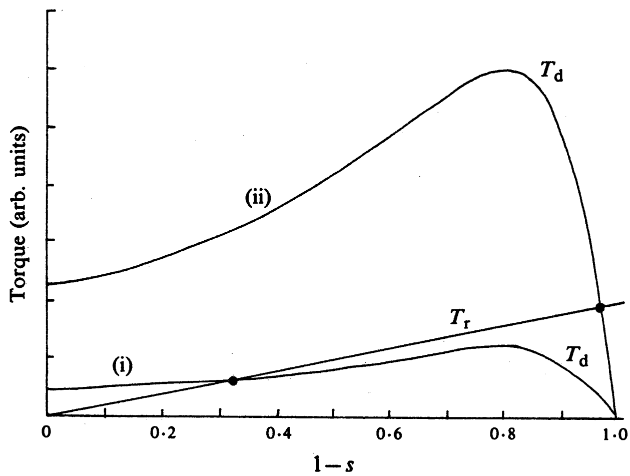


**Fig. 2.** A schematic of the driving torque $T_d$ ($T_{d(i)}$ for large $B_\omega$ and $T_{d(ii)}$ for small $B_\omega$) and the retarding torque $T_r$ as functions of the slip $s$.

penetrates the plasma. The motion of the electron fluid is exactly flux preserving in this case. However, this motion is not possible because of the finite retarding torque applied by the ion fluid via momentum transfer collisions. Thus the slip between the electron fluid and the RMF cannot be exactly zero. The actual value of the slip depends on the magnitude of the rotating field as well as the collision frequency and the plasma radius. Fig. 2 shows a schematic of the retarding torque $T_r$ as a function of the slip ($T_r \sim 1-s$), and the driving torque $T_d$ for (i) small RMF and (ii) large RMF. It is seen that the slip can be very small and hence the motion of the electron fluid is approximately flux preserving provided that the magnitude of the RMF is larger than a certain threshold. The screening current is small in this case and the RMF penetrates the plasma. For small values of the RMF, the slip is large and hence the screening currents are large and the RMF is 'screened' from the interior of the plasma. For very small values of the RMF the electron fluid is almost stationary ($s \sim 1$) and the RMF is confined to a skin layer on the surface of the plasma (the skin effect).

We now use the results of this simple model to make a better estimate of the condition for approximate flux preservation for the electron fluid motion (and hence efficient RMF current drive). Neglecting the motion of the ion fluid, the $z$ component of the equation of motion is

$$(\omega/m)rB_\omega-(1-s)(\omega/m)rB_\omega = \frac{m_e}{e}(\nu-\mathrm{i}\,\omega)v_z, \tag{28}$$

where we have used $E_z = (\omega/m)rB_\omega$, $B_r = B_\omega$ and $v_\theta = (1-s)(\omega/m)r$. It follows that

$$v_z = s(\omega/m)r\frac{\omega_{ce}}{\nu-\mathrm{i}\,\omega}, \tag{29}$$

where $\omega_{ce} = eB_\omega/m_e$, and so

$$\langle v_z B_r\rangle = \tfrac{1}{2}s(\omega/m)r\frac{\nu\omega_{ce}}{\nu^2+\omega^2}B_\omega. \tag{30}$$

Using the $\theta$ component of the equation of motion we obtain

$$\langle v_z B_r\rangle = \frac{\omega m}{e}(1-s)(\omega/m). \tag{31}$$

Solving (30) and (31) for $s$ we obtain

$$s = \frac{\nu^2+\omega^2}{\nu^2+\omega^2+\frac{1}{2}\omega_{ce}^2}. \tag{32}$$

It is seen from (32) that the slip $s$ is much smaller than unity and hence the motion is flux preserving, provided that $(\omega^2+\nu^2) \ll \omega_{ce}^2$, where $\omega_{ce}$ is the electron cyclotron frequency calculated using the magnitude of the rotating field.

We now summarise the properties of RMF current drive in an infinitely long plasma cylinder with zero toroidal (azimuthal) steady magnetic field:

(1) The only flux preserving motion is rigid rotation about the axis at $\omega/m$.

(2) The electron fluid is approximately flux preserving provided that $\omega^2+\nu^2 \ll \omega_{ce}^2$ $[=(eB_\omega/m_e)^2]$.

(3) The RMF penetrates the plasma when the motion of the fluid is approximately flux preserving and is confined to a skin layer if the motion is not flux preserving.

(4) A steady azimuthal current is driven as a result of the differential motion between the electron and ion fluids. Larger values of the steady current correspond to smaller values of the slip and hence improved penetration of the RMF into the plasma.

*(d) RMF Current Drive in Cylindrical Plasma with Toroidal (Azimuthal) Field*

The azimuthal field can be generated by means of a thin current carrying conductor along the $z$-axis. We will only consider the special case where the toroidal field $B_T$ is much larger than the rotating magnetic field $B_\omega$. It is clear that a rigid rotational motion at $\omega/m$ is exactly flux preserving. However, for this special case, approximate flux preservation can be achieved by oscillatory motion corresponding to plasma waves related to the whistler mode. This situation can be explained using the induction motor model. If a steady azimuthal field $B_T$ is superimposed on the conceptual induction motor shown in Fig. 1, the coil will be subject to an oscillatory radial force ($2LIB_T$ where $L$ is the length of the coil). Provided that the mechanical structure allows for radial shifts and the mass of the coil is very small, the radial velocity will be such that

$$v_r B_T \sim -E_z, \tag{33}$$

and the current in the coil will be negligibly small,

$$I \sim I_0(E_z + v_r B_T)/E_z, \tag{34}$$

where $I_0$ is the current that would flow if the coil was not allowed to execute the radial oscillatory motion. Note that, the coil motion described above is approximately flux preserving since the magnetic flux linking the coil is approximately constant. The other mode of flux preserving motion, namely rotation at $\omega/m$ is inhibited because the driving torque $T_d \sim 2LrIB_\omega$ is much smaller than the starting torque for $B_T = 0$ (or if the oscillatory radial motion is not allowed).

