Aust. J. Phys., 1991, **44**, 181–99

Composite Mesons in Self-confining Chiral Solitons*

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Abstract

Most quark–meson models for formation of a baryon as a bag or soliton solution begin with elementary local meson fields including a classical scalar configuration that provides repulsion of valence quarks from the vacuum. We explore aspects of the very different formation mechanism that operates in a model where chiral effective meson fields are composite objects generated from bilocal $q\bar{q}$ fluctuation fields and the dynamical quark mass can be self-confining. Speculations are made on whether this viewpoint can motivate meson–nucleon relativistic field models containing intrinsic cutoffs for use in nuclear physics.

1. Introduction

Nontopological soliton models provide a phenomenological picture of baryon substructure in terms of quarks coupled to self-interacting boson fields. A recent review (Birse 1990) summarises the large variety of such models and their applications in a nuclear physics context. In one of the earliest models (Friedberg and Lee 1977), a scalar field ϕ has a Yukawa coupling $g\bar{q}\phi q$ to quarks and a quartic self-interaction $U(\phi)$. The latter is chosen to have an absolute minimum $U(\phi_V)=0$ at a constant vacuum value ϕ_V, while having a local minimum at $\phi=0$. With a semi-classical treatment, this provides a self-consistent mechanism for an energetically stable configuration in the presence of a localised quark density. Here the ϕ field is to represent non-perturbative gluon effects and provides an attraction mechanism to localise quarks to a region where $\phi<\phi_V$. Quark–antiquark fluctuations in the form of effective meson fields are expected to be important collective degrees of freedom in any low energy modeling of QCD. Because of the dominant role of the pion, chiral extensions of soliton models have been given a great deal of attention (Birse *et al.* 1984, 1985; Kahana *et al.* 1984). There is no unique way to make a chiral extension of the Friedberg–Lee soliton (Williams and Dodd 1988). The original confining field ϕ may be kept as a chiral invariant or may be chosen as the chiral partner of the pion. The latter approach is not available to bag models where the scalar confining field is implemented in the form of a boundary condition to join the cavity interior $\phi=0$ [having bag constant $U(0)=B$] with the vacuum region $\phi=\phi_V$. With $(\sigma,\boldsymbol{\pi})$ as the chiral four-vector, chiral solitons have been studied in the format of the Gell-Mann and Lévy (1960) linear σ

* Dedicated to Professor Ian McCarthy on the occasion of his sixtieth birthday.

0004-9506/91/020181\$05.00

model where the self-interaction is $U(\sigma^2+\pi^2)$, the quark–meson coupling is $g\bar{q}(\sigma+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})q/f_\pi$ and there are the usual kinetic terms for σ and π.

The soliton model we wish to explore here is of the linear sigma type except that the fields σ and π are not elementary fields but are generated from bilocal $\bar{q}q$ collective modes. Such a model has been developed previously (Cahill and Roberts 1985) from an action containing a quark kinetic term and a quartic term describing finite range gluon exchange coupling of quark currents. After Fierz-reordering the quartic term there are many structures, two of which have the form $\bar{q}(x)\sigma(x,y)q(y)$ and $\bar{q}(x)i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x,y)q(y)$ where σ and π transform as $\bar{q}(y)q(x)$ and $\bar{q}(y)i\gamma_5\boldsymbol{\tau}q(x)$ respectively. All structures that contribute to the vacuum quark self-energy are kept so that there is the translationally invariant form $\Sigma(p)=i\gamma\cdot p(A(p)-1)+B(p)$. Fluctuations above the vacuum configurations are kept only for the σ and π channels to produce the simplest chiral model. The only bose fields that enter in this approach are those that can be made from $\bar{q}q$. No scalar field to simulate glueball effects is introduced. The composite fields $\sigma(x,y)$ and $\boldsymbol{\pi}(x,y)$ contain an internal form factor. The underlying chiral symmetry dictates that in the limit of zero meson momentum, the pseudoscalar form factor is $B(x-y)$ the scalar part of the self-energy (Delbourgo and Scadron 1979). The quark meson coupling term in this model is then of the form $\bar{q}(x)B(r)[\sigma(R)+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}(R)]q(y)$ where $r = x-y$ and $R = (x+y)/2$. Here $\sigma(R)$ and $\boldsymbol{\pi}(R)$ are localised fields for motion of the centre of mass of the composites. Because of the space and time translation invariance of the dynamical self-energy, and also the nonlocality of the couplings, this generalised soliton model has a different dynamical content than more standard models whose solutions are well studied. Here we describe some results from an initial study that ignores the pion field and adopts a gaussian form for the scalar field $\sigma(R)$. A fully self-consistent treatment in which the scalar field is obtained from the quark source will be reported elsewhere.

We also explore a property of this type of model that can impose absolute confinement. Soliton models are often endowed with absolute confinement through the device of a colour dielectric in a manner motivated by the work of Nielsen and Patkos (1982). A typical example is the chromo-dielectric model (CDM) (Fai *et al.* 1988; Krein *et al.* 1988*a*) in which the inverse gluon propagator is suppressed at large distances by a dielectric function $K(\phi)$ which tends to zero where ϕ, the auxiliary chiral singlet scalar field, approaches the vacuum value at the absolute minimum of $U(\phi)$. The resulting quark self-energy amplitude B scales as $K^{-1/2}$ and absolute confinement is implemented through the divergence of B toward the edge of the soliton. Absolute confinement implies dynamical breaking of chiral symmetry and the corresponding Goldstone pion must couple to quarks through the dielectric-dependent amplitude B. It is difficult to have the intrinsic properties of this pion field independent of the dielectric medium.

In contrast here, we explore the implications of having an absolute confining property embodied in the translation invariant quark self-energy amplitudes $A(x-y)$ and $B(x-y)$. Thus free propagation is prohibited and confinement is implemented in a way which is independent of absolute position and does not require an additional scalar field. The quarks interact with dynamical fluctuations in the field variables whose vacuum configurations generate the self-energy amplitudes A and B. These interactions are what make quark propagation possible so that eigenenergies and a nontopological soliton solution may be defined. The amplitudes A and B are in general given by a Schwinger–Dyson equation which requires knowledge of the two-point gluon propagator. We rely upon recent studies which indicate that the resulting A and B can be confining if there is sufficient infra-red strength in the effective gluon propagator (Krein *et al.* 1988*b*; Roberts and McKellar 1990; Bannur *et al.* 1990). At this stage we employ a simple model for A and B that is confining in order to explore the properties of quark eigenstates bound by a scalar meson field in such a circumstance. If the quark energies and spatial distributions are suitably well behaved as functionals of the meson fields, then a very efficient self-confining chiral soliton model will have been obtained.

In Section 2 we briefly review the features of the soliton model and the connection between the dynamical quark self-energy and vacuum $\bar{q}q$ configurations on the one hand and between meson fields and $\bar{q}q$ fluctuations on the other. The transition to an energy functional for a mean field soliton with evident quark and meson contributions is outlined in Section 3. The results of recent numerical studies are presented in Section 4. A characteristic feature of the distributed nucleon–meson form factor that arises from the composite nature of the meson is considered in Section 5. A summary is presented in Section 6.

