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The Sigma Term: Leading Non-analytic Behaviour of Decuplet Contributions

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Abstract

We examine the analytic structure, in powers of the quark mass ($\hat{m}$), of the chiral loop corrections to the pion–nucleon sigma term arising from coupling to the lowest mass baryon decuplet. The leading non-analytic term is found to go like $\hat{m}^2 \ln(\hat{m}/2a)$ (with a the delta–nucleon mass splitting), and to be quite significant numerically.

1. Introduction

For more than a decade, one of the central problems in relating QCD to observables in intermediate energy physics has been the outstanding discrepancy in the pion–nucleon sigma term. This quantity is of fundamental theoretical importance because it is a direct measure of the chiral symmetry-breaking terms in QCD:

$$\sigma = \tfrac{1}{3}\langle N|[Q_{i5},[Q_{i5},H_{QCD}]]|N\rangle\,. \tag{1}$$

A naive quark model estimate yields σ [$\sigma(t=0)$] of order 26 MeV (Li and Pagels 1971), but this increases to about 35 MeV when the effect of the leading chiral correction is taken into account (Gasser 1981). This leading correction, which is associated with a nucleon–pion (N–π) loop, introduces terms that are formally non-analytic in the small (non-strange) quark masses—in particular terms like $\hat{m}^{\frac{3}{2}}$ and $\hat{m}^2 \ln \hat{m}$.

For a long time the experimentally deduced value (at a momentum transfer $t=+2m_\pi^2$), $\Sigma(2m_\pi^2)$, has been around 65 MeV (Höhler *et al.* 1979). This was thought to reduce to about 60 MeV at $t=0$ (Gasser 1981), which is a long way from the theoretical value of 35 MeV quoted above. However, it has been recently been shown that the scalar, isoscalar form factor is much softer than previously thought (primarily because of the nearby Δ resonance) and a more accurate 'experimental' value would be $\Sigma(0) \simeq 45 \pm 6$ MeV (Gasser *et al.* 1991*a*, 1991*b*; Speth *et al.* 1992). While this clearly reduces the discrepancy with the theoretical value quoted above (namely 35 MeV), there is still a sizable difference which merits investigation.

One of the major findings of the chiral quark models, and particularly the cloudy bag model (CBM) (Théberge *et al.* 1980*a*, 1980*b*, 1982; Thomas 1984) has been the important role played by the Δ resonance in determining nucleon

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properties. For example, its inclusion was crucial in establishing the convergence of the perturbative treatment of pionic corrections (Dodd *et al.* 1981) and in calculating the nucleon and octet magnetic moments (Théberge and Thomas 1983). It was therefore natural to examine the role of Δ–π loops in the σ term, and Jameson *et al.* (1992) showed that they give an extra 7–12 MeV (depending on the bag size). This essentially removes the residual discrepancy between theory and experiment and obviates the need for an undesirably large strange quark component in the nucleon. (While the emphasis here is on the Δ–π contribution, the explicit calculation included *all* octet pseudo-Goldstone boson–decuplet baryon configurations. However, the K and η loops each contributed less than 1 MeV.) Very recently, Jenkins and Manohar (1991) have also stressed the importance of the decuplet in chiral perturbation theory.

The difference between the theoretical estimate of Gasser and Leutwyler (1982) (35 MeV) and Jameson *et al.* (1992) (37 to 47 MeV) raises an interesting question. The former authors have argued that the decuplet contributions to the σ term are not leading non-analytic contributions (LNAC) and therefore should already be included in their phenomenological treatment. In this work we calculate the detailed quark-mass dependence of the Δ–π correction. Our finding is that its LNAC piece goes like $\hat{m}^2 \ln(\hat{m}/2a)$ (where $a = M_\Delta - M_N$) and is numerically quite significant. This is inconsistent with the argument of Gasser and Leutwyler (1982) and suggests further work.