Now we consider RMF current drive in a cylindrical plasma. The axial screening current is obtained using the $z$ component of the electron fluid equation of motion:

$$\frac{(\nu_{ei}+i\,\omega)m_e}{ne^2} J_z = E_z + v_r B_T. \tag{35}$$

Far from the plasma surface, the radial velocity is approximately given by

$$nm_e(i\,\omega+\nu_{ei})v_r \sim J_z B_T. \tag{36}$$

Using (35) and (36) we obtain

$$v_r B_T \sim -\frac{E_z}{1+(\nu_{ei}+i\,\omega)^2/\Omega_{ce}^2}, \tag{37}$$

produce beneficial effects such as reducing the current density and power dissipation at the plasma edge.

## 5. Double Helix Current Drive

We consider an infinitely long cylindrical plasma. The applied magnetic field consists of a steady azimuthally symmetric part,

$$\boldsymbol{B}_0 = B_\theta(r)\hat{\boldsymbol{\theta}} + B_{\mathrm{a}}(t)\hat{\boldsymbol{z}}, \tag{44}$$

and a radio frequency travelling wave part,

$$\boldsymbol{B}_{\mathrm{R}} = \boldsymbol{B}_\omega \, \mathrm{e}^{\mathrm{i}(\omega t - m\theta - kz)}, \tag{45}$$

where we use the standard phasor notation. The experimental arrangement corresponding to this idealised model was described by Dutch and McCarthy (1986). In the absence of the travelling wave field, any rigid rotation about the $z$-axis, rigid translation along the $z$-axis, or a combination thereof, is flux preserving. In the presence of the travelling wave field, there is a single parameter family of flux preserving motions:

$$\boldsymbol{v} = \alpha(\omega/m)r\hat{\boldsymbol{\theta}} + (1-\alpha)(\omega/k)\hat{\boldsymbol{z}}, \tag{46}$$

where $\alpha$ is an arbitrary constant. (Note that for the standard RMF current drive $k = 0$ and $\alpha = 1$, and the velocity in the $z$ direction is in this case arbitrary.) The flow described by (46) is shearless; hence, any closed curve moving with $\boldsymbol{v}$ maintains its shape and orientation with respect to the unit vectors of the cylindrical coordinates: $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$ and $\hat{\boldsymbol{z}}$. It is also observed that the phase of the RMF is constant at any point moving with this flow velocity. It follows that the flux in any closed curve moving with the flow is constant and (46) indeed describes flux preserving motions if the magnetic field is given by (45). If the RMF excites low frequency waves in the equilibrium plasma, the oscillatory motion associated with these waves is approximately flux preserving. Thus equation (46) represents only a subset of the set of all flux preserving motions for this particular geometry. It is also obvious that the velocity given by (46) is flux preserving in the presence of both the steady and travelling wave fields. Such motion is 'kinematically' possible for any value of $\alpha$.

It was shown by Klima (1973, 1974) that, for magnetic fields given by (44) and (45), the power dissipation (per unit length) $P$ is related to the $z$ component of the force acting on the plasma (per unit length) $F_z$, and the $z$ component of the torque (per unit length) $T_z$ by the equations

$$P = (\omega/k)F_z, \qquad P = (\omega/m)T_z. \tag{47, 48}$$

Provided that the frequency of the RMF is sufficiently high that the oscillatory ion motion can be neglected, $F_z$ and $T_z$ act predominantly on the electron fluid and make it move in the $z$ direction and rotate about the $z$ axis. Again the motion of the electron fluid has to be approximately flux preserving, hence the velocity of the electron fluid is given by (46). The value of $\alpha$ is determined by the momentum relaxation mechanism. If we assume that the number density $n$ is uniform and that the electron–ion momentum transfer collision frequency $\nu_{ei}$ is constant, we obtain

$$F_z = \pi r_0^2 \, n m_e \, \nu_{ei}(1-\alpha)(\omega/k), \tag{49}$$

$$T_z = \tfrac{1}{2}\pi r_0^4 \, n m_e \, \nu_{ei} \, \alpha\omega/m, \tag{50}$$

where $r_0$ is the plasma radius. Using equations (47)–(50) we obtain

$$\begin{aligned} v = &\{ m^2/(m^2+0\cdot 5k^2 r_0^2)\}(\omega/m)r\hat{\theta} \\ &+\{0\cdot 5k^2 r_0^2/(m^2+0\cdot 5k^2 r_0^2)\}(\omega/k)\hat{z}. \end{aligned} \tag{51}$$

Equation (51) is similar to the results obtained by Dutch *et al.* (1986) for $m = 1$. However, this mode of electron motion, and the accompanying steady current, occur only in the special case where the screening current is exactly parallel to the steady magnetic field. Otherwise an oscillatory, flux preserving, wave motion will be excited (Bertram 1988*a*). As shown above, the presence of such oscillatory wave motions can lead to a substantial reduction in power dissipation, and hence to an equivalent reduction in the transfer of momentum to the electron fluid.

## 6. Conclusions

We have applied the concept of flux preserving motion to the theory of RMF current drive. It appears that this approach provides a unified physical picture for the RMF current drive. The experimental results obtained so far for different configurations as well as the various theoretical studies appear to agree with the predictions made using this simple model. It should be emphasised, however, that our physical model is purely qualitative. It is hoped that quantitative theoretical studies of various aspects of the RMF current drive, and continued experimental investigations will improve our understanding and support, refine or refute the model developed in this paper.

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