2. Generalised Soliton Model

In order to indicate the sense in which the chiral meson fields employed in this soliton model are composites, we review the transformation from quark field variables to $\bar{q}q$ field variables. A more comprehensive review is available (Cahill 1991; present issue p. 105) that treats a transformation to meson, diquark and eventually baryon fields. The Euclidean space generating functional for fermion Green's functions is taken to be

$$Z[\bar{\eta},\eta] = N\int D\bar{q}Dq \exp\{S[\bar{q},q] + \int d^4x(\bar{\eta}q + \bar{q}\eta)\}, \tag{1}$$

where the action is that of the global colour symmetry model (GCM) (Praschifka *et al.* 1987) given by

$$S[\bar{q}q] = -\int d^4x d^4y[\bar{q}(x)(\gamma\cdot\partial + m)\delta(x-y)q(y) + \tfrac{1}{2}g^2 j^a_\mu(x)D_{\mu\nu}(x-y)j^a_\nu(y)], \tag{2}$$

where the quark current is $j^a_\mu(x) = \bar{q}(x)\frac{1}{2}\lambda^a\gamma_\mu q(x)$. In the limit as the small current quark mass $m \to 0$, the GCM has, for the case of two quark flavours employed here, $SU(2)_L \otimes SU(2)_R$ chiral symmetry. Also, a global $SU(3)$ colour symmetry is employed. Some consequences of ignoring the local manifestation of colour symmetry are discussed elsewhere (Cahill 1991). The colour algebra, chiral symmetry, and the association of the function $D_{\mu\nu}$ with an effective two-point gluon function make this model capable of describing some aspects of QCD (Cahill and Roberts 1985; Praschifka *et al.* 1989). For convenience we take $D_{\mu\nu}(x-y) = \delta_{\mu\nu}\ D(x-y)$. The point of view of the GCM is that $D(x-y)$ is a parameter function for the model which is to contain at least a running coupling constant $\alpha_s(q^2) \sim 1/\ln(q^2/\Lambda^2)$ to incorporate the asymptotic freedom of QCD at large Euclidean momenta. A variety of forms have been employed for numerical work within the GCM (Cahill and Roberts 1985; Praschifka *et al.* 1989) and in other studies (Krein *et al.* 1988*b*; Roberts and McKellar 1990; Bannur *et al.* 1990) and such an approach has proved successful in the description of the low mass meson spectrum and dynamics.

Fierz reordering applied to the quark Grassmann fields transforms the current–current term through

$$\left\{\frac{\lambda^a}{2}\gamma_\mu\right\}_{ij}\left\{\frac{\lambda^a}{2}\gamma_\mu\right\}_{lm} = (\Lambda^\theta)_{im}(\Lambda^\theta)_{lj}\,. \tag{3}$$

The discrete index θ ranges over the terms of distinct transformation character in Lorentz, flavour and colour space. The Λ are direct products of Lorentz, flavour and colour matrices. With $D_{\mu\nu} \propto; \delta_{\mu\nu}$ the four Lorentz invariants scalar, vector, pseudoscalar and axial vector are produced. With two flavours of quarks each Λ is either isoscalar or isovector. The Fierz reordering of the colour λ matrices yields colour singlet and colour octet terms. We follow the bosonisation procedure (Shrauner 1977; Munczek 1982; Cahill and Roberts 1985) in which the quartic term in quark fields produced by (3) is reformulated as a functional integration over auxiliary bose fields $\mathcal{B}^\theta(x,y)$ having the transformation properties of $\bar{q}(y)\Lambda^\theta q(x)$. Fluctuations in these fields will be interpreted as effective meson fields. For the fluctuations we will ignore the colour octet sector and deal only with colour singlet effective meson fields. This is not completely satisfactory since part of the colour structure of the model action is thereby discarded. However, it has been shown (Cahill 1989) that the complete colour structure may be kept by the use of a further Fierz reordering so that bilocal combinations of quark fields that are not colour singlets appear only in the form of diquark fields $q(x)q(y)$ and $\bar{q}(x)\bar{q}(y)$. In this work we treat a static mean field quark–meson model of a baryon. The retention of just colour singlet effective meson fields can be viewed as ignoring correlations that are expressible as diquark degrees of freedom. At this level, the Fierz reordered form of (2) is essentially a nonlocal version of the Nambu–Jona-Lasinio (NJL) (1961) model. The limit $D(x-y) \propto \delta(x-y)$ recovers the local NJL model.

To deal with a ground state configuration with valence quarks, we add a constraint on the baryon number through a chemical potential μ via the canonical transformation of quark fields

$$q(x) \rightarrow q'(x) = e^{\mu x_4} q(x)\,. \tag{4}$$

With the quartic quark terms replaced by bose field integrations, the remaining bilinear quark field term may be handled by Grassmann integration in the standard way. The result is a generating functional given by

$$Z[\mu,\bar{\eta},\eta] = N\int D\mathcal{B}\,\exp\{S[\mu,\mathcal{B}] + (\bar{\eta}G[\mu,\mathcal{B}]\eta)\}\,, \tag{5}$$

where the bosonised action is

$$S[\mu,\mathcal{B}] = Tr\Big[LnG^{-1}[\mu,\mathcal{B}] - LnG^{-1}[\mu=0,\mathcal{B}]\Big] + S[\mathcal{B}] \tag{6}$$

with the vacuum action given by

$$S[\mathcal{B}] = TrLnG^{-1}[\mu = 0, \mathcal{B}] - \tfrac{1}{2}\int d^4x d^4y\, \frac{\mathcal{B}^\theta(x,y)\mathcal{B}^\theta(y,x)}{g^2 D(x-y)}. \tag{7}$$

We choose the separation in (6) in order to define effective meson fields from an expansion of the vacuum action, and to isolate the contribution from valence quarks. The meson kinetic terms are produced at leading order in the derivative expansion of the fermion loop term of $S[\mathcal{B}]$. The inverse propagator is

$$\begin{aligned} G^{-1}(\mu; x,y) &= e^{\mu x_4} G^{-1}(x,y) e^{-\mu y_4}. \\ &= (\gamma\cdot\partial + m - \gamma_4\mu)\delta(x-y) + e^{\mu x_4}\Lambda^\theta \mathcal{B}^\theta(x,y) e^{-\mu y_4}. \end{aligned} \tag{8}$$

The quarks are Yukawa coupled to the auxiliary bose field variables of integration with bare vertices Λ^θ. Besides the familiar shift of the time derivative, the additional μ dependence in (8) is due to the nonlocality of the bose fields. In later developments, a propagator G associated with (8) will be required. With appropriate boundary conditions, the μ dependence of G will serve to shift the pole structure in the momentum component conjugate to $x_4 - y_4$ so that valence and vacuum configurations are treated together in the usual way. Previous work on the connection of these methods to soliton models, and static bag models in particular, introduced valence quarks without relying upon a chemical potential (Cahill and Roberts 1985). There the baryon number constraint is imposed through construction of an approximate three-quark Green's function to identify the minimising energy for a nucleon state through a stationary phase argument. We use a chemical potential which constrains the baryon number after a functional Legendre transformation to the standard effective action Γ. This quantity is proportional to the energy functional for a static system (Cornwall *et al.* 1974).

