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# The Physics of Tachyons
# III. Tachyon Electromagnetism*

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*Abstract*

A new formulation of the theory of tachyons using the same two postulates as in special relativity is applied to electromagnetism. Tachyonic transformations of the electromagnetic fields **E** and **B** are rigorously derived from Maxwell's equations and are shown to be the same as for bradyonic transformations. Tachyonic transformations of current density, charge density, scalar and vector potentials are also derived and discussed. Tachyonic optics and the four-potential of a moving tachyonic charge are also discussed, along with generalised four-vector transformations and electromagnetic four-tensors in extended relativity. Use is made of a switching principle to show how tachyons automatically obey the law of conservation of electric charge in any inertial reference frame, even though the observed tachyon electric charge is not an invariant between observers. The electromagnetic field produced by a charged tachyon takes the form of a Mach cone, inside which the electromagnetic field is real and detectable, while outside the cone the field generated by the tachyon is imaginary and undetectable.

## 1. Introduction

The aim of this paper is to build upon the work of Dawe and Hines (1992*a*, 1992*b*) in the previous papers in this series on tachyon kinematics (hereafter referred to as Paper I) and tachyon dynamics (Paper II). The conclusion of Paper II on dynamics clearly pointed to a study of electromagnetism for tachyons as the next stage in the overall development of a comprehensive and viable formulation which is logical and internally consistent. The material in this paper extends the theory of tachyons to cover the subject of electromagnetism and discusses topics such as the tachyonic transformations of the electric and magnetic fields, charge density and current density, conservation of tachyonic electric charge, the electromagnetic field radiated by a charged tachyon, the Doppler effect, four-potentials, generalised four-vector transformations and the electromagnetic field and stress-energy tensors.

The development of tachyon mechanics in Papers I and II showed that the framework of special relativity (SR) can be extended to include particles having a relative speed greater than the speed of light in free space. The requirements necessary to allow this extension of special relativity into extended relativity (ER)

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are the switching principle (expressed mathematically as the '$\gamma$-rule'), a standard convention for decomposing imaginary square roots and the minor modification of some familiar definitions such as 'source' and 'detector'. The results and definitions of ER automatically reduce to those of SR as soon as the objects appear to the observer to be bradyons.

Some terms will be in common usage throughout this work, so they will be defined here. 'Special relativity' (SR) refers to all currently accepted physics for particles which travel more slowly relative to the observer than the speed of light. These particles will henceforth be called 'bradyons'. A 'tachyon' is defined to be a particle which is travelling relative to the observer at a speed greater than the speed of light. 'Extended relativity' (ER) is the theoretical framework which describes the motion and interactions of tachyons. A 'bradyonic observer' travels at a speed less than $c$, while a 'tachyonic observer' travels at a speed greater than $c$ relative to the inertial reference frame of the laboratory.

## 2. Summary of ER Developed Thus Far

The theory being developed in this series of papers is founded upon the philosophy of Corben (1978) who has argued that tachyons, should they exist, 'are basically the same objects as ordinary particles (they just look different because they are moving so fast).' With this in mind, the same two postulates apply in ER as in SR:

*Postulate 1: The laws of physics are the same in all inertial systems.*

*Postulate 2: The speed of light in free space has the same value c in all inertial systems.*

The term 'inertial system' now refers to any system travelling at a constant velocity with respect to an inertial observer, irrespective of whether the system is travelling slower than or faster than the speed of light. The postulates lead to the Lorentz transformations if the relative speed $u$ between the two inertial reference frames is such that $u^2 < c^2$, and to the following superluminal Lorentz transformations (SLTs) if the relative speed between the two inertial reference frames is such that $u^2 > c^2$ (see Paper I):

$$x' = i\gamma_u(x - ut), \qquad y' = iy, \qquad z' = iz, \qquad t' = i\gamma_u(t - ux/c^2)\,. \tag{1}$$

Here $u$ is the relative speed along the common $x, x'$ axes of an inertial reference frame $\Sigma'$ with respect to an inertial reference frame $\Sigma$ and

$$\gamma_u = (1 - u^2/c^2)^{-1/2} \tag{2}$$

for both $u^2 < c^2$ and $u^2 > c^2$.

Inverse tachyonic transformations can be obtained using the following rules: (i) interchange primed and unprimed quantities, (ii) reverse the sign of $u$ and (iii) reverse the sign of $i$. A fourth rule which applies only to transformations involving the proper mass is discussed in Paper II.

When $u^2 > c^2$ the inertial frames $\Sigma$ and $\Sigma'$ are on opposite sides of the light barrier and a particle seen by $\Sigma$ as a bradyon would be seen as a tachyon by $\Sigma'$ and vice versa. Even though tachyonic transformations such as (1) indicate that the axes transverse to the boost are imaginary while the axis parallel to the boost remains real, an inertial observer in the rest frame of the tachyon considers all of the axes to be real.

In Paper I it was shown that tachyons can logically and consistently obey the conservation laws of energy, momentum and electric charge through the use of a 'switching principle' (expressed mathematically as the '$\gamma$-rule'). A detailed numerical example was used to demonstrate that *'for switched tachyons the negative root of $\gamma_u$ is used and for unswitched tachyons the positive root of $\gamma_u$ is used.'* Written explicitly, this rule is

$$\gamma_u = \mathrm{sign}(\gamma_u)(1 - u^2/c^2)^{-1/2}\,, \tag{3}$$

where $\mathrm{sign}(\gamma_u) = +1$ if the particle appears to an observer to be an unswitched tachyon or a bradyon, and $\mathrm{sign}(\gamma_u) = -1$ if the particle appears to that observer to be a switched tachyon.

The switching principle in ER is not just a gimmick, but is the physical means which allows tachyons, if they exist, to obey the laws of physics and to interact with bradyonic matter at the classical relativistic level. Note that as there is no switching for a particle viewed by an observer to be a bradyon, then $\mathrm{sign}(\gamma_u)$ is always $+1$ and the standard result of SR is automatically recovered. Here the speed $u$ is the relative speed between two inertial reference frames and should not be confused with the speed of the particle in the observer's inertial reference frame.

As mentioned briefly in Paper I, Schwartz (1982) has replaced the application of a reinterpretation or switching principle by a study of the integration of certain components of the stress energy–momentum tensor over particular three-dimensional surfaces in four-dimensional space–time. A detailed discussion of the work of Schwartz will be reserved for a later stage of the present series when the quantum formulation of tachyon properties is treated.

The tachyonic velocity transformations, which are exactly the same in ER and SR, automatically show whether the particle is switched or unswitched relative to a particular observer. Let $v_x$ be the speed of the particle in the initial frame $\Sigma$, while $u$ is the speed of the final frame $\Sigma'$ relative to $\Sigma$ along the common $x, x'$ axes. The particle will appear to $\Sigma'$ to be switched if

$$c > u > c^2/v_x \text{ for } v_x > c \text{ and } |u| < c, \text{ or} \tag{4}$$

$$c < u < c^2/v_x \text{ for } v_x < c \text{ and } |u| > c\,. \tag{5}$$

To be consistent in the calculations of ER, a convention is used to deal with imaginary square roots such that when $u^2 > c^2$ then

$$(1 - u^2/c^2)^{\frac{1}{2}} = i(u^2/c^2 - 1)^{\frac{1}{2}}\,, \tag{6}$$

so that $\gamma_u$ can be written as

$$\gamma_u = -\mathrm{sign}(\gamma_u)i|\gamma_u|\,. \tag{7}$$

The metric used throughout this paper is $(+1, +1, +1, +1)$ as detailed on p. 593 of Paper I and a discussion of the reason for this choice of metric is also to be found there.

## 3. Electric and Magnetic Field Transformations

Vital components of any viable theory of tachyons are the transformations of the electric and magnetic fields. The form of these transformations in ER affects the way all other electromagnetic quantities are dealt with and, since charged tachyons should be able to emit photons, there is also an effect on how tachyons, if they exist, may possibly be detected through their electromagnetic interactions with photons and bradyonic matter. There are two choices in how to proceed with a development of electromagnetism in ER, both of which start with Maxwell's equations, which in bradyonic frame $\Sigma$ are

$$\nabla . \mathbf{B} = 0, \tag{8}$$

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t, \tag{9}$$

$$\nabla . \mathbf{E} = \rho/\epsilon_0, \tag{10}$$

$$\nabla \times \mathbf{B} = \mu_0(\mathbf{j} + \epsilon_0 \partial \mathbf{E}/\partial t). \tag{11}$$

Here $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic field vectors respectively, while $\mathbf{j}$ and $\rho$ are the current density and charge density and $\epsilon_0$ and $\mu_0$ are the permittivity and permeability constants of free space.

