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Kinetic Equation with Neutrino Oscillations in the Early Universe

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Abstract

This paper considers the kinetic equation for interacting neutrino gas in the context of an expanding early universe. It is suggested that if neutrino oscillations are present and CP violations occur prior to the decoupling of the neutrino gas from the rest of the universe, then lepton number may not be conserved and, in principle, significant permanent neutrino chemical potentials may develop and survive until the present day. This would lead to the well known effect that if the electron neutrino chemical potential is significantly non-zero, then the primordial abundances of the light elements are affected and differ from those of the standard model. Numerical computation is required to examine the parameter ranges leading to a significant non-zero electron neutrino chemical potential.

1. Introduction

Over the last 15 years there has been a major upsurge of both theoretical and experimental research into neutrino oscillations. One of the major aims of this research has been an attempt to explain the discrepancy [Davis *et al.* 1968; Davis 1980; also more recent results from Hirata *et al.* 1988–KAMIOKANDE-II; Aglietta *et al.* 1988–LSD at Mont Blanc and several new experiments proposed Pakvasa 1988–Borex; Abazov 1988–SAGE, and others; see also Bahcall (1989) for review of the solar neutrino experiments and the supernova explosion SN1987A] between the observed and the predicted neutrino fluxes from the sun,* but other work (McKellar and Granek 1980, 1982; Khlopov and Petcov 1981; Dolgov 1981; Zel'dovich and Khlopov 1981; Manohar 1987; Barbieri and Dolgov 1990 and a general review by Denegri *et al.* 1990) attempted to determine the effects of such oscillations on the evolution of the universe and the relaxation of the constraints placed on the number of neutrino families (e.g. see the review by Boesgaard and Steigman 1985) imposed by the standard model of the universe. In addition to the neutrino oscillations in vacuum proposal for the solution of the solar neutrino puzzle, a solution involving neutrino oscillations in matter (MSW oscillations) has been suggested (Mikheyev and Smirnov 1985, 1988; Wolfenstein 1979; Zaglauer and Schwarzer 1987; Bahcall and Haxton 1989; Rosen and Gelb 1989).

* In fact, recent data from these experiments confirm the neutrino problem and imply new neutrino physics [Acker *et al.* 1990; see also the references therein to the talks presented at the NEUTRINO'90 (CERN) and 25th International Conference on High Energy Physics (Singapore) by the experimental groups].

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In addition, since the neutrino mass is not well known and, further, it is still not clear if it is non-zero, a number of laboratory experiments have been done to place some upper limits on the value of an electron neutrino (Boris *et al.* 1985, 1987; see however Simpson 1986*b*; Bergkvist 1985*a*, *b*; Wilkerson *et al.* 1987–LANL group; Fritschi *et al.* (1986)–Zurich group; INS group 1987; and further claims by the Russian group–Lubimov 1988). Further improvements in various direct measurements have been obtained (INS group 1988; Kundig *et al.* 1988; Lubimov 1988; Daniel *et al.* 1988; Nakamura 1991). A possibility of existence of a 17 keV neutrino has also been investigated (Simpson 1984, 1986*a*, see however Borge *et al.* 1986; Hetherington *et al.* 1987; Zlimen *et al.* 1988). The experimental situation has been reviewed recently by Barish (1992). One method of determining if the neutrino has a non-zero mass is via other possible properties such as decay products, if it is unstable (Matsumoto *et al.* 1988) and the constraints on such decays (Krauss 1984; Sarkar and Cooper 1984; Lindley 1985; Granek 1988; Granek and McKellar 1990), arrival delay following supernova explosion or its magnetic moment etc. The supernova explosion in the Large Magellanic Cloud, SN1987A, resulted in some data, which, after analysis, led to some additional constraints on neutrino properties such as mass, lifetimes and magnetic moment (Dar *et al.* 1987; DeLeener-Rossier *et al.* 1987; von Feilitzsch and Oberauer 1988; Mohapatra 1988; Barbieri and Mohapatra 1988; Lattimer and Cooperstein 1988; Mohapatra and Nussinov 1989; Cowsik 1988; Krivoruchenko 1988; Zhaol *et al.* 1988; Adams 1988; Burrows 1988).

Another way to establish whether the neutrino has a non-vanishing mass is through a related effect of neutrino oscillations, where a neutrino of one type is transformed into another type without interacting with a second particle. Such an effect, if present in the physical world, may have a dramatic effect on nucleosynthesis in the standard model of the universe (see e.g. Weinberg 1972) through its effect on the balance between neutron and protons just prior and during nucleosynthesis, and consequently will affect the synthesis of the elements like deuterium, ^{3}He, ^{4}He and other light elements. The standard model places fairly rigid constraints on a number of physical parameters of the model, some of which are the neutrino chemical potential, mass and number of neutrino species. These constraints may be circumvented in the presence of neutrino oscillations, provided the oscillations are able to generate an effective chemical potential. In order to test the effect of this assumption on the early universe scenario, a nucleosynthesis model based on the model originally developed by Wagoner (1969) was built independently (McKellar and Granek 1980; Granek and McKellar 1981; Granek 1988). Since the primordial abundances of the elements heavier than ^{4}He are very uncertain (the possible evolution of lithium abundance has been discussed by Hobbs 1987; Deliyannis *et al.* 1989; Molaro 1987; Schramm 1987; while that of ^{3}He by Rood *et al.* 1987), the model is limited to nuclei with atomic number $A < 5$. It is tested on a range of parameters and the results are compared with those of the standard model, with and without degenerate neutrinos but excluding neutrino oscillations, as given by David and Reeves (1980), Olive *et al.* (1981) and Boesgaard and Steigman* (1985 and references therein). The results indicate

* Boesgaard and Steigman (1985) presented a general review of the accumulated work in this area by a number of researchers done over a period close to a decade.

that, when oscillations are included, the predictive power of the standard model is significantly reduced and consequently additional constraints must be imposed if more rigorous model predictions are expected. A recent review by Denegri *et al.* (1990) addresses most of the experimental results as well as those concerning the cosmological calculations mentioned above.

In this paper we would like to derive and discuss a kinetic equation for a system of interacting neutrinos, or neutral particles in the context of an expanding universe.* The derived equation may, in principle, be used to obtain the phase space density functions as a function of time, as they evolve in and out of equilibrium, provided the neutral particles undergo interactions similar to those of neutrinos and only the mass eigenstates and the strength of the interactions may vary. From now on we shall refer to these neutral particles as neutrinos.

In McKellar and Granek (1980) and Granek and McKellar (1981) we assumed that the electron neutrino chemical potential was generated during the expansion of the universe and prior to the commencement of nucleosynthesis. Here we formalise the mechanism, given in McKellar and Granek (1982), and propose a method of calculation of a neutrino chemical potential excess from the mass mixing matrix U and the kinetic equations governing the interactions of the weak neutrino eigenstates and the expansion of the universe.

In Section 2, starting with a general relativistic kinetic equation, we apply the relevant conservation laws and symmetries of the model in order to reduce the number of independent variables. The equations are then simplified to a more conventional form of the Boltzmann equation (25). By introducing a set of new variables, the part of the kinetic equation responsible for the expansion of the universe is decoupled, leaving an explicit expression for the total and non-adiabatic terms of the kinetic equation (31).

In Sections 4 and 5 we propose a scenario where various interactions go in and out of equilibrium conditions. Therefore, it is proposed that evolution out of equilibrium from the point of view of the mass eigenstates will lead to generation of chemical potential excesses with the aid of the CP violating processes. It is also expected that, with appropriate tuning, these processes may be sufficiently strong to generate significant $\nu-\bar{\nu}$ asymmetry or effective chemical potentials of sufficient magnitude.

Finally, before the work is summarised, the kinetic equation is reformulated in Sections 6 and 7 so that time relaxation techniques may be applied and the problem may be better adapted for possible computer solutions that would confirm or disprove the original assumptions.

2. Relativistic Kinetic Equation in the Early Universe

Let us consider a particle in the Friedmann universe, described by the Robertson–Walker metric (Weinberg 1972)

$$g_{tt} = 1\,, \tag{1}$$

* This equation has been derived before in Granek (1988) and also in Granek and McKellar (1990), but in the latter paper the oscillation term was omitted.

$$g_{rr} = \frac{R^2(t)}{1 - kr^2}, \tag{2}$$

$$g_{\theta\theta} = R^2(t)\, r^2, \tag{3}$$

$$g_{\phi\phi} = R^2(t)\, r^2 \sin^2\theta, \tag{4}$$

where $R(t)$ is the time dependent radius of curvature of the universe, k is the trichotomic constant, while r and θ are polar coordinates.