The treatment (Cahill and Roberts 1985) followed here is to expand the vacuum action about the saddle point $\mathcal{B}_0(x,y)$, defined by $\delta S/\delta\mathcal{B}_0 = 0$. To lowest order in $\hbar$, $\mathcal{B}_0$ is the vacuum expectation value. Translationally invariant solutions for each $\mathcal{B}_0^\theta$ produce the quark self-energy $\Sigma(x-y) = \Lambda^\theta\mathcal{B}_0^\theta(x-y)$ which in momentum space satisfies

$$\begin{aligned} \Sigma(p) &= i\gamma\cdot p[A(p^2) - 1] + B(p^2) \\ &= g^2\int \frac{d^4q}{(2\pi)^4} D(p-q)\frac{\lambda^a}{2}\gamma_\mu \frac{1}{i\gamma\cdot q + m + \Sigma(q)}\frac{\lambda^a}{2}\gamma_\mu, \end{aligned} \tag{9}$$

an equation of Schwinger–Dyson form. Numerical solutions for the amplitudes A and B have been obtained in recent studies (Praschifka *et al.* 1989; Krein *et al.* 1988*b*; Roberts and McKellar 1990). For the remaining formal developments we shall assume that convenient forms for the amplitudes A and B are available. The propagating meson fields are identified through functional expansion of the vacuum action $S[\mathcal{B}]$ in powers of the fluctuations $\hat{\mathcal{B}}$ defined by $\hat{\mathcal{B}}^\theta(x,y) = \mathcal{B}^\theta(x,y) - \mathcal{B}_0^\theta(x-y)$. With retention of just scalar–isoscalar and pseudoscalar–isovector terms, we may write

$$\Lambda^\theta\hat{\mathcal{B}}^\theta(x,y) = \hat{\sigma}(r;R) + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}(r;R). \tag{10}$$

It is convenient to use a relative coordinate $r = x - y$ and a centre of mass coordinate $R = (x+y)/2$ for the bilocal meson fields. The second order terms in the expansion of $S[\mathcal{B}]$ identify propagators from which on-mass-shell eigenstates can be identified. The zero mass pole in the pion channel identifies $B(r)$ as the mass shell form factor. This is also consistent with the Ward identity that produces $B(r)$ as the residue at the zero mass pole in the axial vector vertex (Delbourgo and Scadron 1979). Since the fluctuation part of the chiral partner must have the same form factor, fluctuation fields with the form

$$\Lambda^{\theta}\hat{\mathcal{B}}^{\theta}(x,y) = B(r)\{\hat{\sigma}(R) + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}(R)\}, \tag{11}$$

used in an earlier consideration (Cahill and Roberts 1985) of soliton models of the present type, are seen to be appropriate for the zero momentum Goldstone modes. The factorised form in $x - y$ and $(x+y)/2$ is an approximation when the meson modes are off-mass-shell.

Contact with local models of the linear sigma (Gell-Mann and Lévy 1960) type may be made through expansion of the vacuum action in terms of the fluctuation fields $\hat{\sigma}$ and π with neglect of all but the lowest-order derivative terms. The fermion loop term of the action (7) has the nonlocal structure $TrLn[\gamma\cdot\partial A + m + B(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})]$ where $\sigma(R) = 1 + \hat{\sigma}(R)$ is the chiral partner of the pion. With a zero current quark mass, a chirally symmetric expansion (Roberts *et al.* 1988) of this quantity can be carried out in terms of $U(R) = \sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi} = \chi\, e^{i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}}$ where $\chi^2 = \sigma^2 + \pi^2$ is a chiral singlet. Only the real part of the $TrLn$ term is retained and the summation of all non-derivative terms may be evaluated as a functional of χ^2. The second term of (7) is also a functional of χ^2. The lowest order derivative terms of the fermion loop are quadratic. The result is (Cahill and Roberts 1985)

$$S[\sigma,\pi] - S[1,0] = -\int d^4R\left\{\frac{f_\pi^2}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\boldsymbol{\pi})^2] + V(\chi^2(R))\right\}, \tag{12}$$

where $V(\chi^2)$ is the meson self-interaction given by

$$V(\chi^2) = -12\int\frac{d^4q}{(2\pi)^4}\left\{ln\left[\frac{q^2A^2(q^2) + B^2(q^2)\chi^2}{q^2A^2(q^2) + B^2(q^2)}\right] - \frac{B^2(q^2)[\chi^2 - 1]}{q^2A^2(q^2) + B^2(q^2)}\right\}.$$

The meson masses can be obtained by differentiating the potential $V(\chi^2)$ twice with respect to $\hat{\sigma}$ or π. The pion mass is zero corresponding to the exact chiral limit while the $\hat{\sigma}$ mass is finite. The pion decay constant f_π obtained from this expansion can be expressed as a convergent integral involving A and B (Roberts *et al.* 1988). The potential $V(\chi^2)$ has turning points at $\chi^2 = 0$ and at the vacuum configuration $\chi^2 = 1$ corresponding to a local maximum and an absolute minimum respectively. A simplified form that respects these properties is the sigma model form $V(\chi^2) = c(\chi^2 - 1)^2$ where $c = f_\pi^2 m_\sigma^2/8$. With a small current quark mass m included in the quark propagators that make up V, a pion mass term with $m_\pi \propto m$ is generated. Explicit expressions for f_π and the meson masses (Praschifka *et al.* 1987; Roberts *et al.* 1988) in terms of the amplitudes A and B yield the typical values $f_\pi \sim 70 - 85$ MeV, and $m_\sigma \sim 0\cdot5 - 1$ GeV.

When the fields are rescaled to absorb the decay constant f_π and the constant vacuum value of the action is discarded, the complete action for the soliton model can be written as

$$S[\mu,\sigma,\pi] = Tr[LnG^{-1}(\mu,\sigma,\pi) - LnG^{-1}(0,\sigma,\pi)] + \int d^4R\mathcal{L}_m(R)\,, \tag{13}$$

where the localised meson Lagrangian is

$$\mathcal{L}_m = -\tfrac{1}{2}(\partial_\mu\sigma)^2 - \tfrac{1}{2}(\partial_\mu\boldsymbol{\pi})^2 - \tfrac{1}{2}m_\pi^2\boldsymbol{\pi}^2 - U(\sigma^2+\boldsymbol{\pi}^2) \tag{14}$$

and $U(\chi^2) = V(\chi^2/f_\pi^2)$. The chemical potential dependence of the fermion *TrLn* term ensures that a meson source from valence quarks will be generated. The inverse quark propagator occurring in (13) is, for $\mu = 0$,

$$\begin{aligned} G^{-1}(x,y) &= \gamma\cdot\partial_x A(x-y) + m\delta(x-y) + f_\pi^{-1}B(x-y) \\ &\quad\times\left\{\sigma\left(\frac{x+y}{2}\right) + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\left(\frac{x+y}{2}\right)\right\}. \end{aligned} \tag{15}$$

The meson part of the action (13) is of the standard format of a linear sigma model, and the meson component of a soliton energy functional is easily obtained. The non-standard features of this soliton model are contained in the structure of the valence quark contribution.