The first choice as to the method of development is to modify the first postulate's meaning, in much the same way that the definitions of 'source' and 'detector' have been altered (see Paper I). Suppose that a generalised law can be developed which applies to bradyons and tachyons in a somewhat different manner for each object, but is nevertheless covariant with respect to the enlarged class of inertial reference frames, which now includes bradyonic and tachyonic observers. In particular, when considering tachyonic charges, Maxwell's equations in free space can either remain the same or change so as to describe bradyonic and tachyonic charges in a different way, but nevertheless remaining covariant with respect to the enlarged class of inertial reference frames (Recami and Mignani 1974).

This first choice in how to develop electromagnetism in ER, taken by Recami and Mignani (1974, 1976, 1977), is to add terms to Maxwell's equations in order to symmetrise them. To this end, terms $-\rho(S)$ and $j(S)$ are added to the right-hand side of equations (8) and (9) respectively where the extra terms apply only to charged tachyons. This leads to the pairs of components $(E_y, B_z)$ and $(E_z, B_y)$ transforming like the space–time coordinates $x$ and $t$, while the transformations of $E_x$ and $B_x$ behave like the transformation of $y$ and $z$ such that $E_x = iE_{x'}$ and $B_x = iB_{x'}$ when transforming from bradyonic frame $\Sigma$ to tachyonic frame $\Sigma'$. This in turn leads to electrically charged tachyons (in their own rest frame) appearing to bradyonic observers to behave like magnetic monopoles and to the tachyonic current density being imaginary (Recami 1986). The authors choose to disagree with this view of tachyon electromagnetism, preferring instead the views of Corben (1975).

This second choice as to how electromagnetism can be developed in ER, made by Corben (1975), considers Maxwell's equations in a vacuum to be the same for bradyons and tachyons. This method has the advantage of requiring no changes in any of the fundamental laws or constants of physics and places the differences between SR and ER in the transformations of variables. The authors have chosen to follow Corben in making this second choice because of its inherent simplicity and the logical and consistent nature of the results it produces using rigorous derivations. This choice means that Maxwell's equations in a vacuum are to be considered as fundamental laws of physics, as are the conservation laws of energy, momentum and electric charge in a given inertial reference frame, and so postulate 1 of ER is applicable. It should be mentioned at this stage that this question of choice is already foreshadowed in the early review of Recami and Mignani (1974) in which a duality principle is introduced in refutation of the assertion that bradyonic and tachyonic characteristics were absolute.

In order to derive the tachyonic transformations of the electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ it is necessary to consider Maxwell's equations in a vacuum as used in bradyonic frame $\Sigma$ and tachyonic frame $\Sigma'$. For the first postulate of ER to be true an inertial tachyonic observer in tachyonic frame $\Sigma'$ must be able to write Maxwell's equations as

$$\nabla' . \mathbf{B}' = 0 \,, \tag{12}$$

$$\nabla' \times \mathbf{E}' = -\partial \mathbf{B}'/\partial t' \,, \tag{13}$$

$$\nabla' . \mathbf{E}' = \rho'/\epsilon_0' \,, \tag{14}$$

$$\nabla' \times \mathbf{B}' = \mu_0' (\mathbf{j}' + \epsilon_0' \partial \mathbf{E}'/\partial t') \,. \tag{15}$$

Here $\epsilon_0'$ and $\mu_0'$ are the permittivity and permeability constants of free space in tachyonic frame $\Sigma'$. The permittivity and permeability constants in each frame are related by

$$\mu_0' \epsilon_0' = c^{-2} = \mu_0 \epsilon_0 \,, \tag{16}$$

as a consequence of the second postulate.

In order to relate the set of Maxwell's equations in frames $\Sigma$ and $\Sigma'$ to each other, the connection between the partial derivatives in $\Sigma$ with those in $\Sigma'$ must first be known. Applying the chain rule to the tachyonic transformations in (1) produces the transformation of partial derivatives:

$$\frac{\partial}{\partial x} = i\gamma_u \left( \frac{\partial}{\partial x'} - \frac{u}{c^2} \frac{\partial}{\partial t'} \right) , \quad \frac{\partial}{\partial y} = i \frac{\partial}{\partial y'} \,,$$

$$\frac{\partial}{\partial z} = i \frac{\partial}{\partial z'} \,, \quad \frac{\partial}{\partial t} = i\gamma_u \left( \frac{\partial}{\partial t'} - u \frac{\partial}{\partial x'} \right) , \tag{17}$$

where $u$ is the speed of $\Sigma'$ along the common $x, x'$ axes relative to $\Sigma$. The inverse transformations are

$$\frac{\partial}{\partial x'} = -i\gamma_u\left(\frac{\partial}{\partial x} + \frac{u}{c^2}\frac{\partial}{\partial t}\right), \quad \frac{\partial}{\partial y'} = -i\frac{\partial}{\partial y},$$

$$\frac{\partial}{\partial z'} = -i\frac{\partial}{\partial z}, \quad \frac{\partial}{\partial t'} = -i\gamma_u\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right) \tag{18}$$

The corresponding transformations of partial derivatives in SR omit the factor of $i$ in (17) and $-i$ in (18).

Using (17) in the $y$-component of (9) produces an expression which can be compared with the $y'$-component of (13) to give

$$E_{x'} = E_x, \ E_{z'} = \gamma_u(E_z + uB_y), \ B_{y'} = \gamma_u(B_y + uE_z/c^2). \tag{19}$$

Similarly, using (17) in the $z$-component of (9) leads to an expression which can be compared with the $z'$-component of (13) to give

$$E_{x'} = E_x, \ E_{y'} = \gamma_u(E_y - uB_z), \ B_{z'} = \gamma_u(B_z - uE_y/c^2). \tag{20}$$

By using (18) and the newly derived transformations for $B_{y'}$ and $B_{z'}$ in (12) it can be shown that $B_{x'} = B_x$ as expected. Hence for a boost along the common $x, x'$ axes the electromagnetic field components transform between bradyonic and tachyonic inertial reference frames as follows:

$$E_{x'} = E_x, \ E_{y'} = \gamma_u(E_y - uB_z), \ E_{z'} = \gamma_u(E_z + uB_y), \tag{21}$$

$$B_{x'} = B_x, \ B_{y'} = \gamma_u(B_y + uE_z/c^2), \ B_{z'} = \gamma_u(B_z - uE_y/c^2). \tag{22}$$

The inverse transformations are

$$E_x = E_{x'}, \ E_y = \gamma_u(E_{y'} + uB_{z'}), \ E_z = \gamma_u(E_{z'} - uB_{y'}), \tag{23}$$

$$B_x = B_{x'}, \ B_y = \gamma_u(B_{y'} - uE_{z'}/c^2), \ B_z = \gamma_u(B_{z'} + uE_{y'}/c^2). \tag{24}$$

These are exactly the same transformations as those used in SR for transforming between two bradyonic frames, so that (21) to (24) are valid for $-\infty < u < \infty$. Note that the correct invariances for the electric and magnetic fields hold in ER:

$$E^2 - c^2B^2 = E'^2 - c^2B'^2 \quad \text{and} \quad \mathbf{E}.\mathbf{B} = \mathbf{E}'.\mathbf{B}'. \tag{25}$$

This is appropriate since the electromagnetic waves travel at the same speed in free space, regardless of whether the observer's reference frame is bradyonic or tachyonic.

The Lorentz force on a moving electric charge $q$ in frame $\Sigma$ is

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \tag{26}$$

In the previous two papers it was shown that the transformations of velocity and force apply for $-\infty < u < \infty$ and not just for the SR range $-c < u < c$. As the transformations of $\mathbf{E}$ and $\mathbf{B}$ have also been shown to be the same for both $u^2 < c^2$ and $u^2 > c^2$ then the Lorentz force in frame $\Sigma'$ must be

$$\mathbf{F}' = q'(\mathbf{E}' + \mathbf{v}' \times \mathbf{B}')\,, \tag{27}$$

irrespective of whether $\Sigma'$ is a bradyonic or tachyonic reference frame.

The transformations of $\mathbf{E}$ and $\mathbf{B}$ given by (23) and (24) are the same as those of Corben (1975) when allowance is made for the conversion between gaussian and SI units. The transformations of $\mathbf{E}'$ and $\mathbf{B}'$ given by (21) and (22) are different to those of Recami and Mignani (1974, 1976, 1977) in which the pairs of components $(E_y, B_z)$ and $(E_z, B_y)$ transform like the space–time coordinates $x$ and $t$, while the transformations of $E_x$ and $B_x$ behave like the transformation of $y$ and $z$ such that $E_x = iE_{x'}$ and $B_x = iB_{x'}$.