The entropy of the interacting species is given by

$$S = \frac{R^3}{T}[P + \rho]_{\text{int}}, \tag{5}$$

to within an additive constant, where T is the temperature of the interacting gas and the subscript refers to the interacting part of the system. If the universe evolves in thermal equilibrium or adiabatically, then the time derivative of (5) vanishes, i.e. $dS/dt \equiv 0$. However, this is not guaranteed if there are entropy generating processes present. Since radiation effectively remains as part of the interacting energy throughout the period of interest,* then any change in radiation temperature, unaccounted for by the expansion of the universe, is a measure of the entropy generated.

Let us now introduce a heating parameter ϕ, defined initially as the ratio of the temperature of the electromagnetic radiation T to T_N of some non-interacting species expanding adiabatically, e.g. superweakly interacting neutrinos that have already decoupled, i.e. $\phi(t) = T(t)/T_N(t)$. We also define an increasing expansion scale parameter λ as the ratio of the temperature of the non-interacting gas to some fixed reference temperature T_r, i.e. $\lambda(t) = T_r/T_N(t)$. These parameters allow the splitting of the problem into the non-adiabatic and adiabatic parts respectively. The temperature of radiation T at any time t relative to the time t_0 will then satisfy

$$\frac{T}{T_0} = \frac{\phi/\lambda}{\phi_0/\lambda_0}. \tag{6}$$

The average particle momentum p scales with the expansion parameter so that

$$p/p_0 = \lambda_0/\lambda, \tag{7}$$

and since entropy is distributed evenly among the degrees of freedom, then the fractional increase in the entropy of the radiation is a measure of fractional increase in the overall entropy of all the interacting species in the universe.

* This is true even if the universe becomes matter dominated provided radiation is still coupled to it. It is expected that at decoupling any large scale entropy generation will cease so that the adiabatic form of the kinetic equation may be used.

This may be stated as

$$\frac{S}{S_0} = \left(\frac{\lambda T}{\lambda_0 T_0}\right)^3 = (\phi/\phi_0)^3 . \tag{8}$$

This allows us to treat the case where entropy of the universe is not conserved in interactions. It may be observed here that when ϕ is a constant then the equations (6–8) have the usual properties associated with an adiabatic expansion of the universe, i.e. that λT is a constant. In other words, the rate of change of ϕ is a measure of the strength of entropy generating processes. From equations (6–8) it is possible to obtain the relationships for p, ϕ, λ, T and S, so that

$$\frac{dp}{dt} = -\frac{p}{\lambda}\frac{d\lambda}{dt} = -\frac{\dot{R}}{R}p, \tag{9}$$

$$\frac{1}{T}\frac{dT}{dt} = \frac{1}{\phi}\frac{d\phi}{dt} - \frac{1}{\lambda}\frac{d\lambda}{dt}, \tag{10}$$

and

$$\frac{1}{S}\frac{dS}{dt} = \frac{3}{\phi}\frac{d\phi}{dt}, \tag{11}$$

giving

$$\frac{1}{\lambda}\frac{d\lambda}{dt} = \frac{1}{3S}\frac{dS}{dt} - \frac{1}{T}\frac{dT}{dt}. \tag{12}$$

Let us now consider a reference frame fixed to the local fluid, so that the hydrodynamic four-velocity reduces to $U = (1, \mathbf{0})$, and where there is no external force. Then substituting in the continuity equation, as shown by e.g. deGroot *et al.* (1980), gives

$$p^\mu \partial_\mu f(x,p) + F^\mu \partial_{p\mu} f(x,p) = C[x,p] + [p^\mu \partial_\mu f(x,p)]_{\rm osc}, \tag{13}$$

where the last term is the contribution due to neutrino oscillations,

$$\begin{aligned} C[x_a, p_a] = \sum_{a'} \epsilon_{aa'} \int & D(p_{a'}, m_{a'}) D(p_b, m_b) D(p_{b'}, m_{b'}) \\ & \times [f(x_{a'}, p_{a'}) f(x_{b'}, p_{b'}) W(p_{a'}, p_{b'} | p_a, p_b) \\ & - f(x_a, p_a) f(x_b, p_b) W(p_a, p_b | p_{a'}, p_{b'})], \end{aligned} \tag{14}$$

$$\partial_{p\mu} = \frac{\partial}{\partial p^\mu}, \tag{15}$$

and

$$D(p,m) = \frac{d^4 p}{(2\pi)^4} (2\pi)\, \delta(p^2 - m^2)\, \theta(p^0), \tag{16}$$

where $\epsilon_{aa'} = \frac{1}{2}\delta_{aa'}$ guarantees that the transition rate W represents the rate for both cases of identical particles $a' = b'$ and different particles $a' \neq b'$. The vector F^{μ} is the four-vector equivalent of the external force $\mathbf{F}$, where $F_i = p^0\mathbf{F}^i$. We may average equation (14) over energy, by integrating it with respect to p^0 using

$$\int \frac{dp^0}{2\pi}(2\pi)\,\delta(p^2 - m^2)\,\theta(p^0) = \frac{1}{2E}, \tag{17}$$

where E is the energy of a particle with mass m, i.e. $E = \sqrt{\mathbf{p}^2 + m^2}$. Equation (13) then reduces to a more conventional form

$$(\partial_t + \mathbf{u}\cdot\nabla + \mathbf{F}\cdot\nabla_p)f = \int \frac{d^3\mathbf{p}_{a'}}{(2\pi)^3}\frac{d^3\mathbf{p}_{b'}}{(2\pi)^3}\frac{d^3\mathbf{p}_b}{(2\pi)^3}K(\mathbf{p}_a,\mathbf{p}_b|\mathbf{p}_{a'},\mathbf{p}_{b'}) + [(\partial_t + \mathbf{u}\cdot\nabla + \mathbf{F}\cdot\nabla_p)f]_{\rm osc}, \tag{18}$$

where

$$K(\mathbf{p}_a,\mathbf{p}_b|\mathbf{p}_{a'},\mathbf{p}_{b'}) = f_{a'}f_{b'}w(\mathbf{p}_{a'},\mathbf{p}_{b'}|\mathbf{p}_a,\mathbf{p}_b),-f_af_bw(\mathbf{p}_a,\mathbf{p}_b|\mathbf{p}_{a'},\mathbf{p}_{b'}), \tag{19}$$

$$w(\mathbf{p}_a,\mathbf{p}_b|\mathbf{p}_{a'},\mathbf{p}_{b'}) = \frac{W(\mathbf{p}_a,\mathbf{p}_b|\mathbf{p}_{a'},\mathbf{p}_{b'})}{(2E_a)(2E_b)(2E_{a'})(2E_{b'})}. \tag{20}$$

The last term of equation (18) is the oscillation term. Therefore the evaluation of the form of the full kinetic equation reduces now to the evaluation of the $w(\mathbf{p}_a,\mathbf{p}_b|\mathbf{p}_{a'},\mathbf{p}_{b'})$, the transition probability per unit volume per unit time for the processes

$$a + b \to a' + b', \tag{21}$$

and the evaluation of the oscillation term. Since the universe is assumed uniform and isotropic, the spatial derivatives vanish, and with no external force on the system the second and third terms on the left side of (18) vanish, simplifying to

$$\partial_t f(x,p) = C[f(x,p)] + [\partial_t f(x,p)]_{\rm osc}, \tag{22}$$

with the number of variables f which depends on reduced to p and t, where p is the magnitude of the three-momentum vector and t is the proper time. Also because of the time dependence of momentum, we write $p = p(p_0,t)$, where p_0 is the momentum at time $t = t_0$, so that

$$f(x,p) = f(t,p(p_0,t)). \tag{23}$$

This means that

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_p + \left(\frac{\partial f}{\partial p}\right)_t\frac{dp}{dt} = \left(\frac{\partial f}{\partial t}\right)_p - \frac{\dot{R}}{R}p\left(\frac{\partial f}{\partial p}\right)_t \tag{24}$$

[note that the second equality of (24) was used before by Dolgov (1981) in a closely related work], hence using (9–12) we obtain the expanding universe form of Boltzmann's kinetic equation

$$\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial t}\right)_p + \left(\frac{1}{T}\frac{dT}{dt} - \frac{1}{3S}\frac{dS}{dt}\right) p \left(\frac{\partial f}{\partial p}\right)_t = C[f] + \left(\frac{\partial f}{\partial t}\right)_{\rm osc} . \tag{25}$$

