Static and Dynamic Moments of the $^7$Li Nucleus

F. C. Barker$^A$, Y. Konno$^A$ and R. H. Spear$^B$

$^A$ Department of Theoretical Physics, Research School of Physical Sciences, Australian National University, G.P.O. Box 4, Canberra, A.C.T. 2601, Australia. 
$^B$ Department of Nuclear Physics, Research School of Physical Sciences, Australian National University, G.P.O. Box 4, Canberra, A.C.T. 2601, Australia.

Abstract

The data of Weller et al. (1985) on the tensor analysing powers for elastic and inelastic Coulomb scattering of aligned $^7$Li ions have been reanalysed in order to obtain information on the values of the four $^7$Li moments $Q$, $B(E2)_l$, $\tau_{11}$ and $\tau_{12}$. It is shown that a single set of values, chosen primarily to be consistent with the value of $Q$ measured by molecular techniques and the values of $B(E2)_l$ and $\tau_{12}$ required to fit unpolarised $^7$Li data, and also with the theoretical constraint $\tau_{11} = -|\tau_{12}|$, gives a good fit to the aligned $^7$Li data.

1. Introduction

In recent years, measured values of the quadrupole moment ($Q$) of the $\frac{3}{2}^-$ ground state of $^7$Li and the reduced E2 transition probability ($B(E2)_l$) to the $\frac{1}{2}^-$ first excited state have been used to test models describing the low-lying states of $^7$Li and $^7$Be, or at least to select parameter values in the models (Mertelmeier and Hofmann 1986; Kajino 1986; Altmeyer et al. 1988; Buck and Merchant 1988). Such models have been used in the calculation of astrophysical $S$-factors for radiative-capture reactions involving these nuclei. Also Kajino et al. (1988) suggested a direct relationship between the $S$-factor for the reaction $^4$He($t,y$)$^7$Li and the tensor moments ($\tau_{11}$ and $\tau_{12}$) of the nuclear polarisability involving the $^7$Li ground state and first excited state.

A significant discrepancy exists between recent values of $Q$ measured by nuclear and molecular means. From fits to their measurements on Coulomb scattering of aligned $^7$Li ions, Weller et al. (1985) obtained $Q = -3 \cdot 70 \pm 0.08$ efm$^2$, while analysis of data on the molecule LiH gave $Q = -4.06$ efm$^2$ (Sundholm et al. 1984) and on LiF gave $Q = -4.055 \pm 0.080$ efm$^2$ (Diercksen et al. 1988). There are similar but less striking discrepancies between the values obtained by different methods for $B(E2)_l$ and $\tau_{12}$. Weller et al. obtained $B(E2)_l = 8.3 \pm 0.5$ e$^2$ fm$^4$ and $\tau_{12} = 0.23 \pm 0.06$ fm$^3$, while analysis of Coulomb excitation data obtained with unpolarised $^7$Li gave $B(E2)_l = 7.59 \pm 0.10$ e$^2$ fm$^4$ and $\tau_{12} = 0.16 \pm 0.01$ fm$^3$ (Vermeer et al. 1989). Weller et al. also gave $\tau_{11} = 0.23 \pm 0.06$ fm$^3$; this quantity is not obtainable from unpolarised $^7$Li data.
Kajino et al. (1988) criticised the values obtained by Weller et al. on the grounds that model calculations require $\tau_{11}$ to be negative, whereas Weller et al. obtained a positive value.† Calculations based both on an $\alpha$+t cluster model (Mertelmeier and Hofmann 1986; Kajino et al. 1988) and on the shell model (Barker and Woods 1989) predict $\tau_{11} \approx -|\tau_{12}|$. The uncertainty given by Weller et al. would exclude any negative value of $\tau_{11}$. Thus there are problems with the values and uncertainties of the moments given by Weller et al. in comparison both with other experimental values and with model expectations.