## 4. Charge and Current Densities

Conservation of electric charge in tachyonic frame $\Sigma'$ is expressed by the equation of continuity

$$\frac{\partial j_{x'}}{\partial x'} + \frac{\partial j_{y'}}{\partial y'} + \frac{\partial j_{z'}}{\partial z'} + \frac{\partial \rho'}{\partial t'} = 0\,, \tag{28}$$

where $\mathbf{j}' = (j_{x'}, j_{y'}, j_{z'})$ is the current density and $\rho'$ is the charge density within a volume element. It is assumed to have no sources or sinks. Applying (18) to (28) and multiplying through by $-1$ leads to

$$i\gamma_u\left(\frac{\partial}{\partial x}(j_{x'} + u\rho')\right) + i\frac{\partial j_{y'}}{\partial y} + i\frac{\partial j_{z'}}{\partial z} + i\gamma_u\left(\frac{\partial}{\partial t}(\rho' + uj_{x'}/c^2)\right) = 0\,. \tag{29}$$

Conservation of electric charge in bradyonic frame $\Sigma$ is expressed as

$$\frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} + \frac{\partial j_z}{\partial z} + \frac{\partial \rho}{\partial t} = 0\,, \tag{30}$$

and comparing this with (29) shows that the current and charge densities in ER transform as

$$j_x = i\gamma_u(j_{x'} + u\rho')\,,\ j_y = ij_{y'}\,,\ j_z = ij_{z'}\,,\ \rho = i\gamma_u(\rho' + uj_{x'}/c^2)\,. \tag{31}$$

The corresponding transformations from bradyonic frame $\Sigma$ to tachyonic frame $\Sigma'$ are

$$j_{x'} = -i\gamma_u(j_x - u\rho)\,,\ j_{y'} = -ij_y\,,\ j_{z'} = -ij_z\,,\ \rho' = -i\gamma_u(\rho - uj_x/c^2)\,. \tag{32}$$

The transformations of $\mathbf{j}$ and $\rho$ derived here are not equivalent to those of Recami and Mignani due to the different treatment of Maxwell's equations in the two formulations.

The transformations of current and charge densities in SR are given by

$$j_{x'} = \gamma_u(j_x - u\rho)\,,\ j_{y'} = j_y\,,\ j_{z'} = j_z\,,\ \rho' = \gamma_u(\rho - uj_x/c^2)\,, \tag{33}$$

$$j_x = \gamma_u(j_{x'} + u\rho')\,,\ j_y = j_{y'}\,,\ j_z = j_{z'}\,,\ \rho = \gamma_u(\rho' + uj_{x'}/c^2)\,. \tag{34}$$

Hence the square of the charge and current density four-vector $J_\lambda = (\mathbf{j}, ic\rho)$ is

$$\sum_{\lambda=1}^{4}(J'_\lambda)^2 = \pm\sum_{\lambda=1}^{4}(J_\lambda)^2\,, \tag{35}$$

where the upper (+) sign refers to transformations between two bradyonic frames or two tachyonic frames and the lower (−) sign applies when transforming between a bradyonic frame and a tachyonic frame.

The tachyonic transformations of current density and charge density have the opposite sign to the tachyonic space–time transformations given by (1). There are in fact three general types of tachyonic transformations: those with $+i$ when transforming from $\Sigma$ to $\Sigma'$, those with $-i$ when transforming from $\Sigma$ to $\Sigma'$ and those transformations which apply for $-\infty < u < \infty$. This will be discussed further in Section 9.

Knowing the transformations of the current and charge densities between $\Sigma$ and $\Sigma'$ allows the second pair of Maxwell's equations to be used as a check of this formulation's internal consistency. Using (17) and (31) in the $y$-component of (11) and then cancelling the factor of $i$ produces an expression which can be compared with the $y'$-component of (15). This gives the transformations of $B_{x'}$, $B_{z'}$ and $E_{y'}$ derived earlier and also $\mu'_0 = \mu_0$, so that $\epsilon'_0 = \epsilon_0$ from (16). Therefore the fundamental constants $\mu_0$ and $\epsilon_0$ are the same for both bradyons and tachyons. This agrees with having the same metric apply for both $u^2 < c^2$ and $u^2 > c^2$: the fundamental properties of space–time are the same for both bradyonic and tachyonic observers. Further substitutions can be used to confirm that the second pair of Maxwell's equations yield the same transformations of $\mathbf{E}$ and $\mathbf{B}$ that were derived from the first pair, even though the tachyonic transformations of current and charge density contain imaginary factors.

## 5. Conservation of Electric Charge

### *(5a) Transformation of a Volume Element*

A detailed proof of the conservation of electric charge for tachyons in a given reference frame requires the transformation of a cubic volume element between various tachyonic and bradyonic frames.

Consider an element of volume which is at rest in frame $\Sigma_0$, where the volume element is a cube of edge length $l_0$ so that its proper volume is $V_0 = l_0^3$. In Paper I it was shown that a rod moving along the common $x, x_0$ axes with speed $v$ appears to undergo Lorentz–Fitzgerald contraction for $0 < v^2 < c^2$ and $c^2 < v^2 < 2c^2$, while the rod appears to be dilated for $2c^2 < v^2$. The apparent length $l$ of the cube in frame $\Sigma$ along the $x$ axis is therefore $l = l_0(1 - v^2/c^2)^{\frac{1}{2}}$ for $v^2 < c^2$ and $l = l_0|(1 - v^2/c^2)^{\frac{1}{2}}|$ for $v^2 > c^2$, where the modulus sign appears because lengths and volumes are intrinsically positive and real, even for tachyons as explained in Paper I. There is no length contraction or dilation in a direction transverse to the boost, so that the apparent length of the rod in frame $\Sigma$ in the transverse direction is simply $l_0$ for bradyons and tachyons alike. Hence the apparent volume of the element as measured in frame $\Sigma$ is

$$V = V_0(1 - v^2/c^2)^{\frac{1}{2}} \quad \text{for} \quad v^2 < c^2\,,$$

$$V = V_0|(1 - v^2/c^2)^{\frac{1}{2}}| \quad \text{for} \quad v^2 > c^2 \tag{36}$$

where $V_0 = l_0^3$. Similarly, the apparent volume of the element as seen in another frame $\Sigma'$ moving along the common $x', x, x_0$ axes with speed $v'$ relative to $\Sigma_0$ is $V' = V_0(1 - v'^2/c^2)^{\frac{1}{2}}$ for $v'^2 < c^2$ and $V' = V_0|(1 - v'^2/c^2)^{\frac{1}{2}}|$ for $v'^2 > c^2$.

In Paper I it was shown that the velocity transformations lead to the following useful identities in both SR and ER:

$$\gamma_v = \gamma_u \gamma_{v'}(1 + uv_{x'}/c^2)\,, \tag{37}$$

$$\gamma_{v'} = \gamma_u \gamma_v (1 - uv_x/c^2)\,. \tag{38}$$

Substituting (37) into (36) and then using the appropriate expression for $V'$ gives the volume transformation as

$$V = V'(1 - u^2/c^2)^{\frac{1}{2}}(1 + uv_{x'}/c^2)^{-1} \quad \text{for} \quad u^2 < c^2 \text{ and } v'^2 < c^2\,,$$

$$V = V'|(1 - u^2/c^2)^{\frac{1}{2}}|(1 + uv_{x'}/c^2)^{-1} \quad \text{for} \quad u^2 > c^2 \text{ and } v'^2 < c^2\,,$$

$$V = V'|(1 - u^2/c^2)^{\frac{1}{2}}(1 + uv_{x'}/c^2)^{-1}| \quad \text{for} \quad v'^2 > c^2\,, \tag{39}$$

where $u$ is the speed of $\Sigma'$ relative to $\Sigma$.

### *(5b) Conservation of Electric Charge in Each Reference Frame*

Let $V_0$ be the volume of a small element of charge as measured in an inertial frame $\Sigma_0$, relative to which the charge is instantaneously at rest. The current density is $j_0 = 0$ and the total charge within the element is equal to $\rho_0 V_0$ where $\rho_0$ is the proper density of proper charge. Now assume that there is a second inertial reference frame $\Sigma$ which is travelling with speed $v = v_x$ along the common $x, x_0$ axes relative to $\Sigma_0$. Frame $\Sigma$ is bradyonic and measures a charge density in the volume element of $\rho$. Using the transformations of charge density in the previous section gives $\rho = \gamma_v \rho_0$ if $v^2 < c^2$ or $\rho = i\gamma_v \rho_0$ if $v^2 > c^2$.

The total electric charge within the volume element as measured by bradyonic frame $\Sigma$ is

$$\rho V = \gamma_v \rho_0 V_0 (1 - v^2/c^2)^{\frac{1}{2}} = \rho_0 V_0 \text{ for } v^2 < c^2\,,$$

$$\rho V = i\gamma_v \rho_0 V_0 |(1 - v^2/c^2)^{\frac{1}{2}}| = \mathrm{sign}(\gamma_v)\rho_0 V_0 \text{ for } v^2 > c^2\,, \tag{40}$$

where use has been made of the $i - \gamma$ convention given in Section 2. Hence for $v^2 < c^2$ the familiar result of SR has been obtained, which is that electric charge is an invariant when the rest frame and the final frame are both bradyonic reference frames. For $v^2 > c^2$ the apparent sign of the electric charge depends upon the sign of the square root in $\gamma$, which is positive for unswitched tachyons and negative for switched tachyons. Thus the $\gamma$-rule correctly gives the apparent sign of the charge: unswitched tachyons appear to the observer to have the same charge as the particle's proper charge (i.e. rest frame charge), while switched tachyons appear to the observer to have the opposite charge to their proper charge. This agrees with the worked example in paper I of the exchange of a charged tachyon.