If the momentum of a particle p is now replaced by a dimensionless momentum parameter q, where the time dependent part of p is absorbed into the temperature so that

$$p(t) = qT(t), \tag{26}$$

then the derivatives at fixed p may be written as

$$\left(\frac{\partial f}{\partial t}\right)_p = \left(\frac{\partial f}{\partial T}\right)_{q,t} \left(\frac{dT}{dt}\right)_p + \left(\frac{\partial f}{\partial q}\right)_{T,t} \left(\frac{dq}{dt}\right)_p + \left(\frac{\partial f}{\partial t}\right)_{q,T} , \tag{27}$$

and

$$\left(\frac{\partial f}{\partial t}\right)_q = \left(\frac{\partial f}{\partial T}\right)_{q,t} \left(\frac{dT}{dt}\right)_p + \left(\frac{\partial f}{\partial t}\right)_{q,T} , \tag{28}$$

and from (26)

$$\left(\frac{dq}{dt}\right)_p = \frac{d}{dt}(p/T) = -\frac{q}{T}\frac{dT}{dt} . \tag{29}$$

Then substituting in equation (25) from equations (28) and (29) we find

$$\begin{aligned} \left(\frac{\partial f}{\partial t}\right)_p + \left(\frac{1}{T}\frac{dT}{dt} - \frac{1}{3S}\frac{dS}{dt}\right) p \left(\frac{\partial f}{\partial p}\right)_t &= \left(\frac{\partial f}{\partial t}\right)_q - \frac{q}{3S}\left(\frac{dS}{dt}\right)\left(\frac{\partial f}{\partial q}\right)_{T,t} \\ &= \left(\frac{\partial f}{\partial t}\right)_q - \frac{q}{\phi}\left(\frac{d\phi}{dt}\right)\left(\frac{\partial f}{\partial q}\right)_{T,t} \\ &= \hat{D}f , \end{aligned} \tag{30}$$

which is the differential operator we are looking for.

This term is the one that replaces now the usual time derivative in Boltzmann's equation, but the neutrino oscillation term is still to be included explicitly to form the complete equation. This then becomes

$$\left(\frac{\partial f}{\partial t}\right)_q - \frac{q}{\phi}\left(\frac{d\phi}{dt}\right)\left(\frac{\partial f}{\partial q}\right)_{T,t} = C[f] + \left(\frac{\partial f}{\partial t}\right)_{\rm osc} \tag{31}$$

Note that in the case of adiabatic expansion, ϕ is a constant so that the second term of (31) vanishes identically and the conventional Boltzmann's equation results when the oscillation term is ignored.* Also the collision

* When the right side of (31) is set to zero, the left side becomes the collision-free Boltzmann equation or a form of the Vlasov equation (see e.g. Tremaine and Lee 1987).

term vanishes if there are no collisions or if the system remains in thermal equilibrium during expansion. This means that if the function f under these circumstances is purely a function of q then its shape is time independent. Thus equation (31) allows us to determine the number densities of some particles during the kinetic evolution of the universe, without having to solve the kinetic equation for each of the species present.

Let us now look at the two terms on the right-hand side of (25) in detail.

(2a) The Collision Term

We now come back to equation (14). The expression for the transition probability W is given by

$$W(p_a,p_b|p_{a'},p_{b'}) = (2\pi)^4\delta^4(p_{a'}+p_{b'}-p_a-p_b)m_a^2m_b^2(1-f_a{}')(1-f_b{}') \times\sum_{s'}|M(p_a,p_b|p_{a'},p_{b'})|^2, \qquad (32)$$

where the factors $(1-f)$ are the fermion statistical factors of the state into which the particles a and b may scatter (in the case of bosons the sign of f in the corresponding factor must change to +) and s' is a group spin label for all the unobserved final spin states. The interaction matrix element is

$$M = \frac{G}{\sqrt{2}}[\bar{u}_{a'}\gamma_\mu(1\mp\gamma_5)\bar{u}_a][\bar{u}_{b'}\gamma_m u(1\mp\gamma_5)\bar{u}_b], \qquad (33)$$

where G is the relevant coupling constant ($G = G_L$ the Fermi constant for left-handed $V{-}A$ interactions and $G = G_R$ for the right-handed $V{+}A$ interactions), while the minus is for left-handed current and plus for the right-handed current. This reduces to

$$|M|^2 = 128G^2\frac{(p_a\cdot p_{b'})(p_{a'}\cdot p_b)}{m_a^2m_b^2}; \qquad (34)$$

note that to get the reaction $a+\bar{a}\to b+\bar{b}$ we exchange $a'\leftrightarrow b'$ getting $\bar{a}$ and $\bar{b}$, which requires the exchange $p_{a'}\leftrightarrow p_b$. This exchange leaves the expression (34) invariant (after application of the energy–momentum conservation laws), so that the scattering matrix element is the same for both processes and the rate only depends on number densities of the particles in question.

(2b) The Oscillation Term

An anti-particle (or charge conjugate) wave function, corresponding to a left-handed neutrino wave function ν_L is given by (Bjorken and Drell 1964; Kayser 1984)

$$(\nu_{\alpha L})^c \equiv C\bar{\nu}_{\alpha L}^T, \qquad (35)$$

which satisfies*

$$(\nu^c_\alpha)_L = (\nu_{\alpha R})^c, \tag{36}$$

where

$$(\nu^c_\alpha)_L = \tfrac{1}{2}(1-\gamma_5)(\nu^c_\alpha); \tag{37}$$

hence $(\nu^c_\alpha)_L$ is really a left-handed particle. The weak currents, described in the previous section, couple only to the left-handed particles or right-handed anti-particles. We may, however, also introduce the mass term to the Lagrangian as well. The various Lagrangian components corresponding to the Dirac and Majorana masses are as follows:

1. $\mathcal{L}^D_{\alpha\beta} = -m^D_{\alpha\beta}\bar{\nu}_{\alpha R}\nu_{\beta L} + H.c.$
   This is the Dirac mass coupling left-handed neutrino to the right-handed anti-neutrino, and similarly coupling right-handed neutrino to the left-handed anti-neutrino.
2. $\mathcal{L}^M_{\alpha\beta} = -m^M_{\alpha\beta}(\overline{\nu^c_\alpha})_R\nu_{\beta L} + H.c.$
   This is the Majorana mass term coupling left-handed neutrino to the left-handed anti-neutrino and coupling right-handed neutrino to the right-handed anti-neutrino.

The couplings can be combined into a single general expression, where the matrix M contains both the Dirac and Majorana terms, and all left-handed neutrinos and anti-neutrinos are grouped into a single multi-generation vector, e.g.

$$\nu_L = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \cdot \\ \cdot \\ \cdot \\ \nu^c_e \\ \nu^c_\mu \\ \nu^c_\tau \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}_L \qquad \nu_R = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \cdot \\ \cdot \\ \cdot \\ \nu^c_e \\ \nu^c_\mu \\ \nu^c_\tau \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}_R \tag{38}$$

Then the most general Lagrangian will have the form

$$\mathcal{L} = -\bar{\nu}_R M \nu_L, \tag{39}$$

where

$$M = \begin{pmatrix} M^D & M^M_L \\ M^M_R & M'^D \end{pmatrix}, \tag{40}$$

* The Dirac γ matrix convention used here is that of Bjorken and Drell (1964), where the charge conjugation operator $C = -C^{-1} = -C^T = -C^H = i\gamma^2\gamma^0$ and $\gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

and $M_R^M = (M_L^M)^*$ (Rosen 1984). The $n \times n$ mass matrices M^D and M'^D are the Dirac mass term matrices and $M_{L,R}^M$ are the Majorana mass term matrices. If in the above case $M_{L,R}^M \equiv 0$, then we have n Dirac neutrinos (i.e. neutral particles with weak interaction properties similar to that of electron). If $M^D = M'^D \equiv 0$, we have n Majorana particles, which are their own anti-particles. In the most general case there are $2n$ Majorana neutrinos, made up from $4n$ basic spinor solutions of the Dirac equation for each lepton generation in e, μ, τ, ... etc., i.e. the solutions are chosen in such a way so as to satisfy the charge conjugation requirement (35). In the above three cases only the Dirac neutrino conserves the total lepton number, since it has the same charge conjugation properties as that of charged leptons, i.e. is invariant under the global gauge transformation, while the Majorana neutrino violates this conservation requirement and is not invariant under such an operation (e.g. Kayser 1984). This means that if neutrinos are of Dirac type, then the total lepton number of the universe will remain fixed, while only the conventional interactions take place. In general, if we define the neutrino mass eigenstates ν_i and the weak eigenstates ν_α then we have

$$\nu_\alpha = \sum_{i=1}^{2n} U_{\alpha i} \nu_i , \tag{41}$$

where U is a unitary matrix, and in the case of pure Dirac neutrinos the $U_{\alpha i}$ vanish for $i > n$ if $\alpha \leq n$, or for $i \leq n$ if $\alpha > n$, while for pure Majorana neutrinos the $U_{\alpha i}$ vanish for $i \leq n$ if $\alpha \leq n$, or for $i > n$ if $\alpha > n$ (Bilenky and Petcov 1987).