Kajino et al. (1988) reanalysed the data of Weller et al. (1985), assuming that the parameters satisfied relations suggested by model calculations‡:

$$B(E2)l = \frac{25}{16\pi}Q^2(1 + \delta), \quad \delta = 0.085,$$

$$\tau_{12} = \tau_{11}(1 + \eta), \quad \eta = 0.073. \quad (1)$$

They also fixed $Q$ at its molecular value, $Q = -4.06 \text{ efm}^2$, giving $B(E2)l = 8.90 \text{ e}^2 \text{ fm}^4$, so that only one adjustable parameter remained. Then their best fit was obtained with $\tau_{11} = -0.269 \text{ fm}^3$ and $\tau_{12} = -0.289 \text{ fm}^3$. All four of these values differ considerably from those of Weller et al. (1985). Also the values of $B(E2)l$ and $\tau_{12}$ are inconsistent with the values obtained from unpolarised $^7\text{Li}$ data (Vermeer et al. 1989). The value of $\delta$ in equation (1a) was obtained from $\alpha$+t cluster calculations (Kajino 1986); however, as discussed in Vermeer et al. (1989), there is a wide spread of $\delta$ values obtained from other calculations, ranging from $-0.092$ to $0.29$. Similarly, shell model calculations (Barker and Woods 1989) gave $\eta = -0.08$. Thus, use of the relations and values (1) is open to question.

Since neither of the previous analyses of the aligned $^7\text{Li}$ data of Weller et al. (1985), by Weller et al. and by Kajino et al. (1988), seems to be entirely satisfactory, we here reanalyse the same data, to see if a fit is possible that is consistent with results from other data and with model expectations.

### 2. Fitting Procedure

The data of Weller et al. (1985) consist of angular distributions of the tensor analysing power for scattering of aligned $^7\text{Li}$ ions by $^{58}\text{Ni}$ at four sub-Coulomb energies and by $^{120}\text{Sn}$ at three sub-Coulomb energies, both for elastic scattering, $T_{20}^\text{el}$, and for elastic plus inelastic scattering, $T_{20}^\text{el+in}$.

As in Weller et al. (1985) and Kajino et al. (1988), we use the program ECIS79 (Raynal 1972) to calculate the Coulomb scattering of the aligned $^7\text{Li}$, including only the ground and first excited states, for given values of the four parameters $Q$, $B(E2)l$, $\tau_{11}$ and $\tau_{12}$. Calculated values of $T_{20}^\text{el}$ and $T_{20}^\text{el+in}$ depend on the tensor part of the polarisation potential, which in the adiabatic approximation

† In Egelhof et al. (1987), it is said that Weller et al.’s best fit gives $\tau_{11} = -0.23 \text{ fm}^3$. Our calculations, however, indicate that Weller et al. actually used $\tau_{11} = +0.23 \text{ fm}^3$, so that we agree with the criticism by Kajino et al.

‡ Although Kajino et al. (1988) implied that $\tau_{12}$ must be negative, the sign of $\tau_{12}$ is actually a matter of convention, as was discussed in Barker and Woods (1989). Kajino et al. used the minus sign in the expression (4b) below, consistent with their negative value of $\tau_{12}$. 

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has the radial dependence [Alder and Winther 1975; see equation (1) in Kajino et al. 1988]

\[
\frac{4\pi Z_t e}{5} \frac{1}{r^3} \langle I_i \parallel M(\text{E2}) \parallel I_f \rangle - \left( \frac{9\pi}{5} \right)^{\frac{1}{2}} \frac{Z_t^2 e^2}{r^4} \tau_{if}, \tag{2}
\]

where

\[
\tau_{if} = \frac{3}{5} \pi^{\frac{1}{2}} \sum_n W(111 I_i, 2 I_n) \langle I_i \parallel M(\text{E1}) \parallel I_n \rangle \langle I_n \parallel M(\text{E1}) \parallel I_f \rangle/(E_n - E_i). \tag{3}
\]