Now suppose there is a third inertial reference frame $\Sigma'$ moving along the common $x', x, x_0$ axes with speed $v' = v_{x'}$ relative to frame $\Sigma_0$ and with speed $u$ relative to frame $\Sigma$. The speeds $v'$, $v$ and $u$ are related by the velocity transformation $v' = (v-u)/(1-uv/c^2)$. The current density $j'$ in frame $\Sigma'$ is given by $j' = \rho' v'$ for both bradyons and tachyons. A particular case of interest is when inertial reference frame $\Sigma_0$ is tachyonic and frames $\Sigma$ and $\Sigma'$ are both bradyonic, so that $v^2 > c^2$, $v'^2 > c^2$ and $u^2 < c^2$. Combining (39) with (34) gives

$$\rho V = \gamma_u V'(\rho' + uj'/c^2)|(1-u^2/c^2)^{\frac{1}{2}}(1+uv'/c^2)^{-1}| = \mathrm{sign}(\gamma_u)\rho' V'$$

using the $\gamma$-rule and the fact that $1 + uv'/c^2 > 0$ in this case. From (40) the total apparent electric charge measured in frame $\Sigma'$ is

$$\rho' V' = \mathrm{sign}(\gamma_u)\rho V = \mathrm{sign}(\gamma_u)\mathrm{sign}(\gamma_v)\rho_0 V_0 \,. \tag{41}$$

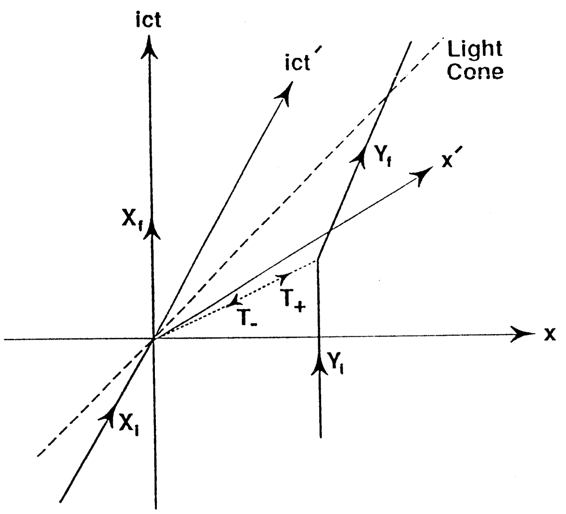


**Fig. 1.** Exchange of a tachyon $T$ between two bradyonic objects $X$ and $Y$. Observer $\Sigma$ using axes $(x, ict)$ sees $T$ as an unswitched tachyon $T_+$ carrying charge $+q$ from $X$ to $Y$, while observer $\Sigma'$ using axes $(x', ict')$ sees $T$ as a switched tachyon $T_-$ travelling from $Y$ to $X$ with apparent charge $-q$.

An example of this particular case is the hypothetical exchange of an electrically charged tachyon between two electrically charged bradyons, as illustrated in the Minkowski diagram of Fig. 1. The incoming bradyon $X_i$ has charge $+4q$ and emits an unswitched tachyon $T_+$ with charge $+q$ according to observer $\Sigma$ using axes $(x, ict)$, so the outgoing bradyon $X_f$ has charge $+3q$. Observer $\Sigma$ later sees bradyon $Y_i$ carrying charge $+3q$ absorb $T_+$ to become the outgoing bradyon $Y_f$ with charge $+4q$. In the rest frame of the exchanged tachyon it carries charge

$+q = \rho_0 V_0$ throughout its existence. Hence the total apparent electric charge in the system at any given $t$-time is $+7q$. According to observer $\Sigma'$ using axes $(x', ict')$, the exchanged tachyon in Fig. 1 appears to be switched and travels from $Y$ to $X$. As the tachyon appears to be unswitched in frame $\Sigma$ but switched in frame $\Sigma'$ then $\mathrm{sign}(\gamma_v) = 1$ and $\mathrm{sign}(\gamma_u) = -1$, and so the measured apparent electric charge of the tachyon as given by (41) is $\rho V = \rho_0 V_0 = +q$ in frame $\Sigma$ and $\rho' V' = -\rho V = -q$ in frame $\Sigma'$. Thus the apparent total electric charge as measured by $\Sigma'$ at any $t'$-time is still conserved at $+7q$, even though there is a switched tachyon present in the system for part of the $t'$-time.

This example shows that the $\gamma$-rule, which was originally developed in Paper I as a device to allow tachyons automatically to obey the laws of conservation of energy and momentum, also allows tachyons to obey the law of conservation of electric charge in a given inertial reference frame.

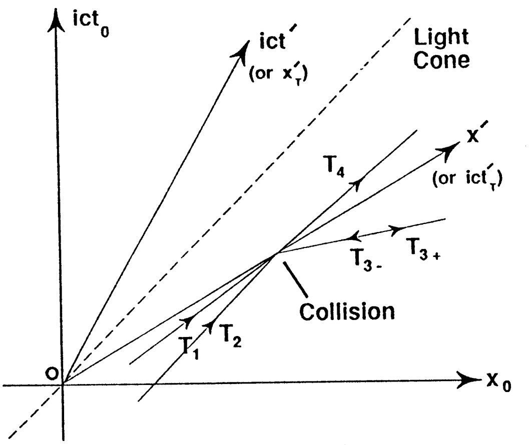


**Fig. 2.** Hypothetical collision between tachyons as represented by a Minkowski diagram. Observer $\Sigma_0$ using axes $(x_0, ict_0)$ sees the collision $T_1 + T_2 \to T_{3+} + T_4$, while observer $\Sigma'$ using axes $(x', ict')$ sees the reaction $T_1 + T_2 + T_{3-} \to T_4$ and observer $\Sigma'_T$ using axes $(x'_T, ict'_T)$ sees the reaction $T_1 + T_2 \to T_{3+} + T_4$.

*(5c) Nonconservation of Total Electric Charge and Particle Number for Tachyons between Reference Frames*

Consider a hypothetical collision between two charged tachyons, as represented by the Minkowski diagram in Fig. 2. Observer $\Sigma_0$ using axes $(x_0, ict_0)$ sees two unswitched tachyons, $T_1$ and $T_2$, enter the collision and later sees two unswitched tachyons, $T_3$ and $T_4$, exit the collision. A second observer $\Sigma'$ using axes $(x', ict')$ sees the unswitched tachyons $T_1$ and $T_2$ and the switched tachyon $T_3$ enter the collision but later sees only one unswitched tachyon, $T_4$, exit the collision. Hence the apparent number of tachyons at any given time is not necessarily the same for different observers in a system involving collisions.

In the tachyonic frame $\Sigma'_T$, using axes $(x'_T, ict'_T)$, all of the particles in Fig. 2 appear to be bradyons, with $T_1$ and $T_2$ colliding to produce $T_3$ and $T_4$. Therefore the number of particles observed before or after the collision is an invariant between frames for which all of the particles are either bradyons or unswitched tachyons, but is not invariant for frames in which some tachyons appear to be switched. Some of the consequences of this point will be discussed shortly.

Now suppose that the incoming particles $T_1$ and $T_2$ each have electric charge $+q$, and that both $T_3$ and $T_4$ have charge $+q$ in their own rest frames. Therefore the total apparent incoming charge in frames $\Sigma_0$ and $\Sigma'_T$ is $+2q$, which exactly matches the net apparent outgoing charge in each frame. Observer $\Sigma'$ sees three tachyons enter the collision, but as $T_3$ is switched it has apparent charge $-q$ and so the net apparent incoming charge measured by $\Sigma'$ is $+2q - q = +q$. This exactly matches the charge $+q$ on tachyon $T_4$, which is the only product of the collision according to observer $\Sigma'$. Thus the total electric charge is conserved in each of the frames $\Sigma_0$, $\Sigma'$ and $\Sigma'_T$, even though observers in those frames may measure different total amounts of electric charge and also measure different numbers of particles before and after the collision.

As a second example based on Fig. 2, suppose $T_1$ and $T_4$ both have electric charge $+q$, while $T_2$ and $T_3$ both have charge $-q$ in their own rest frames. Observers $\Sigma_0$ and $\Sigma'_T$ would measure a total electric charge before and after the collision of $+q - q = 0$. Observer $\Sigma'$ would measure an initial charge of $+q + q - q = +q$ as $T_3$ is switched, while the outgoing charge carried by $T_4$ is $+q$. Again it can be seen that the total electric charge is conserved within each inertial reference frame, regardless of whether it is bradyonic or tachyonic, and that the observed total electric charge is different for different observers if some of the particles are switched tachyons. It must be stressed that this effect is purely an artefact of the observer's relative motion, and does not mean that some of the electric charge has been 'destroyed' or 'created' simply by transforming from one reference frame $\Sigma_0$ to another reference frame $\Sigma'$. Transforming to any reference frame in which all of the particles appear to be either bradyons or unswitched tachyons will 'recover' the apparent difference in charge. What must be remembered is that the conservation laws and Maxwell's equations all hold true in any given inertial reference frame, regardless of whether it is a bradyonic or a tachyonic frame. This result agrees with the conclusion of Recami and Mignani (1974), who have shown that electric charge is always conserved in each frame, but is no longer an invariant between frames when dealing with tachyons.