It is now necessary to establish the time evolution of the weak eigenstates ν_α for an arbitrary particle phase space distribution given by $f_{\alpha\beta}(t,p)$. To do this we first consider a general density operator $\hat{\rho}(t_0)$ defining an initial neutrino state at $t = t_0$. Then at a later time t the evolution of this state operator is given by

$$\hat{\rho}(t) = e^{-i\hat{\mathcal{H}}_0(t-t_0)} \hat{\rho}(t_0) e^{i\hat{\mathcal{H}}_0(t-t_0)} , \tag{42}$$

where $\hat{\mathcal{H}}_0$ is the part-Hamiltonian including only the mass interaction and not the collision term. Also, if $\hat{\rho}$ does not depend explicitly on time then this may be expressed alternatively as

$$\frac{\partial \hat{\rho}}{\partial t} = i[\hat{\rho}, \hat{\mathcal{H}}_0] . \tag{43}$$

Let this operator now be the phase space density operator $\hat{f}$ containing particle statistics information, i.e. $\hat{\rho} = \hat{f}$. The matrix element $f_{\alpha\beta}$ is then given by $\langle \nu_\alpha | \hat{f} | \nu_\beta \rangle$. Applying this definition to the expression (43) gives

$$\frac{\partial f_{\alpha\beta}}{\partial t} = \frac{d}{dt} \langle \nu_\alpha | \hat{f} | \nu_\beta \rangle = i[\langle \nu_\alpha | \hat{f} \hat{\mathcal{H}}_0 | \nu_\beta \rangle - \langle \nu_\alpha | \hat{\mathcal{H}}_0 \hat{f} | \nu_\beta \rangle] , \tag{44}$$

and since

$$\begin{aligned} \langle \nu_\alpha | \hat{f} \hat{\mathcal{H}}_0 | \nu_\beta \rangle &= \sum_{\gamma,i,j} \langle \nu_\alpha | \hat{f} | \nu_\gamma \rangle \langle \nu_\gamma | \nu_i \rangle \langle \nu_i | \hat{\mathcal{H}}_0 | \nu_j \rangle \langle \nu_j | \nu_\beta \rangle \\ &= \sum_{\gamma,i,j} f_{\alpha\gamma} U_{\gamma i}^* \delta_{ij} E_j U_{\beta i} , \end{aligned} \tag{45}$$

because $|\nu_i\rangle$ are the energy eigenstates of the Hamiltonian $\hat{\mathcal{H}}$ and are an orthogonal set, then using (45) equation (44) may be rewritten as

$$\frac{\partial f_{\alpha\beta}}{\partial t} = i\sum_{\gamma,i} E_i(f_{\alpha\gamma}U^*_{\gamma i}U_{\beta i} - f_{\gamma\beta}U^*_{\alpha i}U_{\gamma i}). \tag{46}$$

Therefore, together with equation (30), the complete kinetic equation for neutrino interactions may be written as

$$\left(\frac{\partial \hat{f}}{\partial t}\right)_p + \left(\frac{\partial \hat{f}}{\partial p}\right)_t \left(\frac{1}{T}\frac{dT}{dt} - \frac{1}{3S}\frac{dS}{dt}\right) = C[\hat{f}] + i[\hat{f}, \hat{\mathcal{H}}_0]. \tag{47}$$

The right-hand side of (47) looks identical to that of Dolgov (1981). However, he considers the case with CP conservation where no permanent chemical potential may develop, although, if the conditions are right, an excess of neutrinos over anti-neutrinos (or vice versa) may be present at nucleosynthesis, leading to abundances of light elements different to those in the standard model.

The energy E_i in equation (46) is quite clearly defined because the $|\nu_i\rangle$ are the eigenstates of the mass matrix and have the same momentum, and consequently the usual energy momentum relationship $E_i^2 = p^2 + m_i^2$ applies, where p is the same for all particles i. From here on, the convention $f_{\alpha\alpha} \equiv f_\alpha$ will be used in both mass and weak interaction eigenstate projection.

(2c) Reduction of the Collision Term

We shall now return to the collision term in (18) which, even with explicit insertion of the scattering matrix given by equations (34) and (32), still remains a cumbersome integral:

$$I_{\text{col}} = \int \frac{d^3\mathbf{p}_{a'}}{(2\pi)^3}\frac{d^3\mathbf{p}_b}{(2\pi)^3}\frac{d^3\mathbf{p}_{b'}}{(2\pi)^3}(2\pi)^4\delta^4(p_{a'} + p_{b'} - p_a - p_b)m_a^2 m_b^2(1 - f_{a'})(1 - f_{b'})f_a f_b$$
$$\times \sum_{s'} |M(p_a, p_b | p_{a'}, p_{b'})|^2 . \tag{48}$$

This becomes after substitution for M and integration over $d^3\mathbf{p}_{a'}$

$$I_{\text{col}} = 16G^2 \int \frac{d^3\mathbf{p}_{b'} d^3\mathbf{p}_b}{(2\pi)^5}\delta(E_{a'} + E_{b'} - E_a - E_b)\frac{(p_a \cdot p_{b'})(p_{a'} \cdot p_b)}{E_{a'}E_{b'}E_a E_b}$$
$$\times f_a f_b(1 - f_{a'})(1 - f_{b'}). \tag{49}$$

This integral remains very awkward to evaluate numerically, even when the kinematic angles are defined, because of the interdependence of the integration limits, in addition to its multidimensionality. It is therefore necessary to convert the integrand into a form where the integration limits and the integrand may be expressed more simply. This may be done in the centre of mass reference frame. Since the integral (14) is a four-scalar, then equation (48) must transform like the inverse of the zeroth component of a four-vector, hence the integral (49) multiplied by E_a must be a four-scalar and is therefore invariant under Lorentz transformations. This means that the required transformation may be achieved using the Lorentz momentum transformation equations, where the variables in the centre of mass reference frame are denoted by an asterisk, and

$$\mathbf{p}_a^* + \mathbf{p}_b^* = \mathbf{p}_{a'}^* + \mathbf{p}_{b'}^* = 0\,. \tag{50}$$

The Lorenz transformation parameters β and γ are defined by

$$\beta = \mathbf{p}_{\rm tot}/E_{\rm tot} = (\mathbf{p}_a + \mathbf{p}_b)/(E_a + E_b)\,, \tag{51}$$

$$\gamma = (1 - \beta \cdot \beta)^{-1/2}\,, \tag{52}$$

$$\beta_i = \mathbf{p}_i/E_i \qquad \text{etc.} \tag{53}$$

From the definition of the Mandelstam variables s, t and u, the value of $(\mathbf{p}^*)^2$ may be calculated using

$$(\mathbf{p}^*)^2 = [s - (m_a + m_b)^2][s - (m_{a'} - m_{b'})^2]/4s\,, \tag{54}$$

$$(\mathbf{p}'^*)^2 = [s - (m_{a'} + m_{b'})^2][s - (m_{a'} - m_{b'})^2]/4s\,, \tag{55}$$

and the product

$$p_{a'} \cdot p_b = \tfrac{1}{2}(m_a^2 + m_{b'}^2 - m_{a'}^2 - m_b^2) + p_a \cdot p_{b'}\,. \tag{56}$$

In the integration over the $\mathbf{p}_b$ the phase direction of the z-axis may be chosen along the direction of $\mathbf{p}_a$ without loss of generality, so that the phase integral contributes just 2π to the total integral. Let us choose the direction of $\mathbf{p}_b$ from the z-axis (the azimuth angle) to be θ_s, so that the vector $\mathbf{p}_b$ is given by

$$\mathbf{p}_b = p_b(\sin\theta_s, 0, \cos\theta_s)\,, \tag{57}$$

and the direction of the vector $\mathbf{p}_{b'}^*$ in the centre of mass frame is given by the azimuth angle θ^* and phase ϕ^* relative to the z-axis, so that

$$\mathbf{p}_{b'}^* = p'^*(\sin\theta^* \cos\phi^*, \sin\theta^* \sin\phi^*, \cos\theta^*)\,. \tag{58}$$