Since \( Q = \frac{2}{3} \sqrt{\pi} \left( \frac{3}{2} \parallel M(\text{E2}) \parallel \frac{3}{2} \right) \) and \( B(\text{E2}) \parallel = \frac{1}{4} \left( \frac{3}{2} \parallel M(\text{E2}) \parallel \frac{1}{2} \right)^2 \), the expression (2) becomes

\[
2\sqrt{\pi} \frac{Z_t e}{r^3} \left( Q - \frac{3}{2\sqrt{5}} \frac{Z_t e}{r} \tau_{11} \right) \tag{4a}
\]

for the ground state diagonal matrix element, and

\[
\frac{8\pi Z_t e}{5} \frac{1}{r^3} \left( \pm B(\text{E2}) \parallel - \frac{3}{8} \frac{5}{\pi} \frac{Z_t e}{r} \tau_{12} \right) \tag{4b}
\]

for the transition matrix element coupling the ground state and the first excited state. The \( \pm \) ambiguity in expression (4b) originates from the sign of the off-diagonal matrix element \( \langle \frac{3}{2} \parallel M(\text{E2}) \parallel \frac{1}{2} \rangle \), and this depends on the choice of the relative sign of the ground state and first excited state wave functions; we assume the plus sign in (4b), which is the convention adopted in Barker and Woods (1989) but opposite to that of Kajino et al. (1988). From (4a), it follows that there are strong correlations between the values of \( Q \) and \( \tau_{11} \), and that the negative signs of both \( Q \) and \( \tau_{11} \), as given by model calculations, imply destructive interference between the two contributions. Similarly from (4b), \( B(\text{E2}) \parallel \) and \( \tau_{12} \) are strongly correlated, and the predicted positive sign of \( \tau_{12} \) implies destructive interference. Thus larger magnitudes of \( Q \) can be more or less compensated in fits by larger magnitudes of \( \tau_{11} \), and larger values of \( B(\text{E2}) \parallel \) by larger values of \( \tau_{12} \).

Weller et al. (1985) pointed out that the quantity \( T^{\text{el,in}}_{20} \) is sensitive to the values of \( Q \) and \( \tau_{11} \), but insensitive to \( B(\text{E2}) \parallel \) and \( \tau_{12} \), while \( T^{\text{el}}_{20} - T^{\text{el,in}}_{20} \) is sensitive to \( B(\text{E2}) \parallel \) and \( \tau_{12} \), but insensitive to \( Q \) and \( \tau_{11} \). Weller et al. (1985) and Kajino et al. (1988) fitted values of \( T^{\text{el,in}}_{20} \) and also of \( T^{\text{el}}_{20} \), which is sensitive to all four parameter values. In order to simplify the analysis, we fit values of \( T^{\text{el,in}}_{20} \) and of the inelastic cross section to the first excited state, \( \sigma^{\text{in}} \), which is essentially proportional to \( T^{\text{el,in}}_{20} - T^{\text{el,in}}_{20} \), as is demonstrated below. This decouples the four parameters into two pairs: values of \( Q \) and \( \tau_{11} \) are determined by fitting the \( T^{\text{el,in}}_{20} \) data, with \( B(\text{E2}) \parallel \) and \( \tau_{12} \) taken to have reasonable fixed values, and values of \( B(\text{E2}) \parallel \) and \( \tau_{12} \) are determined by
fitting $\sigma^{\text{in}}$, where we again adopt reasonable values of $Q$ and $\tau_{11}$. In order to determine $\sigma^{\text{in}}$ we use the definition

$$T_{20}^{\text{el+in}} = \frac{\sigma^{\text{el}} T_{20}^{\text{el}} + \sigma^{\text{in}} T_{20}^{\text{in}}}{\sigma^{\text{el}} + \sigma^{\text{in}}}$$  \hspace{1cm} (5)$$

and assume that

$$\sigma^R = \sigma^{\text{el}} + \sigma^{\text{in}},$$  \hspace{1cm} (6)$$

where $\sigma^R$ is the Rutherford cross section. Equation (6) is sufficiently accurate at the present sub-Coulomb energies. Hence

$$\sigma^{\text{in}} = \sigma^R \frac{T_{20}^{\text{el}} - T_{20}^{\text{el+in}}}{T_{20}^{\text{el}} - T_{20}^{\text{in}}}.$$  \hspace{1cm} (7)$$

The quantity $T_{20}^{\text{in}}$ depends very little on the parameter values (and is independent of them in first order perturbation theory—Zupranski et al. 1979). Since, for the backward angles at which data are available, $T_{20}^{\text{in}} = O(1)$, while $T_{20}^{\text{el}} = O(10^{-2})$, it follows that the deduced values of $\sigma^{\text{in}}$ depend on the experimental values essentially only through the combination $T_{20}^{\text{el}} - T_{20}^{\text{el+in}}$. The consequent sensitivity

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**Fig. 1.** Scatter diagram showing correlation between $(\Delta T_{20}/\epsilon)^{\text{el}}$ and $(\Delta T_{20}/\epsilon)^{\text{el+in}}$, for the data of Weller et al. (1985), where $\Delta T_{20} = T_{20}(\text{calc}) - T_{20}(\text{expt})$. 