The lack of invariance of particle number between reference frames has important consequences for the studies of statistical mechanics, thermodynamics and plasma physics for tachyons. For example, in SR one of the standard techniques in dealing with a gas of colliding particles is to normalise the particle distribution by assuming that the total number of particles is constant. In ER, the noninvariance of particle number between reference frames implies that any normalisation procedure must be carried out in a reference frame in which none of the tachyons appear to the observer to be switched. In the case of a gas of tachyons the reference frame to use for normalisation is that comoving with the centre of mass of the gas, as this is the only frame in which all of the tachyons are unswitched, regardless of how they collide with each other or of their speed distribution. Moreover, using a frame in which none of the tachyons are switched has enabled Dawe *et al.* (1989) to derive the statistical and thermodynamic properties of a tachyonic Maxwell–Boltzmann gas.

## 6. Electric and Magnetic Fields of a Tachyonic Charge

In this section the electromagnetic field produced by a uniformly moving tachyonic point charge in a vacuum is investigated. The following discussion is adapted from the SR case given by Rosser (1964) and Resnick (1968).

Consider a point particle of electric charge $+q'$ at rest in an inertial reference frame $\Sigma'$. The electric and magnetic fields produced by the charge are given by $\mathbf{E}' = q'\mathbf{r}'/4\pi\epsilon_0 r'^3$ and $\mathbf{B}' = 0$ respectively, where the electric field lines diverge from the charge with spherical symmetry. Here

$$r' = (x'^2 + y'^2 + z'^2)^{\frac{1}{2}} \tag{42}$$

is defined to be the distance from the origin $O'$ coincident with the charge in frame $\Sigma'$ to the point at which the field strength is measured.

Now suppose that frame $\Sigma'$ is moving with speed $u > c$ along the common $x', x$ axes with respect to a bradyonic frame $\Sigma$, so that the charge is tachyonic relative to observer $\Sigma$. Using the tachyonic transformations (1) in (42) gives $r' = i(\gamma_u^2(x-ut)^2 + y^2 + z^2)^{\frac{1}{2}}$ and as $E_{x'} = E_x$ the longitudinal component of the electric field as measured in frame $\Sigma$ is

$$E_x = q'x'/4\pi\epsilon_0 r'^3 = \frac{q|\gamma_u|(x-ut)}{4\pi\epsilon_0 i^3(\gamma_u^2(x-ut)^2 + y^2 + z^2)^{\frac{3}{2}}}, \tag{43}$$

where $q' = \mathrm{sign}(\gamma_u)q$ and use has been made of the $\gamma$-rule and the $i-\gamma$ convention in the form of (7). Note that any sign change due to switching in the gamma factor and the electric charge cancels out. In frame $\Sigma$ the transverse electric field components are

$$E_y = \gamma_u(E_{y'} + uB_{z'}) = \frac{qy|\gamma_u|}{4\pi\epsilon_0 i^3(\gamma_u^2(x-ut)^2 + y^2 + z^2)^{\frac{3}{2}}}, \tag{44}$$

$$E_z = \gamma_u(E_{z'} - uB_{y'}) = \frac{qz|\gamma_u|}{4\pi\epsilon_0 i^3(\gamma_u^2(x-ut)^2 + y^2 + z^2)^{\frac{3}{2}}}. \tag{45}$$

From (24) and the above expressions for the components of $\mathbf{E}$ the magnetic field measured in frame $\Sigma$ is

$$B_x = 0,\ B_y = -uE_z/c^2,\ B_z = uE_y/c^2. \tag{46}$$

If the moving charge is considered to be at the origin of $\Sigma$ at the instant $t = 0$, then the electric field is

$$\mathbf{E} = \frac{q|\gamma_u|\mathbf{r}}{4\pi\epsilon_0 i^3(\gamma_u^2 x^2 + y^2 + z^2)^{\frac{3}{2}}}. \tag{47}$$

At other times the field will look the same but translated along the $x$-axis by the distance $ut$. By defining $\theta$ to be the angle made by $\mathbf{r}$ with the $x$-axis then

$$x = r\cos\theta \quad \text{and} \quad y^2 + z^2 = r^2\sin^2\theta. \tag{48}$$

This gives

$$\gamma_u^2 x^2 + y^2 + z^2 = \gamma_u^2 r^2(1 - u^2\sin^2\theta/c^2) \tag{49}$$

and so (47) leads to

$$\mathbf{E} = \frac{\mathrm{sign}(\gamma_u) q\mathbf{r}(u^2/c^2 - 1)}{4\pi\epsilon_0 r^3 (1 - u^2 \sin^2\theta/c^2)^{\frac{3}{2}}}\,, \tag{50}$$

where (7) has been used to simplify the expression.

The equivalent expression for the electric field in SR when $u^2 < c^2$ is

$$\mathbf{E} = \frac{q\mathbf{r}(1 - u^2/c^2)}{4\pi\epsilon_0 r^3 (1 - u^2 \sin^2\theta/c^2)^{\frac{3}{2}}}\,, \tag{51}$$

so that the form of the expressions describing the field in SR and ER are similar. In both cases the field in frame $\Sigma$ is still an inverse square one, as its strength is proportional to $r^{-2}$ in any direction.

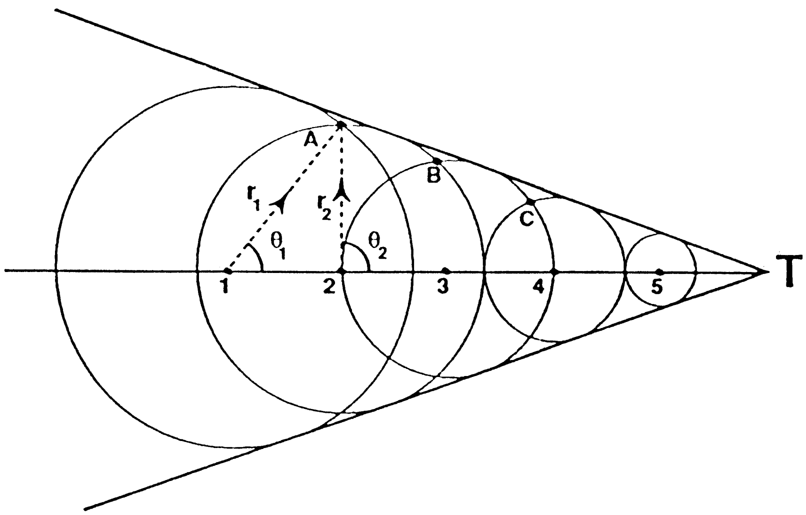


**Fig. 3.** An electromagnetic envelope of a charged tachyon travelling with constant speed forms a 'Mach cone'. The circles represent the wavefronts of the field generated by the tachyon $T$ as it passed through the points $1, 2, 3, \ldots$. For all points inside the cone **E** and **B** are real, whereas for all points outside the cone **E** and **B** are imaginary as the field has not yet arrived at that point. Observers at A, B and C all detect wavefronts radiated from two positions.

Examining (50) shows that when $c^2 > u^2 \sin^2\theta$ all the components of **E** and **B** are real, while for $c^2 < u^2 \sin^2\theta$ all the components of **E** and **B** are imaginary. As $\Sigma$ is an inertial frame used by a bradyonic observer who can only detect real quantities with his instruments, then the charged tachyon's electromagnetic field is only detectable for $|c/u| > |\sin\theta|$ and is undetectable for $|c/u| < |\sin\theta|$. These cases are demonstrated in Fig. 3, which represents a Mach cone similar to those generated by aircraft travelling at supersonic speeds. The circles in Fig. 3 represent electromagnetic wavefronts emitted by the charged tachyon as it passes through a numbered sequence of points. Inside the cone the field is real and detectable, whereas outside the cone the wavefronts have not yet arrived and so the tachyon's field is imaginary and undetectable. When $|c/u| = |\sin\theta|$

the observer in frame $\Sigma$ is at the edge of the cone and $\mathbf{E}$, $B_y$ and $B_z$ are instantaneously infinite. At any later $t$-time $|c/u| > |\sin\theta|$ and the field is real and finite. In analogy with the 'sonic boom' produced by aircraft flying faster than the speed of sound, the observer's initial contact with the cone of electromagnetic wavefronts is described as an 'optic boom' (Recami and Mignani 1972; Barut *et al.* 1982).