Consequently the scalar product $p_a^* \cdot p_{b'}^*$ becomes

$$\begin{aligned} p_a^* \cdot p_{b'}^* &= E_a^* E_{b'}^* (1 - \beta_a^* \cdot \beta_{b'}^*) \\ &= E_a E_{b'}^* \gamma[1 - \beta \cdot \beta_a - \gamma(\beta_a - \beta) \cdot \beta_{b'}^*] \end{aligned} \tag{59}$$

now

$$\beta = (E_a\beta_a + E_b\beta_b)/(E_a + E_b)\,, \tag{60}$$

and

$$1-\beta\cdot\beta_a = E_b(1-\beta_a\cdot\beta_b)/(E_a+E_b), \tag{61}$$

$$\beta-\beta_a = E_b(\beta_b-\beta_a)/(E_a+E_b), \tag{62}$$

so that

$$\begin{aligned} p_a^*\cdot p_{b'}^* &= \frac{E_aE_bE_{b'}^*}{E_a+E_b}\gamma\{1-\beta_a\cdot\beta_b-\gamma(\beta_a-\beta_b)\cdot\beta_{b'}^*\} \\ &= \frac{E_aE_bE_{b'}^*}{E_a+E_b}\gamma\{1-\beta_a\beta_b\cos\theta_s-\gamma\beta_{b'}^*[\beta_a\cos\theta^* \\ &\quad -\beta_b(\cos\theta_s\cos\theta^*+\sin\theta_s\sin\theta^*\cos\phi^*)]\} \end{aligned} \tag{63}$$

It is now possible to reduce the integral (48) to a form with explicitly specified variables of integration

$$\begin{aligned} I_{\mathrm{col}} = \frac{16G^2}{(2\pi)^5}\int d\phi d(\cos\theta_s)p_b^2dp_bd(\cos\theta^*)d\phi^*\frac{p'^*(p_a^*\cdot p_{b'}^*)(p_{a'}^*\cdot p_b^*)}{E_aE_{a'}E_b} \\ \times f_af_b(1-f_{a'})(1-f_{b'}), \end{aligned} \tag{64}$$

where in equation (49) the last δ function removes the factor $(p'^*/E_{b'}^*)dp'^*$ from the integral. The four-vector scalar products may be expressed in the form

$$p_a^*\cdot p_{b'}^* = A+B\cos\phi^*, \tag{65}$$

and

$$p_{a'}^*\cdot p_b^* = C+B\cos\phi^*, \tag{66}$$

where from (63)

$$A=\frac{E_aE_bE_{b'}^*}{E_a+E_b}\gamma[1-\beta_a\beta_b\cos\theta_s-\beta_{b'}^*\cos\theta^*(\beta_a-\beta_b\cos\theta_s)], \tag{67}$$

$$B=\frac{E_aE_bE_{b'}^*}{E_a+E_b}\gamma\beta_b\beta_{b'}^*\sin\theta_s\sin\theta^*\cos\phi^*, \tag{68}$$

and from (56)

$$C=\tfrac{1}{2}(m_a^2+m_{b'}^2-m_{a'}^2-m_b^2)+A. \tag{69}$$

Also, since the energies of the outgoing particles a' and b' are given by

$$E_{a'}=\gamma E_{a'}^*(1-\beta\beta_{a'}^*\cos\theta^*), \tag{70}$$

$$E_{b'}=\gamma E_{b'}^*(1-\beta\beta_{b'}^*\cos\theta^*), \tag{71}$$

which are independent of the phase angle ϕ^*, then that ϕ^* integration may be done immediately, returning the expression

$$\int d\phi^* (p_a^* \cdot p_{b'}^*)(p_{a'}^* \cdot p_b^*) = 2\pi(AC + \tfrac{1}{2}B^2). \tag{72}$$

The collision integral may now be written as

$$I_{\rm col} = \frac{16G^2}{(2\pi)^3}\int_{(p_b)_{\min}}^{\infty} p_b^2 dp_b \int_{-1}^{u} d(\cos\theta_s)\int_{-1}^{1} d(\cos\theta^*)\frac{p'^*(AC+\frac{1}{2}B^2)}{E_a E_{a'} E_b}$$
$$\times f_a f_b (1-f_{a'})(1-f_{b'}), \tag{73}$$

where the integration limits $(p_b)_{\min}$ and u are the limits on the magnitude of momentum of the second incoming particle and the collision angle θ_s. Physically, these limits define the critical incoming angles for any given set of momenta p_a and $p_{b'}$, where u corresponds to the cosine of the smallest angle that the particle b's line of flight may make with the z-axis. Similarly, $(p_b)_{\min}$ gives the critical value of momentum p_b below which the collision cannot proceed. The value of u is one and that of $(p_b)_{\min}$ is zero if the sum of the masses of the incoming particles is the same as that for the outgoing ones, otherwise the values of these parameters are given by

$$u = \min\begin{cases} 1, \\ [m_a^2 + m_b^2 - (m_{a'} + m_{b'})^2 + 2E_a E_b]/2p_a p_b, \end{cases} \tag{74}$$

and

$$\{m_a^2 + m_b^2 - (m_{a'} + m_{b'})^2 + 2[E_a(E_b)_{\min} + p_a(p_b)_{\min}]\} \geq 0. \tag{75}$$

The integral (73) therefore reduces to, at most, a three-dimensional integral.

(2d) Full Neutrino Kinetic Equation without Oscillations

The integral (73) cannot, in general, be evaluated analytically. In the case of spectra of the charged particles in (73), these may be assumed to be the Fermi–Dirac distributions if they are fermions. This means that the collision integral may be written in the form

$$I_{\rm col} = \int p_b^2 dp_b \int d(\cos\theta_s)\int d(\cos\theta^*) T_r(p_a, p_b, p_{b'}, \theta_s, \theta^*) f_a f_b (1-f_{a'})(1-f_{b'})$$
$$= c_{ra}^{(-)} f_a, \tag{76}$$

where $c_{ra}^{(-)}$ is a function of all the kinematic variables and the phase spaces of the unobserved particles, r is a reaction type label and

$$T_r = \frac{16G^2}{(2pi)^3}\frac{p_b^2 p_{b'}}{E_a E_{a'} E_b}\left(AC + \tfrac{1}{2}B^2\right). \tag{77}$$

The term (76) is the rate integral for incoming particles a and b. If the particles a and b are outgoing, then the integral is very similar, except that the labels on the spectral functions are changed, so that the total collision term for the process $a+b \to a'+b'$ is given by

$$C_r[f_a] = \int p_b^2 dp_b \int d(\cos\theta_s) \int d(\cos\theta^*) T_r(p_a, p_b, p_{b'}, \theta_s, \theta^*)$$
$$\times [f_{a'} f_{b'}(1-f_a)(1-f_b) - f_a f_b (1-f_{a'})(1-f_{b'})]$$
$$= c_{ra}^{(+)}(1-f_a) - c_{ra}^{(-)} f_a \,. \tag{78}$$

Note that collision integral coefficients $c_{ra}^{\pm}$ in addition to kinematic factors also depend on the temperature T and the phase space density functions f of the particles involved in the collision.

The quantity of interest, as far as the kinetic equation is concerned, is the total reaction rate for all the processes involving particle a, including annihilations and pair production. This is obtained by summing over all the possible reactions, e.g.

$$a+b \to a+b\,, \tag{79}$$

$$a+\bar{a} \to b+\bar{b}\,, \tag{80}$$

$$a+\bar{a}' \to b+\bar{b}'\,, \tag{81}$$

where (79) is a usual scattering reaction, (80) is an annihilation–pair production reaction, and (81) is a reaction which changes weak flavour of the type $\nu_e + e^+ \to \nu_\mu + \mu^+$ or $\nu_e + \bar{\nu}_\mu \to e^\pm + \mu^\mp$ etc. Therefore we get

$$C[f_a] = \sum_r C_r[f_a]\,. \tag{82}$$

In the case of (79) the summation is over all the particle species present; in reactions (80) the summation is over particle–antiparticle pairs, while in (81) the summation is over all the available charged current reactions.