2. The Sigma Term

Within the CBM the pion loop correction to the σ term is (Jameson *et al.* 1992)

$$\delta\sigma = \frac{m_\pi^2}{2} \int d^3 x \, \langle N | \underline{\phi}^2 | N \rangle \tag{2}$$

$$= \frac{m_\pi^2}{3\pi\mu^2} \int \frac{dk}{\omega_\pi(k)^2} \, k^4 u^2(kR) \left\{ \frac{9 f_{\pi NN}^2}{\omega_\pi^2(k)} + \frac{2 f_{\pi N\Delta}^2}{[M_\Delta - M_N + \omega_\pi(k)]^2} + \frac{2 f_{\pi N\Delta}^2}{\omega_\pi(k)[M_\Delta - M_N + \omega_\pi(k)]} \right\}. \tag{3}$$

Here $\underline{\phi}$ is the pion field operator, $\omega_\pi(k) = \omega_\pi$ is the energy of a pion of 3-momentum $\boldsymbol{k}$, $f_{\pi NB}/\mu$ (with μ fixed at the physical pion mass) are the renormalised coupling constants ($f_{\pi NN}^2 = 0{\cdot}079$ and $f_{\pi N\Delta}^2 = 72/25 f_{\pi NN}^2$), and $u(kR) = 3\, j_1(kR)/kR$ is the natural high-momentum cut-off obtained in the CBM.

The next step in our calculation is to examine the analytic structure, in particular the leading non-analytic terms, of $\delta\sigma$. This information is contained in both the upper and lower limits of the k integration of (3). Instead of using $u(kR)$ for the high-momentum cut-off, we shall for simplicity replace $u(kR)$ with 1 and introduce an ultraviolet cut-off parameter Λ for the upper limit of the integrals. While this does not alter the leading behaviour in powers of $\hat{m}$ it does make it easier to recognise it. By use of *Mathematica* (Wolfram 1988) the integrals of the relevant terms are evaluated and their behaviour is examined. For clarity, we have kept the results from the upper and lower limits of the k

integration separate, so that we can deduce the origin of the non-analytic terms. A series expansion, first in large $\Lambda \gg m_\pi$ and then in small m_π, is carried out.

Rewriting $\delta\sigma$ in terms of the contributions from N–π and Δ–π loops,

$$\delta\sigma = \delta\sigma_N + \delta\sigma_\Delta \,, \tag{4}$$

one finds that the nucleon contribution is

$$\begin{aligned} \delta\sigma_N &= \frac{3m_\pi^2}{\pi} \frac{f_{\pi NN}^2}{\mu^2} \int_0^\Lambda dk \, \frac{k^4}{\omega_\pi^4} \\ &= \frac{3m_\pi^2}{\pi} \frac{f_{\pi NN}^2}{\mu^2} \left(k + \frac{k\, m_\pi^2}{2(k^2+m_\pi^2)} - \tfrac{3}{2}\, m_\pi \arctan\left(\frac{k}{m_\pi}\right)\right)\Bigg|_0^\Lambda \\ &= \frac{3}{\pi} \frac{f_{\pi NN}^2}{\mu^2} \left(m_\pi^2\,\Lambda - \tfrac{3}{4}\,\pi\, m_\pi^3 + \frac{2m_\pi^4}{\Lambda} - \frac{m_\pi^6}{\Lambda^3} + O\left(\frac{1}{\Lambda^4}\right)\right). \end{aligned} \tag{5}$$

For the Δ–π contribution we get

$$\begin{aligned} \delta\sigma_\Delta &= \frac{2m_\pi^2}{3\pi} \frac{f_{\pi N\Delta}^2}{\mu^2} \int_0^\Lambda dk \, \frac{k^4}{\omega_\pi^2 (a+\omega_\pi)^2} + \frac{k^4}{\omega_\pi^3 (a+\omega_\pi)} \\ &= \frac{2m_\pi^2}{3\pi} \frac{f_{\pi N\Delta}^2}{\mu^2} \left(2\,k - \frac{m_\pi^4}{a(\omega_\pi^2 + k\,\omega_\pi)} - \frac{(a^2 - m_\pi^2)(a\,k + m_\pi^2 + a\,\omega_\pi)}{a[a\,k + \omega_\pi^2 + (a+k)\omega_\pi]} \right. \\ &\quad \left. - 3a\ln(k+\omega_\pi) + \frac{3\sqrt{a^2 - m_\pi^2}}{2} \ln\left[\left(\frac{a + k - \sqrt{a^2 - m_\pi^2} + \omega_\pi}{a + k + \sqrt{a^2 - m_\pi^2} + \omega_\pi}\right)^2\right]\right)\Bigg|_0^\Lambda . \end{aligned} \tag{6}$$