3. Soliton Energy Functional

If the chemical potential in the action $S[\mu,\sigma,\boldsymbol{\pi}]$ is set to zero, the saddle point configuration will be $\sigma = f_\pi$ and $\boldsymbol{\pi} = 0$. With a finite chemical potential there will be classical field expectation values σ and $\boldsymbol{\pi}$ that reflect the spatial source distribution of valence quarks. To define the mean field values one may introduce external sources $J_\sigma(x)$ and $\mathbf{J}_\pi(x)$ and a new action

$$\hat{S}[\mu,\sigma,\boldsymbol{\pi},J_\sigma,\mathbf{J}_\pi] = S[\mu,\sigma,\boldsymbol{\pi}] + \int d^4x\{J_\sigma(x)\sigma(x) + \mathbf{J}_\pi(x)\cdot\boldsymbol{\pi}(x)\}\,,$$

so that a generating functional W for connected Green's functions is defined by

$$W[\mu,J_\sigma,\mathbf{J}_\pi] = ln\int D\sigma D\boldsymbol{\pi}\,\exp\,\{\hat{S}[\mu,\sigma,\boldsymbol{\pi},J_\sigma,\mathbf{J}_\pi]\} \tag{17}$$

apart from an additive constant. Then the energy functional for a static soliton is given by $-\Gamma[n,\sigma_0,\boldsymbol{\pi}_0]/\int dx_4$ (Cornwall *et al.* 1974), where the effective action Γ is defined by the Legendre transformation

$$\Gamma[n,\sigma_0,\boldsymbol{\pi}_0] = W[\mu,J_\sigma,\mathbf{J}_\pi] - \int d^4x(J_\sigma\sigma_0 + \mathbf{J}_\pi\cdot\boldsymbol{\pi}_0) - \mu n\,, \tag{18}$$

and the classical field expectation values are $\sigma_0 = \delta W/\delta J_\sigma$ and $\boldsymbol{\pi}_0 = \delta W/\delta\mathbf{J}_\pi$. The quantity n is $\delta W/\delta\mu$ from which the baryon number is obtained as $n/\int dx_4$. For a mean field approximation, the lowest level in the loop expansion is used,

W reduces to the action $\hat{S}$ evaluated at its saddle point configuration, and $\sigma_0, \boldsymbol{\pi}_0$ become those configurations. Thus, at the Hartree level of no meson loops and a single fermion loop, the energy functional is

$$E[n,\sigma,\boldsymbol{\pi}] = E_q[n,\sigma,\boldsymbol{\pi}] + E_m[n,\sigma,\boldsymbol{\pi}], \tag{19}$$

where the quark contribution is

$$E_q[n,\sigma,\boldsymbol{\pi}]\left(-\int dx_4\right) = Tr\left[LnG^{-1}[\mu,\sigma,\boldsymbol{\pi}] - LnG^{-1}[0,\sigma,\boldsymbol{\pi}]\right] - \mu n \tag{20}$$

and the meson contribution is

$$E_m[n,\sigma,\boldsymbol{\pi}] = \int d^3x\{\tfrac{1}{2}(\nabla\sigma)^2 + \tfrac{1}{2}(\nabla\boldsymbol{\pi})^2 + \tfrac{1}{2}m_\pi^2\boldsymbol{\pi}^2 + U(\sigma^2 + \boldsymbol{\pi}^2)\}. \tag{21}$$

We denote the classical static fields now by $\sigma(\mathbf{x})$ and $\boldsymbol{\pi}(\mathbf{x})$. The chemical potential μ is to be treated as a functional $\mu[n,\sigma,\boldsymbol{\pi}]$ obtained from inversion of $n = \delta W/\delta\mu$ which at the present level of treatment is

$$n = \frac{\partial}{\partial\mu} TrLnG^{-1}[\mu,\sigma,\boldsymbol{\pi}]. \tag{22}$$

The equation of motion for the classical σ field is $\delta\Gamma/\delta\sigma = -J_\sigma$ which follows from the definition of the effective action. A similar equation holds for the $\boldsymbol{\pi}$ fields. In the physical limit of no external sources, the field equations of motion are therefore

$$\frac{\delta E}{\delta\sigma} = 0 = \frac{\delta E}{\delta\boldsymbol{\pi}}. \tag{23}$$

The contribution to the field equations from the meson energy component is easily evaluated from (21).

The contribution from the quark component (20) of the energy functional is most conveniently expressed in terms of quark energy eigenvalues that can be identified from spectral decompositions of the fermion propagators (Williams and Cahill 1983). With static meson fields, $G^{-1}(x,y)$ depends on time only through the variable $\tau = x_4 - y_4$, and it is convenient to use the Fourier representation

$$G^{-1}(\omega;\mathbf{x},\mathbf{y}) = \int d\tau\, e^{-i\omega\tau} G^{-1}(\tau;\mathbf{x},\mathbf{y}). \tag{24}$$

The time-translation invariance of $G^{-1}(x,y)$ allows stationary eigenstates of the form $u_j(\mathbf{x})e^{i\omega x_4}$ which satisfy

$$\int d^3y\, G^{-1}(\omega;\mathbf{x},\mathbf{y})u_j(\mathbf{y}) = i\gamma_4\lambda_j(\omega)u_j(\mathbf{x}). \tag{25}$$

The eigenvalues have the form $\lambda_j(\omega) = \omega - i\epsilon_j(\omega)$ where ϵ_j is the quark energy eigenvalue. The ω dependence of $\epsilon_j(\omega)$ arises from the dynamical nature of the self-energy $\Sigma(\omega,\mathbf{x}-\mathbf{y})$. The index j labels the set of distinct states of the spectrum for a given value of ω. The standard Feynman boundary conditions are incorporated in the Euclidean form of $G(x,y)$ with the integration contour C taken along the real ω axis with closure in the lower half plane. In the presence of a chemical potential μ, the appropriate boundary conditions for $G(\mu;x,y)$ are implemented through use of the same contour with the replacement $\lambda_j(\omega) \;\rightarrow\; \lambda_j(\omega+i\mu)$. The *TrLn* term of the valence quark energy functional becomes

$$Tr[LnG^{-1}(\mu)-LnG^{-1}(0)] = \sum_j \int \frac{d\omega}{2\pi}\, e^{-i\omega\eta} ln\left\{\frac{\lambda_j(\omega')}{\lambda_j(\omega)}\right\}\int dx_4\,, \tag{26}$$

where $\omega' = \omega + i\mu$, and the limit $\eta \rightarrow 0^+$ is implied.

The term μn, which must be subtracted from (26) to obtain the valence quark energies, can be cast as a contour integral with use of (22) and (26). The resulting valence quark energy functional is

$$E_q[n,\sigma,\boldsymbol{\pi}] = \sum_j \int_{C'-C} \frac{d\omega}{2\pi}\, e^{-i\omega\eta}\,\frac{\omega\lambda'_j(\omega)}{\lambda_j(\omega)}\,, \tag{27}$$

where $\lambda'_j(\omega) = d\lambda_j(\omega)/d\omega$. The contour C' is along the line $\mathrm{Imag}(\omega) = \mu$ with closure in the lower half plane. The net contour $C'-C$ encloses poles in the upper half plane given by $\lambda_j(\omega_p) = 0$ which is equivalent to $\omega_p = i\epsilon_j(\omega_p)$. This condition identifies the physical positive eigenvalues $\epsilon_j = -i\omega_p$ to be obtained from the self-consistent Dirac equation, which is

$$\begin{aligned}0 = \int d^3y \Big\{&(-\gamma_4\epsilon_j + \boldsymbol{\gamma}\cdot\nabla)A(-\epsilon_j^2;\mathbf{x}-\mathbf{y}) \\ &+ \frac{1}{f_\pi}B(-\epsilon_j^2;\mathbf{x}-\mathbf{y})\left[\sigma\left(\frac{\mathbf{x}+\mathbf{y}}{2}\right) + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\left(\frac{\mathbf{x}+\mathbf{y}}{2}\right)\right]\Big\}u_j(y)\,.\end{aligned} \tag{28}$$

Since $\lambda'_j(\omega)/\lambda_j(\omega)$ has unit residue at the poles, (27) yields

$$E_q[n,\sigma,\boldsymbol{\pi}] = \sum_j \epsilon_j^{(+)}\theta(\mu - \epsilon_j^{(+)})\,. \tag{29}$$

The required chemical potential μ is identified as the highest occupied single particle level consistent with the baryon number, spin and isospin of the system. The Dirac sea contributions present in the individual terms of the formal expression (20) have cancelled out to produce the finite result in (29).