From equation (78) it is quite clear that the collision term is a non-linear function of the spectral particle distributions functions f_a and may be written as

$$C[f_a] = c_a^{(+)}(1-f_a) - c_a^{(-)} f_a\,, \tag{83}$$

where

$$c_a^{(\pm)} = \sum_r c_{ra}^{(\pm)} \tag{84}$$

are functions of the f_a, so that (83) is in fact non-linear in the f_a. The $c_a^{(+)}$ are the rates of creation of particle type a in vacuum, while $c_a^{(-)}$ are the destruction rates from an occupied state. Therefore equation (31), without oscillations may be written as

$$Df_{ab} = \delta_{ab}[c_a^{(+)}(1-f_a(q)) - c_a^{(-)} f_a(q)]\,. \tag{85}$$

(2e) Explicit Form for the Full Kinetic Equation

It is now possible to combine equations (46) and (85) to form the complete expression that needs to be solved in order to investigate the effects of neutrino oscillation in detail during the expansion of the universe. The expression is

$$\begin{aligned} Df_{\alpha\beta} &= i\sum_{\gamma,k} E_k(f_{\alpha\gamma}U_{\gamma k}U^*_{\beta k} - f_{\gamma\beta}U_{\alpha k}U^*_{\gamma k}) \\ &\quad + \delta_{\alpha\beta}[c^{(+)}_{\alpha}(1-f_{\alpha}(q)) - c^{(-)}_{\alpha}f_{\alpha}(q)], \end{aligned} \tag{86}$$

For purposes of computation it may be assumed that $E_k \gg m_k$ to give the approximation

$$E_k \approx p + m_k^2/2p \tag{87}$$

and, using the unitarity relation for the matrix $U_{\alpha\beta}$, (86) may be reduced to

$$\begin{aligned} Df_{\alpha\beta} &\approx \frac{i}{2p}\sum_{\gamma,k} m_k^2(f_{\alpha\gamma}U^*_{\gamma k}U_{\beta k} - f^*_{\gamma\beta}U_{\alpha k}U_{\gamma k}) \\ &\quad + \delta_{\alpha\beta}[c^{(+)}_{\alpha}(1-f_{\alpha}(q)) - c^{(-)}_{\alpha}f_{\alpha}(q)]. \end{aligned} \tag{88}$$

The general case of equation (86), when there are only two weak flavours, leads to a 4×4 mass coupling matrix from equations (39) and (40), and therefore 16 spectral functions $f_{\alpha\beta}$ describe this system. Consequently, the problem as defined above, is highly computation intensive, as each function $f_{\alpha\beta}$ is a function of the scaled momentum q.

It must be noted here that the collision term only involves the usual neutrino eigenstates, and is either absent from the left-handed anti-neutrino and right-handed neutrino states, or the coupling constant G is much weaker for those collisions, so these terms *practically* vanish at the time of interest. In the general case with two generations, only the first two equations of the system (86) will include the collision term, while in the case with n generations, there are $2n$ mass eigenstates and only the first n of the equations (86) include the collision term.

3. Time Scales

The combined kinetic equation (86) has three fundamental time scales associated with it. These are the shortest time scale for the oscillations, the mean interval between collisions and the time scale of the expansion of the universe, or the age of the universe. This last factor is of principal importance in establishing the scenario since if the expansion of the universe is too fast, then there is no time for any temporary equilibrium to occur. The approximate values and relationships to the age of the universe for these time scales are given below (and in McKellar and Granek 1982).

(3a) Age of the Universe

The expansion rate of the universe is given by the equation for the Hubble parameter

$$H = \frac{1}{R}\frac{dR}{dt} = \sqrt{\frac{8\pi G_N}{3}\rho - k/R^2}\,, \tag{89}$$

so that the time scale t_U is effectively given by H^{-1}. Since the universe at the time of interest is assumed to be very hot and radiation dominated, then

$$t_U \sim \tau_e \sim \frac{1}{\sqrt{G_N}T^2}\,, \tag{90}$$

where t_U is the age of the universe and τ_e is the expansion scale of the universe.

(3b) The Oscillation Time

As may be easily shown, the shortest time scale for the oscillations is given by the equation

$$\tau_o \sim \frac{T}{\Delta m_*^2}\,, \tag{91}$$

where

$$\Delta m_*^2 = \max_{i>j} |m_i^2 - m_j^2|\,, \tag{92}$$

and this may be written in terms of t_U

$$\tau_o \sim (\Delta m_*^2 t_U^{1/2} G_N^{1/4})^{-1}\,. \tag{93}$$

(3c) The Collision Time

It may be noted from equations (77) and (78) that to the leading order, the collision rate is proportional to T^5 with the coupling strength $g^2 = G_{L,R}^2$ and consequently, with the aid of equation (90), the mean collision time τ_c may be written as

$$\tau_c \sim g^{-2}\left(\sqrt{G_N}t_U\right)^{5/2}\,, \tag{94}$$

where g is the total number of helicity states involved in collisions, counting 1 per ultra-relativistic boson and 7/8 for ultra-relativistic fermion, while the contribution due to non-relativistic particles is reduced appropriately to account for their smaller numbers.

(3d) Rate Comparison

As was mentioned earlier, the time scales for the neutrino oscillations and the collision rate must be shorter than the age of the universe, i.e. τ_c, $\tau_o < \tau_U$

for these effects to play a significant role in the physical situation. Further it may be noted that while the mean time between collisions increases with time, the oscillation period is reduced, or in other words, the oscillations speed up. This means that if the expansion of the universe is not too fast, the neutrinos will be initially in their weak Hamiltonian eigenstate until the oscillations begin to take effect. Furthermore, if CP violations are to occur and the chemical potential asymmetry is to develop, it is essential that throughout most of the period preceding the decoupling of the left-handed neutrinos shortly before nucleosynthesis, the collision rate is fast and only some oscillation modes are *faster*, as will be shown shortly. The large collision rate is necessary to ensure that any deviations from equilibrium due to neutrino oscillations are quickly thermalised.

From the the above argument and using (89) and (93) we may evaluate the lower limit on Δm_*^2, which satisfies the requirement that $\tau_c/\tau_o \geq 1$. This constraint on the left-handed neutrino species turns out to be $\Delta m_*^2 \geq 10^{-9}$ eV, which is well below the experimental limits to date (a number of experimental groups have undertaken to examine the possibility of neutrino oscillations and a range of results was reported at the Fourth Moriond meeting [Lanceri (CHARM collaboration) 1984; Wotschak 1984; Schreckenbach 1984; Conforto 1984; Thenard 1984; Greenwood 1986; Durkin *et al.* 1988]. All the reports, when put together, restrict neutrino oscillations to either very small mixing angles or to $\Delta m^2 \leq 0 \cdot 2$ eV^2, while Vanucci (1984) reduces this constraint to $\Delta m^2 \leq 0 \cdot 01$ eV^2. This is probably also an appropriate place to mention some recent results concerning the number of weakly interacting species. Some recent laboratory experiments reported by the L3 (Adeva *et al.* 1989), ALEPH (Decamp *et al.* 1989), OPAL (Akrawy *et al.* 1989) and DELPHI (Aarnio *et al.* 1989) collaborations (Close 1989; Dydak 1991; Carter 1992) constrain the number of weak neutrino flavours with light masses to $2 \cdot 99 \pm 0 \cdot 05$.

In order that the transition probabilities

$$\sum_{\alpha} P(\nu_\alpha; 0 \to \nu_\beta; t) \neq \sum_{\bar{\alpha}} P(\nu_{\bar{\alpha}}; 0 \to \nu_{\bar{\beta}}; t) \neq 1 \tag{95}$$

may be satisfied, where the sums are taken over the particles present rather that all possible interactions, at least one, but not all, of the Δm_{ij}^2, must satisfy the requirement $\tau_o < \tau_U$ before the neutrino weak interactions decouple.

Let us now consider the right-handed neutrino components. These may (but this is not essential for the model) interact superweakly with the coupling constant $G_R \ll G_L$, then they will have decoupled from the rest of the universe at some early stage at temperature T_R^* and from then on they expand with temperature T_R. Then as the universe cools and expands adiabatically, as in the conventional model, the various species annihilate and convert to photons, the weak neutrino flavours and $e^\pm$. This process heats up this coupled gas to the temperature T_L such that

$$\frac{T_R}{T_L} = \left(\frac{g_B - g_A}{g_B}\right)^{1/3}, \tag{96}$$

where g_B is the number of weakly interacting spin states just after the decoupling of the right-handed interactions and before annihilations, while

g_A is the number of the spin states that have annihilated by the time the oscillations become significant, where we count 1 for each boson state and 7/8 for each fermion state. We also assume that none of the spin states, other than the right-handed neutrinos, decouple before practically complete annihilation.