At the initial contact with the electromagnetic field the observer receives a wavefront from one direction, but at later times the observer receives wavefronts from two separate directions. This 'two source effect' (see also Recami 1986; Recami *et al.* 1986) is shown in Fig. 3 as the intersection of any two wavefronts. For example, an observer at A detects the wavefronts emitted by the tachyon $T$ when it passed through positions 1 and 2, an observer at B detects the wavefronts radiated from positions 2 and 3, etc. Therefore any calculation of the electromagnetic field must account for the fact that the test charge experiences a superposition of fields generated by the tachyon from two separate positions. As Fig. 3 clearly shows, there are two combinations of $r$ and $\theta$ which are permissible in (50). The definitions given by (48) apply to both permissible pairs $(r_1, \theta_1)$ and $(r_2, \theta_2)$, so that the total electric field measured at any point inside the Mach cone is given by the superposition

$$\mathbf{E} = \mathrm{sign}(\gamma_u) q(u^2/c^2 - 1)(\mathbf{R}_1 + \mathbf{R}_2)/4\pi\epsilon_0 \,, \tag{52}$$

where

$$\mathbf{R}_1 = \frac{\mathbf{r}_1}{r_1^3(1 - u^2 \sin^2\theta_1/c^2)^{\frac{3}{2}}} \,, \quad \mathbf{R}_2 = \frac{\mathbf{r}_2}{r_2^3(1 - u^2 \sin^2\theta_2/c^2)^{\frac{3}{2}}} \,. \tag{53}$$

For points outside the cone or on the edge of the cone only one pair of values $(r, \theta)$ applies and so (50) is appropriate in those instances. Bradyons do not exhibit either the 'two source effect' or the 'optic boom' when travelling through a vacuum, so these effects are distinctive features of charged tachyons.

Now let frame $\Sigma'$, relative to which the tachyon is at rest, have velocity $\mathbf{u}$ relative to bradyonic frame $\Sigma$. The magnetic field measured by $\Sigma$ inside the Mach cone can be generalised using (46) to

$$\mathbf{B} = (\mathbf{u} \times \mathbf{E}/c^2) = \mathrm{sign}(\gamma_u) q\mu_0(u^2/c^2 - 1)(\mathbf{u} \times \mathbf{R}_1 + \mathbf{u} \times \mathbf{R}_2)/4\pi \,, \tag{54}$$

where $\mu_0 = 1/\epsilon_0 c^2$. Here again the two source effect causes a superposition of fields generated by the charged tachyon.

In the limit as $u \to \infty$ the electrically charged tachyon exhibits behaviour similar to that of a magnetic monopole. Assuming that $\theta_1$ and $\theta_2$ are not close to either $0°$ or $180°$, the electric field in (52) is proportional to $1/u$ for $u \to \infty$. For $\mathbf{u} \to \infty$ the magnetic field in (54) becomes approximately

$$\mathbf{B} = \frac{iq\mu_0 c}{4\pi} \left( \frac{\hat{\mathbf{u}} \times \mathbf{r}_1}{r_1^3 \sin^3\theta_1} + \frac{\hat{\mathbf{u}} \times \mathbf{r}_2}{r_2^3 \sin^3\theta_2} \right) , \tag{55}$$

where $\hat{\mathbf{u}}$ is a unit vector in the direction of $\mathbf{u}$. Hence $\mathbf{B}$ is not dependent on the magnitude of $\mathbf{u}$ in this limit and so the expression for the magnetic field of the tachyon looks similar in form to an expression describing the field of a magnetic monopole, with the extra term arising from the two source effect. However, because the magnetic field is purely imaginary in this case it is undetectable by a bradyonic observer.

Now consider the same limit $u \to \infty$ but with $\theta_1$ and $\theta_2$ being either 0° or 180°, so that the observer is now on the line of motion of the charged tachyon. In this case the magnetic field is negligible due to the cross products, so that the only remaining field term is purely electric and the tachyon looks like an electric charge with an effective field strength which diminishes as $1/u$ for $u \to \infty$.

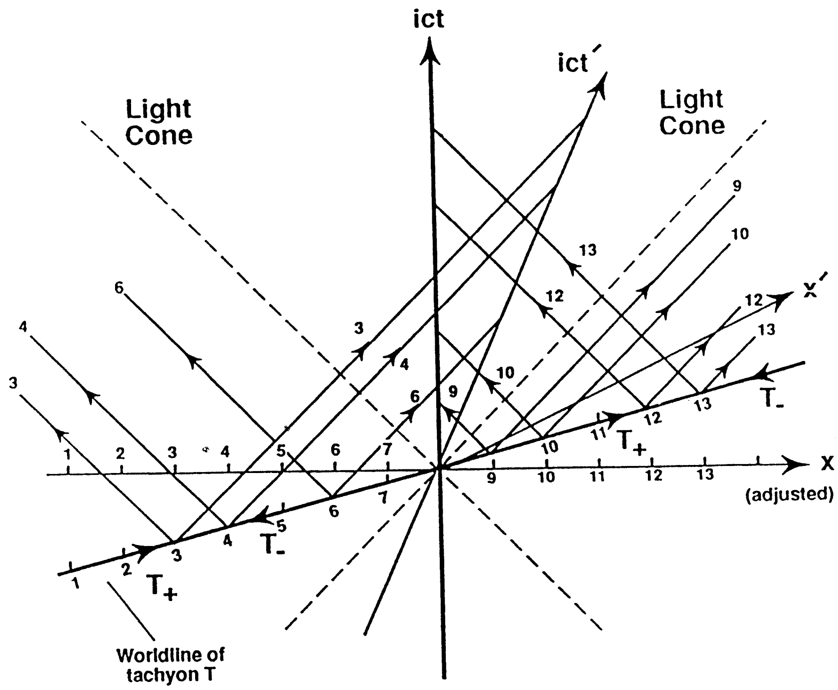


**Fig. 4.** Minkowski diagram showing a tachyon as it travels along the worldline emitting a pair of photons at each point on the numbered sequence. The photons are detected by bradyonic observers $\Sigma$ and $\Sigma'$ using axes $(x, ict)$ and $(x', ict')$ respectively.

The charged tachyon appears to a bradyonic observer to outstrip the electromagnetic fields, which gives rise to further distinctively tachyonic features and complications. One such complication is that in some bradyonic reference frames the tachyon appears to be switched, and so appears to carry the opposite sign of charge and to travel in the opposite direction. To aid in understanding what happens in such cases, Fig. 4 is a Minkowski diagram representing a tachyon emitting photons as it passes through a numbered sequence of points. Bradyonic observer $\Sigma$ uses axes $(x, ict)$, while the second bradyonic observer $\Sigma'$ uses axes $(x', ict')$. The test charge is at rest in frame $\Sigma$ and so it moves along the $ict$ axis. Each photonic worldline is numbered according to the point on the $x$-axis from which it was emitted. In frame $\Sigma$ the test charge receives the photons emitted during the approach of $T_+$ in the order 6, 4, 3 which is in reverse order to that of their emission. According to frame $\Sigma'$ this same section of the tachyon's worldline shows the switched tachyon $T_-$ receding from the test charge, which receives the photons in the order 6, 4, 3. In frame $\Sigma$ the test charge receives photons from the receding tachyon $T_+$ in the order 9, 10, 12, 13, so they are received in the same order as that in which they were emitted. In frame $\Sigma'$ this section of the switched tachyon's worldline represents the approach of $T_-$. An observer in frame $\Sigma'$ sees the test charge receive the photons in the order 9, 10, 12, 13, which is the same order in which they were emitted. Fig. 4, which clearly shows the two source effect experienced by the bradyonic observers $\Sigma$ and $\Sigma'$, is inspired by Recami (1986, Fig. 15). This example shows that a switched tachyon does not reverse the order in which the emitted photons are received by a bradyonic observer. This implies that the presence of tachyons does not alter a sequence of events involving photons and bradyons.

The above results contrast with what the test charge would experience if the charged object travelling past is not a tachyon, but is instead a bradyon B with charge $+q$. In this case the electromagnetic field would always arrive at the test charge before B and so the photons would always be received in the same order as their order of emission. Hence electromagnetic fields can only be received in reverse order to their order of emission if they are generated by charged tachyons. This effect is interpreted as a negative frequency (Mignani and Recami 1973, 1974*a*, 1974*b*) and so any equation describing the observed frequency as a function of initial frequency and relative speed must account for this effect: this will be discussed further in the next section on tachyonic optics. The two source effect does not apply for bradyons, because for any point inside a given field radius there is only one apparent source of the electromagnetic field. The test charge can detect the electromagnetic field generated by any receding charged object, regardless of whether it is bradyonic or tachyonic, but can only detect the approach of charged bradyons and not charged tachyons.