The valence quark contribution to the meson field equations of motion, for example $\delta E_q/\delta\sigma$, can now be evaluated from (27) after accounting for the dependence of μ upon σ and using (28) for the dependence of ϵ_j upon σ (see Frank *et al.* 1991). The resulting equations of motion are

$$-\nabla^2\sigma(\mathbf{z}) + \frac{\delta U}{\delta\sigma(\mathbf{z})} + Q_\sigma(\mathbf{z}) = 0\,, \tag{30}$$

and

$$-\nabla^2\boldsymbol{\pi}(\mathbf{z}) + m_\pi^2\boldsymbol{\pi}(z) + \frac{\delta U}{\delta\boldsymbol{\pi}(\mathbf{z})} + \mathbf{Q}_\pi(\mathbf{z}) = 0\,, \tag{31}$$

where the meson sources provided by valence quarks are

$$Q_{\sigma}(\mathbf{z}) = \sum_j \frac{1}{f_\pi Z_j} \int d^3x d^3y \, \bar{u}_j(\mathbf{x}) B(-\epsilon_j^2; \mathbf{x}-\mathbf{y}) \delta\left(\frac{\mathbf{x}+\mathbf{y}}{2} - \mathbf{z}\right) u_j(\mathbf{y}) \tag{32}$$

and

$$\mathbf{Q}_{\pi}(\mathbf{z}) = \sum_j \frac{1}{f_\pi Z_j} \int d^3x d^3y \, \bar{u}_j(\mathbf{x}) B(-\epsilon_j^2; \mathbf{x}-\mathbf{y}) i\gamma_5 \boldsymbol{\tau} \delta\left(\frac{\mathbf{x}+\mathbf{y}}{2} - \mathbf{z}\right) u_j(\mathbf{y}) \,. \tag{33}$$

In the limit of point coupling where the amplitude B becomes $\bar{B}\delta(\mathbf{x}-\mathbf{y})$, the sources reduce to the local form of conventional soliton models. The set of constants $Z_j = \lambda'_j(\omega_p) = 1 - i\epsilon'_j(\omega_p)$ produce wavefunction renormalisation so that the residue of the propagator $G(x,y)$ involves states $Z_j^{-1/2} u_j(x)$. The frequency dependence of the dynamical quark self-energy is responsible for Z_j. Departures of Z_j from unity are produced when the self-energy amplitude $A(x-y)$ departs from $\delta(x-y)$ and when $B(x-y)$ is not static.

4. Numerical Studies

To investigate the soliton mechanism in the presence of distributed coupling and a confining dynamical self-energy, we consider here the valence quark Dirac equation (28) in momentum space with zero current quark mass and only a scalar meson field. This is

$$[i\gamma \cdot pA(p^2) + B(p^2)]u(\mathbf{p}) + f_\pi^{-1} \int \frac{d^3p'}{(2\pi)^{\frac{3}{2}}} B\left(\frac{p+p'}{2}\right) \hat{\sigma}(\mathbf{p}-\mathbf{p}') u(\mathbf{p}') = 0 \,, \tag{34}$$

where $p_4 = p'_4 = i\epsilon$, ϵ is the energy eigenvalue and $\hat{\sigma} = \sigma - f_\pi$. When absolute confinement is embodied in the self-energy amplitudes $A(p^2)$ and $B(p^2)$, there is no solution (discrete or continuum) to $i\gamma \cdot pA(p^2) + B(p^2) = 0$ for time-like $p^2 < 0$. Hence $p^2 + M^2(p^2) \neq 0$, where $M(p^2) = B(p^2)/A(p^2)$ is the dynamical mass which prevents quark propagation in the vacuum. A simple model with this feature has been employed in a number of previous studies (Munczek and Nemirovsky 1983; Cahill and Roberts 1985; Shakin 1989). In the extreme limit of an effective gluon propagator that has only the zero momentum mode $g^2D(q) = (2\pi)^4 \frac{3}{16}\mu^2\delta^4(q)$, the Schwinger–Dyson equation yields

$$A(p^2) = \begin{cases} 2, & p^2 \le \mu^2/4 \\ \frac{1}{2}[1 + (1 + 2\mu^2/p^2)^{\frac{1}{2}}], & p^2 \ge \mu^2/4 \end{cases}$$

$$B(p^2) = \begin{cases} (\mu^2 - 4p^2)^{\frac{1}{2}}, & p^2 \le \mu^2/4 \\ 0, & p^2 \ge \mu^2/4 \,. \end{cases} \tag{35}$$

We adopt this model here and take the strength parameter μ to be 1 GeV to simulate the strength and range of numerical solutions (Praschifka *et al.* 1989) for $B(p^2)$. For later reference, the dynamical mass associated with (35) is plotted in Fig. 1 as the dot-dashed line.

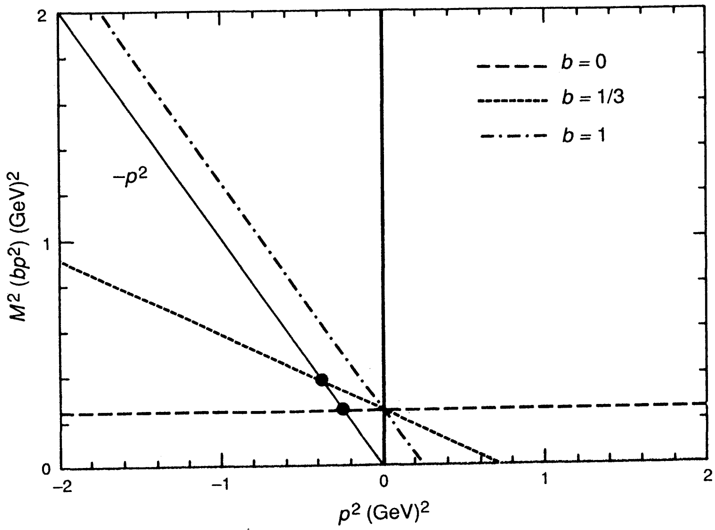

Fig. 1. The square of the dynamical quark mass $M^2(bp^2)$ is plotted versus Euclidean p^2. The dot-dashed line ($b = 1$) illustrates the simple model employed to investigate the effects of absolute confinement upon the solution of the Dirac equation in the presence of a classical scalar meson field. The parameter b is used to define a series of models with reduced dynamical behaviour and the constant mass case is produced by setting $b = 0$.

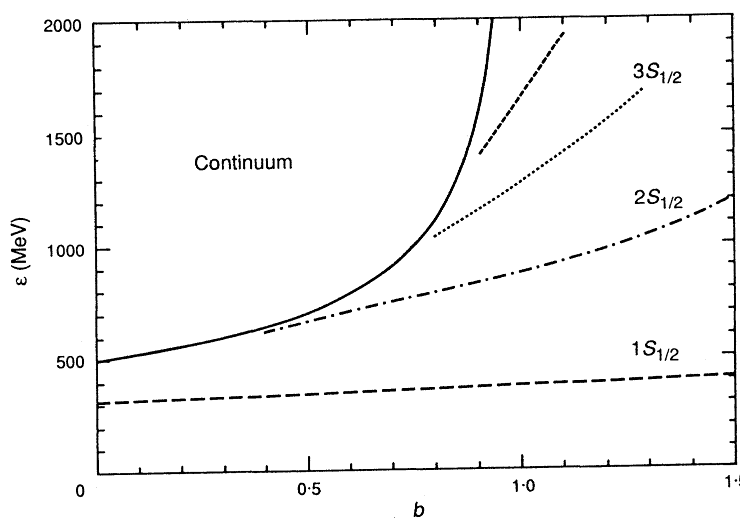

Fig. 2. The $S_{1/2}$ valence quark eigenenergies plotted versus the parameter b which controls the dynamical content of the quark mass function. A gaussian form for the scalar meson field is employed as described in the text. The meson field strength parameter $V_0 = 1407$ MeV. The continuum is taken up by discrete states and disappears at the absolutely confining point $b = 1$.