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of $\sigma^{\text{in}}$ to $B(\text{E2})$ and $T_{12}$, and insensitivity to $Q$ and $T_{11}$, was also pointed out by Vermeer et al. (1984).

Because of correlations in the measured values of $T_{20}^{\text{el}}$ and $T_{20}^{\text{el+in}}$, it is not obvious what uncertainties should be associated with the values of $\sigma^{\text{in}}$. These correlations are shown in Fig. 1, where values of $[T_{20}^{\text{el+in}}(\text{calc})-T_{20}^{\text{el+in}}(\text{expt})]/\epsilon^{\text{el+in}}$ are plotted against the corresponding quantity for elastic scattering for each of the 35 experimental configurations used by Weller et al. (1985) (5 angles, 7 target-energy combinations). Here $T_{20}(\text{expt})$ and $\epsilon$ are the experimental value and uncertainty as given by Weller et al., and $T_{20}(\text{calc})$ is the value calculated for the best-fit parameter set of Weller et al. Due to these correlations, the usual procedure of combining the uncertainties in $T_{20}^{\text{el}}$ and $T_{20}^{\text{el+in}}$ in quadrature would overestimate the uncertainties in $\sigma^{\text{in}}$, and this would be reflected in fits to $\sigma^{\text{in}}$ giving $\chi^2$ values that were too small. For the best-fit parameter set of Weller et al. (1985), the values of $\chi^2$ for the fits to their original $T_{20}^{\text{el}}$ and $T_{20}^{\text{el+in}}$ data are respectively $\chi^2_1 = 29.1$ and $\chi^2_2 = 26.3$, while the fit to $\sigma^{\text{in}}$ with the uncertainties obtained as suggested above gives $\chi^2_3 = 9.0$. In order to obtain a value of $\chi^2_3$ comparable with those of $\chi^2_1$ and $\chi^2_2$, say $\chi^2_3 = 27$, we take the uncertainties in $\sigma^{\text{in}}$ as $\sqrt{\frac{1}{3}}$ times the values obtained by combining the uncertainties in $T_{20}^{\text{el}}$ and $T_{20}^{\text{el+in}}$ in quadrature. To the extent that the latter uncertainties are mainly statistical in nature, this procedure should not unduly bias the determination of the best values of $B(\text{E2})$ and $T_{12}$.

We calculate $\chi^2_2$ for the $T_{20}^{\text{el+in}}$ data on a grid of $Q$ and $T_{11}$ values, and $\chi^2_3$ for the $\sigma^{\text{in}}$ data on a grid of $B(\text{E2})$ and $T_{12}$ values. Because we are looking for acceptable fits with parameter values that are consistent with other information, we take the $Q$ values to be in the range specified by the molecular value of $Q$ and its uncertainty $(-4.06 \pm 0.08 \text{ efm}^2)$, and the $B(\text{E2})$ values to be similarly limited by the values of Vermeer et al. (1989) $(7.59 \pm 0.10 \text{ efm}^4)$.

### 3. Results

Values of $\chi^2_2$ for $Q$ taken equal to the best molecular value $-4.06 \text{ efm}^2$ and to the two extreme molecular values $-3.98$ and $-4.14 \text{ efm}^2$ are shown in Fig. 2a for a range of values of $T_{11}$ [calculated with $B(\text{E2}) = 7.59 \text{ efm}^4$ and $T_{12} = 0.15 \text{ fm}^3$, taken from the values (8) below]. Similarly Fig. 2b shows values of $\chi^2_3$ for $Q$ taken equal to the best value of Vermeer et al. (1989), $B(\text{E2}) = 7.59 \text{ efm}^4$, and to the two values differing from this by their given uncertainty of $\pm 0.10 \text{ efm}^4$, for a range of values of $T_{12}$ (calculated with $Q = -4.06 \text{ efm}^2$ and $T_{11} = -0.15 \text{ fm}^3$).