## 7. Tachyonic Optics

### (7a) Tachyonic Doppler Effect

From Fig. 4 it can clearly be seen that tachyons exhibit a Doppler effect, in which the spacing between wavefronts is compressed in the forward direction of motion and dilated in the opposite direction. The tachyonic Doppler effect and the possibility of superluminal expansions in astrophysics have been considered by Mignani and Recami (1973, 1974*a*, 1974*b*) and later by Recami *et al.* (1986). The following derivation of the tachyonic Doppler effect has been adapted from the SR case given by Helliwell (1966).

Consider a source which is at rest in frame $\Sigma'$, as shown in Fig. 5. The source emits a photon at angle $\theta'$ with respect to the $x'$ axis. In a bradyonic frame $\Sigma$ the photon has angle $\theta$ with respect to the $x$ axis. Frame $\Sigma'$ has speed $u$ relative to frame $\Sigma$ and moves along the common $x, x'$ axes.

For photons in frame $\Sigma'$ the energy $E'_{ph}$ is related to the frequency $\nu'$ by the relation $E'_{ph} = h\nu'$ where $h$ is Planck's constant. As other fundamental constants are the same in SR and ER and the photon's energy is real, then Planck's constant must also be the same for SR and ER. In frame $\Sigma$ the photon's energy is $E_{ph} = h\nu$, where $\nu$ is the observed frequency. The magnitude of the photon's

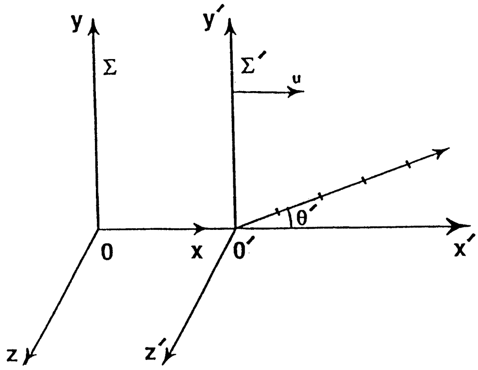


**Fig. 5.** A photon is emitted from a source at rest in frame $\Sigma'$ with energy $E'_{ph}$, frequency $\nu'$ and travels at an angle $\theta'$ with respect to the $x'$ axis. In frame $\Sigma$ the photon has angle $\theta$ with respect to the $x$ axis and has energy $E_{ph}$ and frequency $\nu$.

momentum in frame $\Sigma$ is $|\mathbf{k}| = E_{ph}/c = h\nu/c$ and so $k_x = h\nu\cos\theta/c$. Both the energy and momentum of the photon should transform in the same manner as for the energy and momentum of any other particle. The energy transformation derived in Paper II (p. 732) is

$$\begin{aligned} E'_{ph} &= \gamma_u(E_{ph} - uk_x) \quad \text{for } u^2 < c^2\,, \\ E'_{ph} &= i\gamma_u(E_{ph} - uk_x) \quad \text{for } u^2 > c^2\,, \end{aligned} \tag{56}$$

and substituting expressions for $E'_{ph}$, $E_{ph}$ and $k_x$ leads to the Doppler equations for SR and ER:

$$\begin{aligned} \nu &= \nu'(1-u^2/c^2)^{\frac{1}{2}}(1-u\cos\theta/c)^{-1} \quad \text{for } u^2 < c^2\,, \\ \nu &= \nu'(u^2/c^2-1)^{\frac{1}{2}}(1-u\cos\theta/c)^{-1} \quad \text{for } u^2 > c^2\,. \end{aligned} \tag{57}$$

The corresponding inverse transformations are

$$\begin{aligned} \nu' &= \nu(1-u^2/c^2)^{\frac{1}{2}}(1+u\cos\theta'/c)^{-1} \quad \text{for } u^2 < c^2\,, \\ \nu' &= -\nu(u^2/c^2-1)^{\frac{1}{2}}(1+u\cos\theta'/c)^{-1} \quad \text{for } u^2 > c^2\,. \end{aligned} \tag{58}$$

Once again the $i-\gamma$ convention has been used for $u^2 > c^2$. The expression for the tachyonic Doppler effect given here differs from that given by Mignani and Recami (1973, 1974*a*, 1974*b*) and also Recami and Mignani (1974) by a sign in (58) and also differs due to the possibility of switching changing the effective sign of the square root.

Equations (57) and (58) give real values for $u^2 > c^2$, so the tachyonic Doppler effect is real and therefore measurable in principle by a bradyonic observer. There are three main cases of interest: (i) $\theta = 0$, when the source approaches the observer, (ii) $\theta = \pi$, when the source recedes from the observer, and (iii) $\theta = \pm\pi/2$, when the source moves perpendicular to the observer. In each case $\nu'$ is the proper frequency of the source in its rest frame $\Sigma'$ and $\nu$ is the observed frequency in frame $\Sigma$.

Case (i): $\theta = 0$.

In this case the relative motion results in the source and the observer directly approaching each other, so that (57) reduces to

$$\nu = \nu'\{(1+u/c)/(1-u/c)\}^{\frac{1}{2}} \text{ for } u^2 < c^2\,,$$
$$\nu = -\nu'\{(u/c+1)/(u/c-1)\}^{\frac{1}{2}} \text{ for } u^2 > c^2\,. \tag{59}$$

Hence $|\nu| > |\nu'|$ for $-\infty < u < \infty$, so that relative approach of source and observer always results in the observed light being blueshifted. For an unswitched tachyonic source the positive root of (59) is used and so the observed frequency is negative. This agrees with Fig. 4, which shows that light emitted from the unswitched tachyonic source $T_+$ is blueshifted as it approaches the observer but is received in reverse order to its order of emission. In frames in which the tachyonic source is switched the negative root of (59) is used, so that the observed frequency is positive. This agrees with Fig. 4, which shows that light emitted from the switched tachyonic source $T_-$ is blueshifted as it approaches the observer and is received in the same order in which it was emitted. In the limit as $u \to \infty$ then $\nu \to -\text{sign}(\gamma_u)\nu'$, while in the limit as $u \to c^+$ then $\nu \to -\text{sign}(\gamma_u) \times \infty$, and for $u \to c^-$ then $\nu \to +\infty$.

Case (ii): $\theta = \pi$.

In this case the relative motion results in the source and the observer moving directly away from each other, so (57) gives

$$\nu = \nu'\{(1-u/c)/(1+u/c)\}^{\frac{1}{2}} \text{ for } u^2 < c^2\,,$$
$$\nu = \nu'\{(u/c-1)/(u/c+1)\}^{\frac{1}{2}} \text{ for } u^2 > c^2\,. \tag{60}$$

Hence $|\nu| < |\nu'|$ for $-\infty < u < \infty$, so that the observed light appears to be redshifted regardless of whether the source is bradyonic or tachyonic relative to the observer. For an unswitched tachyonic source the observed frequency is positive, in agreement with Fig. 4 in which the observed fields emitted during relative recession are received in the same order as their order of emission. For a switched tachyonic source the observed frequency of the receding source is negative, in agreement with Fig. 4. In the limit as $u \to \infty$ then $\nu \to \text{sign}(\gamma_u)\nu'$, while for the limits $u \to c^{\pm}$ then $\nu \to 0$.

Case (iii): $\theta = \pi/2$ (transverse motion).

In this case the relative motion results in the source and the observer moving perpendicular to each other, so (57) reduces to the SR and ER expressions for the transverse Doppler effect:

$$\nu = \nu'(1-u^2/c^2)^{\frac{1}{2}} \text{ for } u^2 < c^2\,,$$
$$\nu = \nu'(u^2/c^2-1)^{\frac{1}{2}} \text{ for } u^2 > c^2\,. \tag{61}$$

For both $u^2 < c^2$ and $c^2 < u^2 < 2c^2$ it can be seen that $|\nu| < |\nu'|$, so that the light appears to be redshifted. However, for $u^2 > 2c^2$ the light instead appears to the observer to be blueshifted as $|\nu| > |\nu'|$. This 'colour change' in the transverse Doppler effect is another consequence of having $|\gamma_u| = 1$ when $u^2 = 2c^2$, along with the contraction and dilation effects noted in Paper I. Once again the possibility of switching can change the apparent sign of $\nu$, so that $\nu$ is positive for unswitched sources and negative for a switched tachyonic source. In the limit as $u \to \infty$ then $\nu$ becomes proportional to $\text{sign}(\gamma_u)u\nu'/c$. In the limits $u \to c^{\pm}$ then $\nu \to 0$.