A condensed scalar meson field can modify the mass. Consider the simplified case where the meson field is spatially constant, so that in momentum space $\hat{\sigma}(\mathbf{p}-\mathbf{p}') \to (2\pi)^{\frac{3}{2}} f_\pi \bar{\sigma}\, \delta(\mathbf{p}-\mathbf{p}')$ where $\bar{\sigma}$ is a dimensionless constant to characterise the strength. Then the eigenvalue equation (34) becomes simply

$$p^2 + M^2(p^2)(1+\bar{\sigma})^2 = 0. \tag{36}$$

A physical eigenvalue ϵ is obtained if there is a solution to (36) for time-like $p^2 \equiv -\epsilon^2 + \mathbf{p}^2 < 0$. In this schematic model then, the effective dynamical mass is $\hat{M}(p^2) = M(p^2)(1+\bar{\sigma})$ and a scalar meson field with strength below the vacuum value, i.e. $\bar{\sigma} < 0$, leads to a reduced slope for $\hat{M}^2(p^2)$. The effect would be to rotate the dot-dashed line in Fig. 1 anticlockwise about its intercept on the p^2 axis. This will produce an intercept with the solid line in the time-like $p^2 < 0$ region, and thus an energy eigenvalue is produced. Physical solutions to (36) occur for $-2 < \bar{\sigma} < 0$ at $p^2 = -M_c^2$, where

$$M_c^2 = \frac{\mu^2}{4} \frac{(1+\bar{\sigma})^2}{1-(1+\bar{\sigma})^2} \tag{37}$$

and M_c may be considered a constituent mass. The quark energies in the medium characterised by $\bar{\sigma}$ are given by $\epsilon^2(\mathbf{p}) = \mathbf{p}^2 + M_c^2$.

We consider now solutions of (34) for valence quark states due to a finite range $\hat{\sigma}$ field with the form $\hat{\sigma}(\mathbf{x}) = -V_0 f_\pi \mu^{-1} \exp(-\mathbf{x}^2/r_0^2)$. The strength parameter μ for the amplitude $B(p^2)$, and the σ field range r_0, are fixed at 1 GeV and 1 fm throughout. The confining dynamical mass should induce a large distance decay for the quark wavefunction that is faster than the characteristic exponential form of constant mass solutions and a bag-like behaviour might be expected. In order to explore separately the effect of the dynamical mass and the distributed quark–meson coupling, we use amplitudes $A(bp^2)$ and $B(bp^2)$ for the self-energy and amplitude $B(fK^2)$ for the coupling term where $K = (p+p')/2$ in (34). Here b and f are real parameters in the range [0,1]. The point-coupling limit is achieved by setting $f = 0$ which produces $B(0) = \mu$. As the parameter b is reduced from unity, the dynamical variation of the self-energy amplitudes A and B is reduced and the confining property is removed. This is illustrated in Fig. 1. When $b = 0$, the dynamical mass becomes constant with the value $M(p^2 = 0) = \mu/2$ which is 500 MeV with the present parameters. In Fig. 2 the lowest several $S_{1/2}$ eigenvalues ϵ are plotted versus b for point coupling ($f = 0$) and for $V_0 = 1407$ MeV. As b increases from zero, the continuum is progressively elevated and replaced by discrete states until at $b = 1$ there is only a discrete spectrum. This is because with a confining dynamical mass, there are no solutions to the free wave equation that can be used to describe the large distance behaviour of the states outside the range of the condensed σ field. No solutions with scattering boundary conditions exist. From Fig. 2 the energy of the lowest $S_{1/2}$ state is remarkably insensitive to the parameter b, that is, to the detailed dynamical nature of the mass. This is because we have used a strongly attractive strength for the condensed σ field. The self-consistent solution for the σ field driven by the scalar source term $\bar{q}q$ and the nonlinear self-interacting $U(\sigma)$ is in progress.

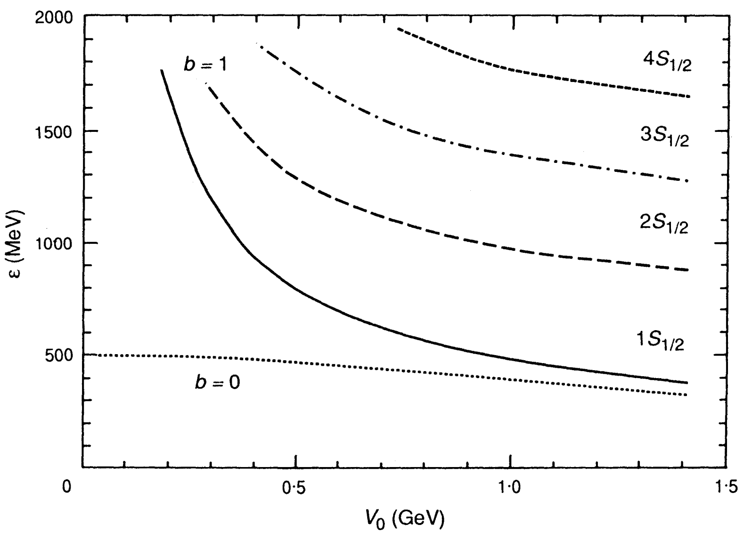

Fig. 3. The $S_{1/2}$ valence quark eigenenergies plotted versus the strength V_0 of the scalar meson field taken to be a gaussian. The divergence at small V_0 is a reflection of the absolutely confining form of the quark mass function when $b = 1$. For a constant mass ($b = 0$) there is only one state.

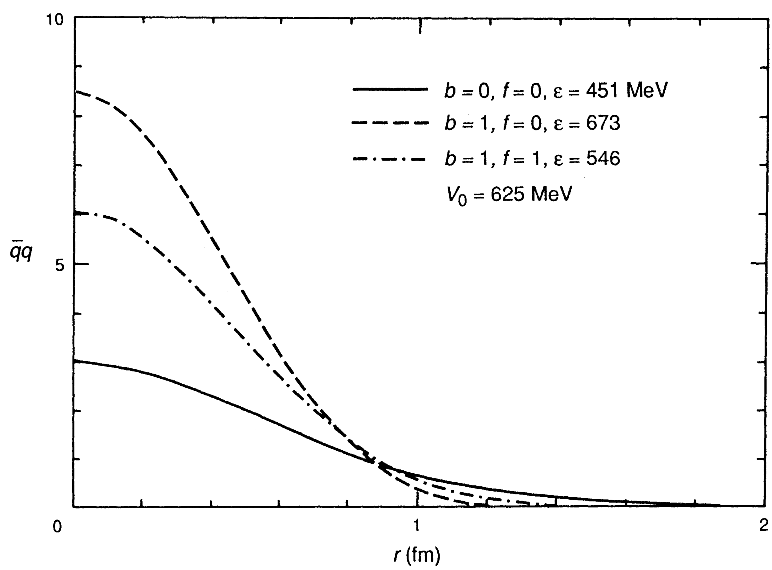

Fig. 4. The quark scalar density is compared for confining ($b = 1$) and non-confining ($b = 0$) models of the quark mass function and also for distributed coupling ($f = 1$) and point coupling ($f = 0$).