The fraction (96) need not be very small (e.g. a value of 0·1 means that in excess of 5000 spin states have annihilated), but since the temperature enters the number densities as T^3, this means that $n_{\nu_L} \gg n_{\nu_R}$ before the commencement of oscillations.

(3e) CP Non-conservation

The transition probability $P(\alpha \to \beta)$ may be derived from equation (41) (e.g. Bilenky and Petcov 1987) using

$$\begin{aligned} P(\alpha \to \beta) &= a^*(\alpha \to \beta) a(\alpha \to \beta) \\ &= \sum_{k,l} |U^*_{\alpha k} U_{\beta k} U^*_{\beta l} U_{\alpha l}| \cos\left(\frac{\Delta m^2_{kl}}{2p} t - \phi_{kl;\alpha\beta}\right), \end{aligned} \tag{97}$$

while

$$P(\bar{\alpha} \to \bar{\beta}) = \sum_{k,l} |U^*_{\bar{\alpha} k} U_{\bar{\beta} k} U^*_{\bar{\beta} l} U_{\bar{\alpha} l}| \cos\left(\frac{\Delta m^2_{kl}}{2p} t + \phi_{kl;\bar{\alpha}\beta},\right), \tag{98}$$

where

$$\phi_{kl;\alpha\beta} = \arg(U^*_{\alpha k} U_{\beta k} U^*_{\beta l} U_{\alpha l}), \tag{99}$$

and the imaginary part may be ignored since it must cancel for P to be real.

It may be noted firstly that the phase $\phi_{kl;\alpha\beta}$ occurs with an opposite sign in (97) and (98) and consequently it is *not* essential that $P(\alpha \to \beta) = P(\bar{\alpha} \to \bar{\beta})$, provided not all ϕ_{kl} and Δm^2_{kl} vanish. It has been shown by several authors (e.g. Kobayashi and Maskawa 1973; Cabbibo 1978) that this effect, which violates CP conservation, requires an additional constraint of at least three non-degenerate mass eigenstates entering the mixing scheme (41), and under such circumstances maximal CP violations *may* occur.

4. Lepton Number Non-conservation during Expansion

In view of the possibly unequal transition probabilities between the transitions $\nu_\alpha \to \nu_\beta$ and $\bar{\nu}_\alpha \to \bar{\nu}_\beta$, it is possible to envisage a scenario where the net lepton number $L = n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}$ for species α is not conserved.

The evolution of the universe is thought to occur as follows. Initially, when the oscillation length of the so-called *sterile* neutrinos becomes shorter than the size of the universe, they freely oscillate into the weakly interacting ones because their typical energy is much lower, and the weakly interacting neutrino oscillations are suppressed by the collisions with other weakly interacting particles. These collisions lead to regeneration of the weak eigenstates, while the subsequent thermalisation results in a very slight increase in the temperature of the universe (Khlopov and Petcov 1981, seem to have changed their mind about this stage of the evolution in their errata). This leads to almost complete extinction of the sterile component, but eventually the temperature of the weakly interacting neutrinos drops sufficiently low so that the neutrinos at the low energy end of the spectrum begin to oscillate faster than they collide and turn back into the sterile component, i.e. they *leak* out of the thermal distribution at an increasing rate as the universe cools. This occurs when the temperature of the universe drops down to, say, below 100 MeV, so that when the oscillations of the weakly interacting neutrinos commence, the left-handed neutrinos will initially oscillate into the *sterile* components at a different rate to the right-handed anti-neutrinos. At the same time inelastic collisions will attempt to thermalise the weak eigenstates by generating more $\nu\bar{\nu}$ pairs, effectively cooling the photon–electron gas and accelerating the oscillations. This process is expected to continue until the number of *sterile* neutrinos has increased sufficiently, so that the reverse oscillations just cancel the oscillations from the weak eigenstates. The situation now is one where the weakly interacting neutrinos have not decoupled from the rest of the universe, but the oscillations are much faster than the collisions.

The scenario described here is that of non-equilibrium evolution, that is, while the neutrino oscillations are developing, the neutrino gas exists mainly in the form of weakly interacting eigenstates. This situation is rapidly rectified as the oscillations develop and speed up as a result of (93) until the system attains new equilibrium form, where number density operator $\hat{f}$ is diagonal with respect to the Hamiltonian $\hat{\mathcal{H}}_0$, i.e.

$$[\hat{f}, \hat{\mathcal{H}}_0] = \hat{0}\,. \tag{100}$$

This eliminates the oscillation term from the kinetic equation (see e.g. 47). The collision term will enforce an equilibrium distribution for the system through collisions. Consequently, the system (86) will be characterised at that stage by mass eigenstate distributions given by the Fermi–Dirac distribution

$$f_{ij} = \delta_{ij}(1 + \exp(E_i/T - \xi_i))^{-1}\,, \tag{101}$$

with ξ_i being the chemical potentials for the various mass eigenstates, not all equal to zero and, in general, different to each other.

It must be noted here that what we call equilibrium before the commencement of the oscillations is *not* the same equilibrium as that attained through oscillations, because effectively, a new type of interaction that ultimately dominates the neutrino interactions, is involved.

5. Mass Eigenstate Representation of Boltzmann's Equation

The kinetic equation (86) or the approximation (88) form a set in the weak interaction basis. As oscillations become the dominant process, however, it becomes very important to know the form of the distribution function in the mass Hamiltonian basis, particularly since in complete thermal and oscillation equilibrium it is the mass eigenstates that acquire the Fermi–Dirac type particle distribution.

The transformation of (88) is done with the aid of the unitary matrix U, so that

$$f_{lm} = \sum_{\alpha,\beta} U^*_{\alpha l} U_{\beta m} f_{\alpha\beta}, \tag{102}$$

and then (88) becomes

$$Df_{lm} = -i\frac{\Delta m^2_{lm}}{2p} f_{lm} + \sum_{\alpha} U^*_{\alpha l} U_{\beta m} c_\alpha (f^0_{\alpha\beta} - f_{\alpha\beta})\delta_{\alpha\beta}, \tag{103}$$

where

$$c_\alpha = c_\alpha^{(+)} + c_\alpha^{(-)}, \tag{104}$$

and

$$f^0_{\alpha\beta} = \frac{\delta_{\alpha\beta}}{1 + c_\alpha^{(-)}/c_\alpha^{(+)}} \tag{105}$$

gives the equilibrium distribution function of the weak eigenstate basis at any given time t. It may also be shown that the collision term is responsible for *regeneration* similar to that of the K meson (Okun 1982, chapter 11), where c_α is equivalent to the scattering amplitude. The transition amplitude is given by the off-diagonal coefficients of the f_{lm}. This could also be made valid for neutrino oscillations in matter, of the MSW type, discussed by (Wolfenstein 1979; Mikheyev and Smirnov 1985; Rosen 1986; Bahcall and Haxton 1989 etc.).

The equation (103) may now be rewritten in another form:

$$\hat{D}\hat{f} + \frac{i}{2p}\hat{M}_\Delta \hat{f} + \hat{T}\hat{f} = \hat{Z}, \tag{106}$$

where $\hat{D}$ is a differential operator, as before, $\hat{f}$ may now be thought of as a vector whose each element has the original matrix f labels lm, $\hat{M}_\Delta$ is a diagonal matrix in the mass eigenstate basis with diagonal elements Δm^2_{lm} and $\hat{T}$ is the hermitean collision matrix operator with elements

$$T_{lm;ij} = \sum_\alpha U_{\alpha l} U^*_{\alpha m} U_{\alpha i} U^*_{\alpha j} c_\alpha = T_{ji;ml}, \tag{107}$$

where ij and lm make up one index each, while

$$\hat{Z} = \hat{T}\hat{f}^0_W, \tag{108}$$

and $\hat{f}^0_W$ is the operator that represents the instantaneous equilibrium tendency of the system in the absence of oscillations.