[Note that in (5) and (6) we have omitted non-analytic terms in the chiral expansion of the pion mass itself which amount to less than 0·5% of the leading term.] Substituting the limits and expanding in large $\Lambda \gg m_\pi$, (6) becomes

$$\begin{aligned} &\frac{2m_\pi^2}{3\pi} \frac{f_{\pi N\Delta}^2}{\mu^2} \left(\left\{ 2\Lambda - 3a\ln(2\Lambda) + \frac{4}{\Lambda}(m_\pi^2 - a^2) + \frac{5}{2\Lambda^2}(a^3 - \tfrac{3}{2} a\, m_\pi^2) + O\left(\frac{1}{\Lambda^3}\right)\right\} \right. \\ &\quad \left. - \left\{ - a - 3\,a \ln m_\pi + \frac{3\sqrt{a^2 - m_\pi^2}}{2} \ln\left[\left(\frac{a + m_\pi - \sqrt{a^2 - m_\pi^2}}{a + m_\pi + \sqrt{a^2 - m_\pi^2}}\right)^2\right]\right\}\right), \end{aligned} \tag{7}$$

and expanding in small m_π one finally gets

$$\begin{aligned} &\frac{2}{3\pi} \frac{f_{\pi N\Delta}^2}{\mu^2} \left\{ m_\pi^2 \left(2\Lambda - 3\,a\, \ln\left[\frac{\Lambda}{a}\right] + a - \frac{4a^2}{\Lambda} + \frac{5a^3}{2\Lambda^2}\right) \right. \\ &\quad \left. + m_\pi^4 \left(- \frac{3}{4a} + \frac{3}{2a} \ln\left[\frac{m_\pi}{2a}\right] + \frac{4}{\Lambda} - \frac{15a}{\Lambda^2}\right) + O\left(\frac{1}{\Lambda^3}, m_\pi^6\right)\right\}. \end{aligned} \tag{8}$$

In fact, we find that the complete non-analytic contribution is

$$\frac{2}{3\pi}\frac{f_{\pi N\Delta}^2}{\mu^2}\left(3am_\pi^2\ln\left[\frac{m_\pi}{2\Lambda}\right]-3m_\pi^2\sqrt{a^2-m_\pi^2}\ln\left[\frac{m_\pi}{2a}\right]\right). \tag{9}$$

In summary, we find that $\delta\sigma$ contains non-analytic terms which behave like

$$m_\pi^3 \simeq \hat{m}^{\frac{3}{2}}, \quad m_\pi^4 \ln m_\pi \simeq \hat{m}^2 \ln \hat{m}, \tag{10}$$

where $m_\pi \propto \hat{m}^{\frac{1}{2}}$ and $\hat{m}$ is the quark mass. The latter term receives a contribution from the Δ–π loop.

3. Discussion

After a detailed calculation of the Δ–π loop contribution to the σ-term we find that its leading non-analytic behaviour involves a term like $\hat{m}^2 \ln \hat{m}$ (or $m_\pi^4 \ln m_\pi$). As m_π has dimensions of energy one can only judge its relative significance once an appropriate energy scale has been set. In this case the natural scale is twice the Δ–nucleon mass splitting (about 300 MeV) as it is the combination $\ln(m_\pi/2a)$ that appears in (8). Evaluating this term (with $\Lambda = 600$ MeV), we find that it is in fact 47% of the total $\delta\sigma_\Delta$ contribution. This is clearly inconsistent with the proposition of Gasser and Leutwyler (1982) that such contributions are well represented by terms linear in the quark mass. We are therefore led to question the use by Gasser and Leutwyler of SU(3) phenomenology supplemented by chiral corrections involving octet but not decuplet baryons. This issue is currently under further investigation.

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