The insensitivity of $\chi^2_2$ to $B(\text{E2})$ and $T_{12}$, and of $\chi^2_3$ to $Q$ and $T_{11}$, is demonstrated by the observation that over the ranges of $B(\text{E2})$ and $T_{12}$ shown in Fig. 2b, $\chi^2_3$ varies by only 1%, and over the ranges of $Q$ and $T_{11}$ in Fig. 2a, $\chi^2_3$ varies by less than 2%. This insensitivity justifies our decoupling procedure.

In Fig. 3, the data of Weller et al. (1985), consisting of values and uncertainties of $T_{20}^{\text{el}}$ and $T_{20}^{\text{el+in}}$, are compared with values calculated for the parameter set (given in equations 8 below) that we take to be the most acceptable, both from fits to these data and from other considerations. Fig. 4 gives a similar comparison of $\sigma^{\text{in}}$ values, where the experimental values and uncertainties are...
derived from the data of Weller et al. as explained in Section 2. It is obvious that there is good agreement in both Figs 3 and 4.

The best-fit parameter values of Weller et al. (1985) give $\chi^2_1 = 29.1$, $\chi^2_2 = 26.3$ and $\chi^2_3 = 27.0$, while those of Kajino et al. (1988) give $\chi^2_1 = 34.4$, $\chi^2_2 = 40.5$ and $\chi^2_3 = 28.6$.

4. Discussion

For the molecular value of $Q = -4.06$ efm$^2$, our best fit to $T_{20}^{\text{in}}$ gives $\chi^2_2 = 31.3$, with $\tau_{11} = -0.16$ fm$^3$. Within the 'allowed range' of molecular $Q$ values, the smallest $\chi^2_2$ is 28.8 for $Q = -3.98$ efm$^2$ and $\tau_{11} = -0.07$ fm$^3$. Likewise, for the best value of $B(E2)_{1} = 7.59$ e$^2$ fm$^4$ obtained from fits to unpolarised $^7$Li data (Vermeer et al. 1989), the best fit to $\sigma^{\text{in}}$ gives $\chi^2_3 = 28.9$, with $\tau_{12} = 0.11$ fm$^3$, and within the 'allowed range' of Vermeer et al. the lowest $\chi^2_3$ is 28.4 for $B(E2)_{1} = 7.69$ e$^2$ fm$^4$ and $\tau_{12} = 0.13$ fm$^3$. After also taking into account the result of Vermeer et al. that $\tau_{12} = 0.16\pm0.01$ fm$^3$, and the expectation from model calculations (Mertelmeier and Hofmann 1986; Kajino et al. 1988; Barker and Woods 1989) that $\tau_{11} \approx -|\tau_{12}|$, we suggest as the most acceptable parameter values for fits to all the available data

$$Q = -4.06 \text{ efm}^2, \quad \tau_{11} = -0.15 \text{ fm}^3,$$

$$B(E2)_{1} = 7.59 \text{ e}^2 \text{ fm}^4, \quad \tau_{12} = 0.15 \text{ fm}^3. \quad (8)$$

They give $\chi^2_1 = 32.3$, $\chi^2_2 = 31.4$ and $\chi^2_3 = 32.0$. The values (8) are used for the calculated curves in Figs 3 and 4.
Fig. 3. Analysing powers $T_{20}$ and $T_{20}^{\text{hin}}$ for the scattering of $^7\text{Li}$ on $^{58}\text{Ni}$ and $^{120}\text{Sn}$ at various bombarding energies. The experimental values and error bars are from Weller et al. (1985). The curves are calculated using the parameter set (8).