Since the operator $\hat{D}$ here is just an ordinary time differential operator, then equation (106) may be immediately written in an integral form

$$\hat{f} = \left(\int_{t_0}^{t} dt' \hat{Z}(t') \exp\left\{\int_{t_0}^{t'} dt'' [i\hat{M}_\Delta/2p + \hat{T}(t'')]\right\}\right)$$
$$\times \exp\left\{-\int_{t_0}^{t} dt' [i\hat{M}_\Delta/2p + \hat{T}(t)]\right\}. \tag{109}$$

It is also easy to show t:

$$(\hat{M}_\Delta)^H_{lm;ij} = (\hat{M}_\Delta)_{ji;ml} = -(\hat{M}_\Delta)_{lm;ij}\,, \tag{110}$$

because the matrix is both symmetric and $\Delta m^2_{lm} = -\Delta m^2_{ml}$, so that the operators inside the square brackets in equation (109) may be diagonalised to give

$$i\hat{M}_\Delta/2p + \hat{T} = \hat{V}\hat{\Lambda}\hat{V}^H = \hat{\mathcal{H}}_i\,, \tag{111}$$

where $\hat{\Lambda}$ is a diagonal matrix operator (with N^2 non-zero elements), while $\hat{V}$ is a $N^2 \times N^2$ unitary matrix. The equation (111) represents the neutrino interaction Hamiltonian H_i, with $\hat{Z}$ representing a forcing term due to interactions with other particles in the system. The instantaneous neutrino state vector is given by

$$\hat{h} = \hat{V}\hat{f}_M\,, \tag{112}$$

and this shows that in the absence of collisions $\hat{h} \equiv \hat{f}_M$, while if the matrix $\hat{M}_\Delta \equiv \hat{0}$, then $\hat{V} \equiv \hat{U}$ and so $\hat{h} \equiv \hat{f}_W$, the weak interaction eigenstate. It may also be shown, by expanding equation (109) with respect to time, that

$$\hat{f}_M(t+\Delta t) = \hat{\mathcal{H}}_i^{-1}\hat{Z} + e^{-\hat{\mathcal{H}}_i\Delta t}(\hat{f}_M(t) - \hat{\mathcal{H}}_i^{-1}\hat{Z}) + O(\Delta t)^2\,, \tag{113}$$

so that when collision rates are fast, the computation of the phase space distribution functions effectively consists of computing the total interaction Hamiltonian consisting of $\hat{\mathcal{H}}_i$ and $\hat{Z}$. Note that equation (113) may be rewritten in a form resembling the relaxation time approximation to the Boltzmann equation (see e.g. Reif 1965, chapter 13), if the equilibrium phase space distribution operator is defined as

$$\hat{f}^0_M \equiv \hat{\mathcal{H}}_i^{-1}\hat{Z}\,, \tag{114}$$

and an operator $\hat{\tau}^{-1}$ as

$$\hat{\tau}^{-1} \equiv \hat{\mathcal{H}}_i\,, \tag{115}$$

resulting in the usual expression

$$\frac{d\hat{f}_M}{dt} \approx -\hat{\tau}^{-1}(\hat{f}_M - \hat{f}^0_M)\,. \tag{116}$$

Now with the aid of equation (108) we find that the operator $\hat{\mathcal{H}}_i^{-1}\hat{T}$ transforms the equilibrium tendency operator in the absence of oscillations to the one in the case where neutrino oscillations are present, i.e.

$$\hat{f}^0_M = \hat{\mathcal{H}}_i^{-1}\hat{T}\hat{f}^0_W\,. \tag{117}$$

6. Simplified Version of Boltzmann's Equation

Let us now return to the kinetic equation (86) to try to simplify it somewhat. Even if some of the assumptions that are made are not entirely true, it is hoped that at least the leading order results may be obtained from such a system.

The collision term in (86) may be written in the form resembling a relaxation equation

$$C[f_\alpha] = c_\alpha^{(+)}(1 - f_\alpha) - c_\alpha^{(-)} f_\alpha = (f_\alpha^0 - f_\alpha)/\tau_c, \tag{118}$$

where $(\tau_c)_\alpha = c_\alpha^{-1}$ is defined by equation (104), $f_\alpha = f_{\alpha\alpha}$ and $f_\alpha^0 = f_{\alpha\alpha}^0$ so that if this system tends to equilibrium, the expression (118) must tend to zero, where $f_\alpha^0 = f_\alpha^0(q,t)$ with the collision rate being given by $\tau_c = (\tau_c)_\alpha(q,t)$. The equation (118) may be simplified for the purpose of computation by assuming that $(\tau_c)_\alpha$ is a constant (at least with respect to q), and also by making some assumption about the form of f_α^0, e.g. by saying that it has some sort of average behaviour of the collision integrals $c_\alpha^{(\pm)}$ given in Sections 2c and 2d:

$$\begin{aligned} c_{r\alpha}^{(+)} &= \langle (1 - f_\beta) f_{\alpha'} f_{\beta'} \sigma(\alpha'\beta' \to \alpha\beta) \rangle \\ &\approx n_{\alpha'} n_{\beta'} \bar{\sigma}(\alpha'\beta' \to \alpha\beta; \hat{\mathcal{H}}) e^{(Q_r - \hat{\mathcal{H}})/T}, \end{aligned} \tag{119}$$

where n_α are the particle number densities, $\bar{\sigma}$ is some average cross section for the process $\alpha'\beta' \to \alpha\beta$, r is its label and $\hat{\mathcal{H}}$ is the Hamiltonian for this process. The remaining factor is the Boltzmann factor for creation of the particle α in the reaction r with a given value of $Q = Q_r$ and temperature T. The other term may be written in a similar manner:

$$\begin{aligned} c_{r\alpha}^{(-)} &= \langle (1 - f_{\alpha'})(1 - f_{\beta'}) f_\beta \sigma(\alpha\beta \to \alpha'\beta') \rangle \\ &\approx n_\beta \bar{\sigma}(\alpha\beta \to \alpha'\beta'; \hat{\mathcal{H}}) e^{-Q_r/T} \end{aligned} \tag{120}$$

and the total rates $c_\alpha^{(+)}$ and $c_\alpha^{(-)}$ are obtained by summing over all possible reactions r. At this stage $\hat{T}$ and $\hat{f}_W^0$ may be evaluated using equations (105) and (106), so that $\hat{\mathcal{H}}_i$ and $\hat{Z}$ may be evaluated at each time step with the aid of equations (108) and (110). Given that the mixing matrix U (equation 41) and the matrix M_Δ are defined, then the iteration, as defined by equation (113) may be computed resulting in the phase space density functions at the new time $t + \Delta t$. This procedure can then be repeated until all reactions of interest decouple.

7. Summary and Discussion

We have dealt here with the possibility of generation of a significant neutrino chemical potential with the aid of neutrino oscillations. The scenario requires neutrinos to have non-zero masses and also that these masses not all be degenerate, but the differences squared between some of them must be $\geq 10^{-9}$ eV. In addition to the usual weakly interacting species, the scheme also requires superweakly interacting neutrinos which decouple from the rest of the universe at a very early time, well before the weakly interacting species begin to oscillate. This leads to the numbers of such neutrino species being very low, much lower than the number of weakly interacting species because of the temperature difference. When an appropriate mass matrix is chosen and mass eigenstates satisfy the above requirements, then the rates of oscillation of the left-handed neutrino species will be different from the rates for the anti-neutrino counterparts. At the same time the weak interactions attempt to compensate for the oscillations by production or annihilation of $\nu\bar{\nu}$ pairs. These interactions also shift the neutrino distributions in momentum space. These CP violating processes will continue until the reaction rates in both directions are in balance, but the excess will not be destroyed, since the system attains a new form of equilibrium that includes the oscillations. A recent argument (Enqvist *et al.* 1990) suggesting that if $B-L$ is conserved [where $B(L)$ is the total baryon (lepton) number of the universe], then large neutrino asymmetry cannot develop is true in this case as well, as they assume only two neutrino species (see Section 3*e*).

In Section 2 mathematical formalism for the kinetic interactions was introduced, based on the electro-weak theory and the theory of neutrino oscillations in vacuum. These were combined into a general expression for the coupled sets of kinetic equations in the weak interaction space. Equation (47) represents the kinetic equation for oscillating neutrinos interacting via collisions with other matter in the expanding universe, while equation (46) gives a low mass approximation for the oscillation term.

The model theory has been developed to the stage where either the suggested collision term approximation may be used to estimate the effect of oscillations on the neutrino chemical potential, or an alternative technique for quick and efficient evaluation of the collision term must be found. One potential deficiency of the collision term approximation suggested in Section 6 is that the flow of particles in momentum space is not well accounted for, and consequently that approximation is unlikely to be adequate on its own. In other words, a more satisfactory approximation will require higher momentum derivatives of f, probably up to at least the second order, since the processes here are similar to some of those discussed elsewhere (Granek 1988; Granek and McKellar 1990). Details of such an expansion are beyond the scope of the present paper; however, we would like to suggest that if the system is considered close to equilibrium for most of its evolution time, then modifications to the linear theories, as discussed by deGroot *et al.* (1980), could lead to a form more reasonable from the computational point of view. The computation itself needs to be done to fully test this model, but we shall not attempt it for now.

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