To gain some insight into how the relation between the quark energies and the σ field is modified by the dynamical content of the quark mass, we show in Fig. 3 the eigenenergies ϵ plotted versus the field strength V_0 for $b = 0$ and $b = 1$, both with point coupling ($f = 0$). As $V_0 \to 0$ the energy of the single bound state that exists in the constant mass ($b = 0$) case goes to the continuum value, but the energies of the confining ($b = 1$) solutions go to infinity, while still providing quark states of finite extent. For the moment, consider V_0 as a variational parameter employed to minimise the soliton energy. The meson component of the energy begins at zero from the vacuum point $V_0 = 0$ and increases quadratically with V_0. For a constant quark mass ($b = 0$), the quark energies begin at that mass value and decrease as V_0 increases away from $V_0 = 0$. If the quark energies do not decrease fast enough, the minimising configuration can occur at or near $V_0 = 0$ giving essentially plane wave quarks and not a soliton solution. This is avoided in standard models by making the coupling constant sufficiently large so that the decrease of the quark eigenvalues with V_0 will dominate to push the total minimum away from the vacuum point $V_0 = 0$ producing well localised quark states. Since the coupling constant and the quark mass are proportional to each other, it is necessary to have a sufficiently large mass parameter to have a soliton that is energetically stable with respect to free quarks.

However, when the p^2 dependence of the dynamical mass is absolutely confining, the $b = 1$ curves in Fig. 3 show that the quark energies will always have a negative slope at small V_0 which overcomes the positive slope of the meson component of the energy. Here the confining dynamical quark mass prevents the minimising configuration from occurring at vanishing meson field strength even if the strength parameter of the dynamical mass ($M(p^2 = 0) = \mu/2$) is very small. There should always be a localised soliton solution which is energetically stable for any choice of parameters.

In Fig. 4 the spatial distribution of the scalar quark density $\bar{q}q$ is displayed for several dynamical choices determined by b and f. The meson field strength is chosen so that without confinement ($b = 0$) the binding is very weak and the quark density is quite diffuse. The effect of including a confining dynamical mass ($b = 1$) is that the quark density becomes significantly more localised towards a cavity-like configuration. A well defined localisation can be produced with relatively weak meson field strengths. Due to the confinement embodied in the dynamical mass, soliton solutions with features approaching a bag model appear possible. Also from Fig. 4, the distributed quark–meson coupling ($f = 1$) is found to have a strong effect on the quark $\bar{q}q$ density in the interior when compared to the point coupling case ($f = 0$). The character of this effect depends upon the meson field strength. In the case shown here, the distributed coupling increases the net size of the quark distribution, while for larger meson field strengths the opposite is true. A fully self-consistent solution is clearly needed.

5. The Nucleon–Meson Form Factor

As is well known, an elementary field that couples to valence quarks within the nucleon will result in a form factor with a range that reflects the size of the quark distribution. The associated momentum variable is the momentum

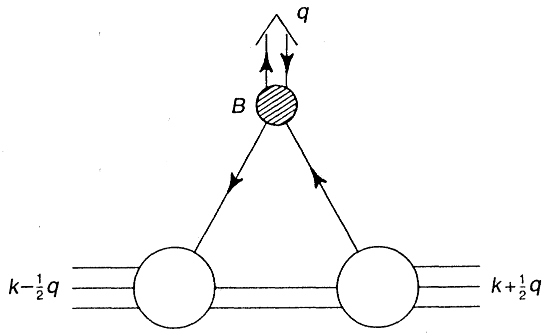

Fig. 5. An illustration of the mechanism employed for the nucleon–meson form factor to estimate the suppression in the nucleon momentum k induced by form factor B of the composite meson.

transfer to the nucleon. We wish to illustrate here that for coupling of a composite meson field to the nucleon, the form factor will have a dependence on the momentum of the nucleon that reflects the size of the meson form factor. We take the case of a static scalar meson field with internal form factor $B(\mathbf{x}-\mathbf{y})$ as illustrated in Fig. 5. The mean-field or independent particle model of the nucleon must be projected to produce a state of definite momentum. The resulting momentum sharing among the valence quarks will induce in the form factor a dependence upon nucleon momentum governed by the range of B. We ignore relativistic boost effects and employ the Peierls–Yoccoz (PY) (1957) projection to produce definite nucleon momentum. If $\psi(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)=u(\mathbf{x}_1)u(\mathbf{x}_2)u(\mathbf{x}_3)$ is the mean field nucleon state for three quarks in the lowest S-state, then the state from PY projection is

$$\psi_k(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)=\frac{1}{n(k)}\int d^3R\; e^{-i\mathbf{k}\cdot\mathbf{R}}\psi(\mathbf{x}_1+\mathbf{R},\mathbf{x}_2+\mathbf{R},\mathbf{x}_3+\mathbf{R}), \tag{38}$$

where the normalisation factor $n(k)$ is chosen to ensure that $\langle\psi_{k'}\mid\psi_k\rangle=\delta(\mathbf{k}'-\mathbf{k})$. This projected state ensures that the dependence upon the centre of mass coordinate $\mathbf{X}=\frac{1}{3}\sum_i x_i$ is $\exp(i\mathbf{k}\cdot\mathbf{X})$. A simple estimate of the nucleon–sigma form factor from

$$\delta(\mathbf{k}'-\mathbf{k}+\mathbf{q})F(\mathbf{k}',\mathbf{k},\mathbf{q})=\int d^3x'd^3x\,\langle\psi_{k'}\mid\bar{q}(\mathbf{x}')B(\mathbf{x}'-\mathbf{x})e^{-i\mathbf{q}\cdot[\frac{1}{2}(\mathbf{x}'+\mathbf{x})]}q(\mathbf{x})\mid\psi_k\rangle \tag{39}$$

leads to

$$\delta(\mathbf{k}'-\mathbf{k}+\mathbf{q})F(\mathbf{k}',\mathbf{k},\mathbf{q})=\frac{3}{n(k')n(k)}\int d^3R'd^3R\; e^{i(\mathbf{k}'\cdot\mathbf{R}'-\mathbf{k}\cdot\mathbf{R})}c^2(\mathbf{R}'-\mathbf{R})f(\mathbf{q};\mathbf{R}',\mathbf{R}), \tag{40}$$

where f is the elementary form factor before projection and is given by

$$f(\mathbf{q};\mathbf{R}'\mathbf{R})=\int d^3x'd^3x\;\bar{u}(\mathbf{x}'+\mathbf{R}')B(\mathbf{x}'-\mathbf{x})e^{-i\mathbf{q}\cdot[\frac{1}{2}(\mathbf{x}'+\mathbf{x})]}u(\mathbf{x}+\mathbf{R}). \tag{41}$$

Here $c(\mathbf{R}'-\mathbf{R})$ comes from integrations over a 'spectator' quark and is given by

$$c(\mathbf{R}'-\mathbf{R}) = \int d^3x u^+(\mathbf{x}+\mathbf{R}')u(\mathbf{x}+\mathbf{R}). \tag{42}$$

After conversion of all quantities to momentum space, e.g. $B(\mathbf{x}) = (2\pi)^{-3}\int d^3P e^{i\mathbf{P}\cdot\mathbf{x}}B(\mathbf{P})$, we obtain

$$F(\mathbf{k}',\mathbf{k},\mathbf{q}) = \frac{3}{n(k')n(k)}\int d^3P\, \bar{u}(\mathbf{P}-\mathbf{q}/2)B(P)u(\mathbf{P}+\mathbf{q}/2)R(\mathbf{P}-\tfrac{1}{2}(\mathbf{k}'+\mathbf{k})), \tag{43}$$

where $R(\mathbf{p})$ is the momentum representation of $c^2(x)$ and is given by

$$R(\mathbf{p}) = (2\pi)^{-3}\int d^3k\, c(\mathbf{p}-\mathbf{k})c(\mathbf{k}), \tag{44}$$

where $c(\mathbf{k}) = u^+(-\mathbf{k})u(-\mathbf{k})$. The normalisation factor $n(k)$ is given by

$$n^2(k) = \int d^3P\, u^+(\mathbf{P})u(\mathbf{P})R(\mathbf{P}-\mathbf{k}). \tag{45}$$

The removal of the PY projection corresponds to the limit $R(p)\to 1$ and $n(k)\to 1$, and then (43) reduces to the form factor for coupling to the quarks of the stationary mean field soliton as in (32). There is then a dependence on only the meson momentum.