The parameter values and uncertainties that Weller et al. (1985) obtained from fitting their aligned $^7\text{Li}$ data are

\[
Q = -3.70 \pm 0.08 \text{ e fm}^2, \quad \tau_{11} = 0.23 \pm 0.06 \text{ fm}^3, \\
B(E2)l = 8.3 \pm 0.5 \text{ e}^2 \text{ fm}^4, \quad \tau_{12} = 0.23 \pm 0.06 \text{ fm}^3.
\]  

The parameter values (8) lie outside the ranges given in (9) (note in particular the different signs of $\tau_{11}$). Weller et al. obtained their uncertainties from the
Fig. 4. Inelastic cross section $\sigma^{\text{in}}$ for the scattering of $^7\text{Li}$ on $^{58}\text{Ni}$ and $^{120}\text{Sn}$ at various bombarding energies. The experimental values and error bars are deduced from the measured values of Weller et al. as explained in Section 2. The curves are calculated using the parameter set (8).

error matrix or equivalently from the $\chi^2+1$ rule (see Egelhof et al. 1987).† This procedure is valid, however, only if all the measured quantities are

† There is an inconsistency in Fig. 3 of Egelhof et al. (1987). From the $1\sigma$ contour shown in the lower part of the figure, the uncertainties in $\tau_{11}$ and $\tau_{12}$ should each be about $\pm 0.12$ fm$^3$, rather than the values $\pm 0.06$ fm$^3$ indicated by the upper part of the figure; it is the latter values that are given by Weller et al. (1985).
uncorrelated (Bevington 1969), and this is certainly not the case for the aligned $^7$Li data, as is evident from Fig. 1. Thus the parameter values (8) should not be ruled out on the grounds that they are incompatible with the values and uncertainties given in (9). In fact, the $\chi^2$ value for the fit to the complete data of Weller et al. with the parameter values (8) is $\chi^2 = \chi_1^2 + \chi_2^2 = 63.7$ (66 degrees of freedom), which suggests that the fit is acceptable. It seems that the uncertainties given by Weller et al. for their parameter values are too small.

$$Q - 0.89\tau_{11} \text{ e fm}^{-1} = -3.9 \text{ e fm}^2.$$  (10)

However, unless the value of one or other of $Q$ and $\tau_{11}$ is restricted by some other means, the value of neither $Q$ nor $\tau_{11}$ will be well determined by fitting the aligned $^7$Li data. Similar remarks apply to $B(E2)$ and $\tau_{12}$. The strong correlation between $B(E2)$ and $\tau_{12}$ was mentioned by both Weller et al. (1985) and Vermeer et al. (1989), but previous contour diagrams similar to Fig. 5 connected only the weakly-correlated quantities $Q$ and $k$, where $k$ is proportional to $\tau_{12}$ (Vermeer et al. 1984), and $\tau_{11}$ and $\tau_{12}$ (Egelhof et al. 1987).

Fig. 5. (a) Contour diagram of $\chi_2^2$ as a function of $Q$ and $\tau_{11}$. (b) Contour diagram of $\chi_3^2$ as a function of $B(E2)$ and $\tau_{12}$. For both (a) and (b), the points correspond to the best-fit parameter set of Weller et al. (1985) and to their best fit with $\tau_{11} = \tau_{12} = 0$. The values of $\chi_2^2$ and $\chi_3^2$ are as indicated.
Fig. 6. Plot of \( P/\phi \) as a function of the quantity \( \kappa \), which occurs in the relation (11). (a) Experimental points derived from the aligned \(^7\text{Li}\) data of Weller et al. (1985). (b) Experimental points from the unpolarised \(^7\text{Li}\) data of Vermeer et al. (1984) (closed circles) and of Bamberger et al. (1972) (crosses), taken from Fig. 1 of Vermeer et al. (1989). Where no error bars are shown, the uncertainties are less than or equal to the size of the data points. For both (a) and (b), the solid line is the best fit of Vermeer et al. (1989) to the unpolarised \(^7\text{Li}\) data, and the dashed line corresponds to the parameter set (8).