To investigate the dominant behaviour in the limit $k'\approx k >> q$, we consider (43) in the form

$$F(\mathbf{k}',\mathbf{k},\mathbf{q}) \sim 3\,\frac{\int d^3P\, \bar{u}(\mathbf{P}-\mathbf{q}/2)B(\mathbf{P})u(\mathbf{P}+\mathbf{q}/2)R(\mathbf{P}-\mathbf{k})}{\int d^3P\, u^+(\mathbf{P})u(\mathbf{P})R(\mathbf{P}-\mathbf{k})}. \tag{46}$$

It is apparent that in the limit of point coupling, where $B(\mathbf{P})$ is independent of P, the recoil correction function $R(\mathbf{P}-\mathbf{k})$ is convoluted with essentially similar quark density distributions in the numerator and denominator of (46). Thus no fall off with the nucleon momentum k is expected for point coupling. However a finite range for the sigma form factor $B(\mathbf{P})$ will induce a suppression for large k. This can be seen explicitly in the case of S-wave orbitals if we describe both $u^+(\mathbf{P})u(\mathbf{P})$ and $\bar{u}(\mathbf{P})u(\mathbf{P})$ by different linear combinations of two gaussians $\exp(-P^2/4\alpha^2)$ and $\exp(-P^2/4\beta^2)$. This can account for a slight difference in range of upper and lower components. One parameter α can characterise the inverse size of the quark distribution, and we take $\beta \gtrsim \alpha$. Then $R(p)$ is a linear combination of three gaussian terms $\exp(-p^2/4p_i^2)$ with range parameters $p_i^2 = 2\alpha^2$, $2\beta^2$, and $(\alpha^2+\beta^2)$. The denominator $n^2(k)$ in (46) becomes a sum of six gaussian terms $\exp(-k^2/4k_i^2)$ with the dominant term at large k being $k_i^2 = 3\beta^2$. For estimation of the numerator in (46), we use $B(P) = B(0)\exp(-P^2/4\lambda^2)$ to obtain again a sum of gaussian terms. Retention of just the dominant terms in both numerator and denominator produces the estimated large k behaviour

$$F(\mathbf{k},\mathbf{k},q=0) \sim \exp\left\{-\frac{k^2}{4(9\lambda^2+6\beta^2)}\right\}. \tag{47}$$

The point coupling limit ($\lambda \to \infty$) for the quark–meson form factor produces no fall-off as expected. If the spatial size of the meson form factor $B(P)$ is significantly less than the soliton size, then $(\beta/\lambda)^2 \ll 1$ and the large k behaviour of the nucleon form factor becomes $B(k/3)$. In this limit deviations away from equal sharing of the nucleon momentum among three quarks become insignificant. If the range of B corresponds to a meson size of $0 \cdot 5$ fm then the large k behaviour of the nucleon form factor corresponds to a range which is reduced by one-third, i.e. about $1 \cdot 2$ GeV/c^2.

The above arguments are a long way from providing a definitive description of the distributed features of the nucleon-meson form factor that arise from underlying QCD degrees of freedom. Even in the simple mechanism considered here, relativistic covariance has been ignored. However, given that one should protect nucleon–meson field theory models from the divergences that arise from point-coupling, phenomenological form factors that simulate substructure from a model with QCD degrees of freedom seem necessary. It seems from the above argument that some general features of the nucleon–meson form factor might be constrained once a consistent model for the substructure of the hadrons is specified.

6. Summary

We have considered aspects of a non-topological soliton model in which the chiral meson fields are generated as $\bar{q}q$ composites. The interesting features that we focus upon here are the dynamical self-energy for quarks and the related distributed vertex for quark meson coupling. Initial numerical work to explore the practical consequences of these features has been presented in the context of a static mean-field soliton. The particular method employed here to identify the energy functional at the mean field or Hartree level is to obtain the standard effective action from the Legendre transformation with the help of a chemical potential constraint for the baryon number. The purpose of this approach is two-fold. First, a possible future consideration of radiative corrections might be undertaken by systematically continuing with the loop expansion beyond the lowest level. A second, more practical reason, is that in the presence of a general space–time dependent dynamical self-energy for quarks there are wavefunction renormalisation effects and energy self-consistencies to be defined and maintained for the valence quark states and eigenvalues. This energy dependence of the self-energy has an important effect when absolute confinement is embodied in the self-energy amplitudes.

The full nonlocal soliton model with self-consistent determination of σ,$\boldsymbol{\pi}$, and q fields has not been solved. Here we have presented and described initial results from solution of the valence quark states in the presence of a σ field taken to be a gaussian form. The quark self-energy amplitudes $A(p^2)$ and $B(p^2)$ have been assumed to be given by a simple absolutely confining form. The amplitude B plays a second role as the distributed form factor for quark–meson coupling in the exact chiral limit. For a strong σ field the confining nature of the quark mass function has little effect upon the lowest quark eigenvalue. However, as the σ field strength decreases, the quark eigenvalues grow rapidly but the corresponding quark distribution remains well localised. It seems that there should always be a soliton solution which is localised and stable energetically (as well as absolutely) with respect to free quarks for any choice of model parameters. The distributed form of the quark–meson coupling is found to have a strong effect upon the quark distribution in the interior of the soliton region compared with the usual point coupling form.

We have presented a simplified view of the nucleon–meson form factor to point out that the composite nature of the meson field will induce a suppression in terms of the nucleon momentum. The associated range is roughly one-third of the range of the meson form factor. Mechanisms that indicate a finite range for the three-point nucleon–meson vertex in each of the two independent momenta are of interest for field theory models of nuclear matter where relativistic quantum loop effects are much too strong with point coupling.

It remains to be seen whether full solutions of such a nonlocal soliton model can be successfully applied towards the calculation of, for example, electric and magnetic form factors of the nucleon where an internal structure for the pion may have a significant role.

Acknowledgments

Some twenty-one years ago, one of us (P.C.T.) began work with Ian E. McCarthy as a student. It is a pleasure to acknowledge his guidance, support and enthusiasm via this paper on the occasion of his birthday celebration. We have benefited greatly from discussions with R. T. Cahill, C. M. Shakin, C. D. Roberts and A. W. Thomas. One of us (P.C.T.) is grateful for the support and hospitality of the University of Adelaide during an extended visit. This work was supported in part by the National Science Foundation under Grant No. PHY88-05633.

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Manuscript received 7 December 1990, accepted 15 January 1991