In order to indicate the relative precision possible in the values of \( B(E2) \) and \( \tau_{12} \) determined from fits to the aligned \(^7\text{Li}\) data and to unpolarised \(^7\text{Li}\) data, we plot the data as in Fig. 1 of Vermeer et al. (1989). This makes use of the approximate relation

\[
P \equiv \frac{\phi \sigma_{\text{in}}}{(\sigma_{\text{el}} + \sigma_{\text{in}})} = fB(E2)(1 - \kappa k),
\]

where \( k = 50 \cdot 3\tau_{12}/\sqrt{B(E2)} \) efm\(^{-1}\). The quantities \( f \) and \( \kappa \) are functions of the experimental conditions (energy, angle, etc.) and are calculated from the Winther–de Boer (1966) program, with the molecular value of \( Q \). We have found
that this program gives values of $P$ agreeing with the results of ECIS79 well within the required limits of accuracy. In Fig. 6, values of $P/f$ are plotted as a function of $\kappa$, both for the unpolarised $^7\text{Li}$ data as in Vermeer et al. (1989) and as derived above from the aligned $^7\text{Li}$ data of Weller et al. (1985).

In each part of Fig. 6, the solid line is the best fit of Vermeer et al. to the unpolarised $^7\text{Li}$ data and the dashed line corresponds to the parameter values (8). In each case the intercept on the $y$-axis is equal to the value of $B(E2)_{l}$ and the slope is $-kB(E2)_{l}$. It is seen that the aligned $^7\text{Li}$ data are consistent with the unpolarised $^7\text{Li}$ data, and should have little influence in the determination of $B(E2)_{l}$ and $\tau_{12}$.

We suggest that the uncertainties in the parameter values (8) for $Q$, $B(E2)_{l}$ and $\tau_{12}$ should be taken from previous work, i.e. $\pm 0.08$ efm$^{2}$, $\pm 0.10$ e$^{2}$ fm$^{4}$ and $\pm 0.01$ fm$^{3}$ respectively. It is difficult to estimate the uncertainty in $\tau_{11}$; from analysis of the aligned $^7\text{Li}$ data alone there could be a large uncertainty (about $\pm 0.1$ fm$^{3}$) in the value of $\tau_{11}$ due to the strong correlation with the value of $Q$, but the theoretical constraint $\tau_{11} \approx -|\tau_{12}|$ leads us to suggest a smaller uncertainty of $\pm 0.05$ fm$^{3}$.

The values of $\tau_{11}$ and $\tau_{12}$ that we recommend are very different from those of Kajino et al. (1988). For this reason, and also because we are not convinced by their arguments relating the tensor moments to the astrophysical $S$-factor (see also the comment in Barker and Woods 1989), we think that the value of the zero-energy $S$-factor for the $^4\text{He}(t,\gamma)^7\text{Li}$ reaction derived by Kajino et al. should be treated with caution.

After this paper was essentially completed, related work by Grawert and Derner (1989) became available. From their analysis of the same aligned $^7\text{Li}$ data of Weller et al. (1985), they also found strong correlations between the values of $Q$ and $\tau_{11}$, and between $B(E2)_{l}$ and $\tau_{12}$; however, their use of semiclassical scattering theory introduces significant errors in the calculation of $T_{20}$.

5. Summary

We have reanalysed the data of Weller et al. (1985) on the tensor analysing powers for Coulomb scattering of aligned $^7\text{Li}$ ions, in order to obtain information on the values of the four $^7\text{Li}$ moments $Q$, $B(E2)_{l}$, $\tau_{11}$ and $\tau_{12}$. We use a fitting procedure that effectively decouples the parameters into two pairs. It is found that a single set of parameter values, chosen primarily to be consistent with the value of $Q$ measured by molecular techniques and the values of $B(E2)_{l}$ and $\tau_{12}$ required to fit unpolarised $^7\text{Li}$ data, and also with the theoretical constraint $\tau_{11} \approx -|\tau_{12}|$, gives a good fit to the aligned $^7\text{Li}$ data. Our recommended values and uncertainties are:

$$Q = -4.06 \pm 0.08 \text{ efm}^{2}, \quad \tau_{11} = -0.15 \pm 0.05 \text{ fm}^{3},$$

$$B(E2)_{l} = 7.59 \pm 0.10 \text{ e}^{2} \text{ fm}^{4}, \quad \tau_{12} = 0.15 \pm 0.01 \text{ fm}^{3}.$$ (12)

It is suggested that previous analyses of the same data that led to very different results either severely underestimated the uncertainties in their parameter values (Weller et al. 1985) or assumed inappropriate relations between the moments (Kajino et al. 1988).
Acknowledgments

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