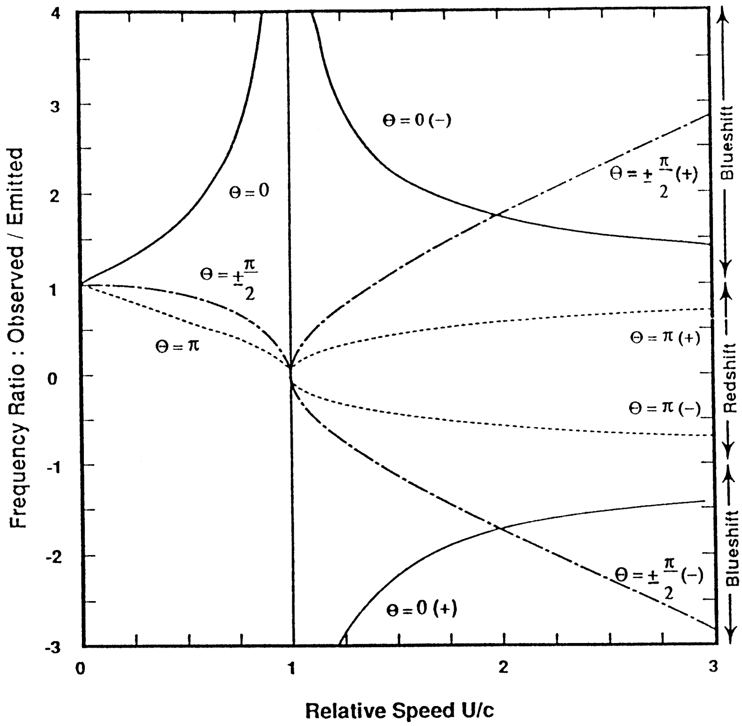


**Fig. 6.** Doppler effect for bradyons, unswitched tachyons (+) and switched tachyons (−). For relative approach $\theta = 0$ and the observed frequency is always blueshifted. For relative recession $\theta = \pi$ and the observed frequency is always redshifted. For transverse motion $\theta = \pm\pi/2$ and the observed frequency is redshifted for $0 < u/c < \sqrt{2}$ and blueshifted for $\sqrt{2} < u/c$. Negative frequency indicates that the light is received in reverse order to its order of emission.

Fig. 6 is a plot of each of these three cases of the Doppler effect for both the relativistic $(u/c < 1)$ and tachyonic $(u/c > 1)$ relative speed ranges. Relative approach $(\theta = 0)$ always results in a blueshift, relative recession $(\theta = \pi)$ always results in a redshift, while transverse motion is a redshift for $u/c < \sqrt{2}$ and a blueshift for $u/c > \sqrt{2}$. The sign in brackets denotes whether the tachyonic source appears to be unswitched (+) or switched (−) according to the observer.

The redshift and blueshift of the mutually receding images in the two source effect can be combined with knowledge of the order in which the fields are received so as to determine which apparent source is the receding tachyon and which apparent source is at the earlier position.

*(7b) Aberration*

Another effect caused by the relative motion of the source and the observer is that of the aberration of light, whereby the apparent direction of propagation of light differs between two different inertial reference frames. The following derivation of tachyonic aberration of light is adapted from the relativistic case given by Muirhead (1973).

Consider a source at rest in frame $\Sigma'$ which emits a photon at an angle $\theta'$ to the $x'$ axis, as shown in Fig. 5. The photon has momentum $\mathbf{k}'$ and energy $E'_{ph} = h\nu' = |\mathbf{k}'|c$. Frame $\Sigma'$ moves along the common $x, x'$ axes with speed $u$ relative to a bradyonic frame $\Sigma$. In this second frame the photon has momentum $\mathbf{k}$ and energy $E_{ph}$. The angle of emission in each frame is determined by the relations $\tan\theta = k_y/k_x$ and $\tan\theta' = k_{y'}/k_{x'}$.

The transformations of $\mathbf{k}$ and $E_{ph}$ from $\Sigma$ to $\Sigma'$ will have the same form as the corresponding transformations of $\mathbf{p}$ and $E$ derived in Paper II, so that for $u^2 > c^2$ the relevant components of $\mathbf{k}$ are $k_x = -i\gamma_u|\mathbf{k}'|(\cos\theta' + u/c)$ and $k_y = -i|\mathbf{k}'|\sin\theta'$. Hence

$$\tan\theta = \sin\theta'(1 - u^2/c^2)^{\frac{1}{2}}(\cos\theta' + u/c)^{-1}, \tag{62}$$

which describes the aberration of light due to a tachyonic source. It is identical to the SR aberration formula except for the speed range $u$. The inverse transformation is $\tan\theta' = \sin\theta(1 - u^2/c^2)^{\frac{1}{2}}(\cos\theta - u/c)^{-1}$, which is also valid for $-\infty < u < \infty$.

As the square root in (62) is imaginary for $u^2 > c^2$ then the expression gives an imaginary value for $\tan\theta$. This imaginary factor is due to the transverse space–time axes being imaginary for tachyonic frames and real for bradyonic frames (see Paper I). A quote by Corben (1978) is appropriate here: 'As before the appearance of imaginary factors is related to the fact that, when tachyons are considered, some of the physical quantities we attempt to measure may in fact not be measurable'. Replacement of $1 - u^2/c^2$ with $u^2/c^2 - 1$ in (62) when $u^2 > c^2$ and taking the arctangent gives the correct magnitude of $\theta$, with the sign of $\theta$ also affected by the possibility of switching in some bradyonic frames.

Another complication arises because of the two source effect discussed earlier. For each point inside the Mach cone there should be two aberration formulae, one for each apparent tachyonic source: $\tan\theta_s = \sin\theta'_s(1 - u^2/c^2)^{\frac{1}{2}}(\cos\theta'_s + u/c)^{-1}$, where $s = 1$ or 2.

*(7c) Plane Waves*

The wavefunction describing a plane wave is of the form (Resnick 1968; Møller 1972):

$$\Psi = A\cos\{2\pi(x\cos\theta/\lambda + y\sin\theta/\lambda - \nu t)\}, \tag{63}$$

where $A$ is the amplitude, $\lambda$ is the wavelength, $\nu$ is its frequency and $\theta$ is the angle in the $x, y$ plane between the wave's direction of propagation and the $x$ axis.

Consider a train of monochromatic light waves of unit amplitude, emitted by a source at the origin $O'$ of frame $\Sigma'$: see Fig. 5. The wavefronts are sufficiently distant from $O'$ to be considered as plane waves. The propagation is of the form

$$\Psi' = \cos\{2\pi(x'\cos\theta'/\lambda' + y'\sin\theta'/\lambda' - \nu' t')\} \tag{64}$$

with the wave speed being $\lambda'\nu' = c$. In a second inertial frame $\Sigma$ the wavefronts are still planes, as both SR and ER transformations turn a plane in $\Sigma'$ into a plane in $\Sigma$. Therefore the propagation of the wavefronts in frame $\Sigma$ is described by (63) with $A = 1$ and $\lambda\nu = c$.

Now suppose that frame $\Sigma'$ moves with speed $u^2 > c^2$ along the common $x, x'$ axes relative to bradyonic frame $\Sigma$. The appropriate transformations for $x'$, $y'$ and $t'$ are the SLTs given in (1), and substituting them into (64) leads to the propagation of the waves being described by

$$\Psi = \cos\{2\pi\,(-i\gamma_u x(\cos\theta' + u/c)/\lambda' - iy\sin\theta'/\lambda' + i\gamma_u\nu' t(1 + u\cos\theta'/c))\}\,. \tag{65}$$

As both (63) and (65) describe the same plane wave as seen in frame $\Sigma$, the coefficients of $x$, $y$ and $t$ from each expression can be equated. Comparing the $x$ and $y$ components respectively gives

$$\cos\theta/\lambda = \;-\,i\gamma_u(\cos\theta' + u/c)/\lambda'\,, \tag{66}$$

$$\sin\theta/\lambda = \;-\,i\sin\theta'/\lambda'\,. \tag{67}$$

Dividing (67) by (66) gives (62), which is the expression for the aberration of light from a tachyonic source. Comparing the $t$ component leads to the inverse transformation of the tachyonic Doppler effect (58) for $u^2 > c^2$. This again demonstrates the internal consistency of this formulation of the theory of tachyons by having derivations based on different methods giving the same results.

There is another result which at first appears to be somewhat peculiar. Using $\nu = c/\lambda$ and $\nu' = c/\lambda'$ in (58) gives $1/\lambda = -i\gamma_u(1 + u\cos\theta'/c)/\lambda'$ and using this relation to eliminate $\lambda$ and $\lambda'$ from (66) produces

$$\cos\theta = (\cos\theta' + u/c)(1 + u\cos\theta'/c)^{-1}\,. \tag{68}$$

This expression describes the transformation of cosines between frames $\Sigma'$ and $\Sigma$ and is the same for $u^2 < c^2$ and $u^2 > c^2$. Inserting test values for $u$ and $\theta'$ into (68) shows that $|\cos\theta| \leq 1$ for $u^2 < c^2$ and $|\cos\theta| \geq 1$ for $u^2 > c^2$. Hence the equation produces a mathematical absurdity for $u^2 > c^2$, although in spite of this, (68) appears to be the 'correct' transformation, in that it gives the correct conversion for the tachyonic Doppler equation when exchanging reference frames.

Both $\lambda$ and $\lambda'$ can be eliminated from (67) to give the sine transformation

$$\sin\theta = \sin\theta'(1 - u^2/c^2)^{\frac{1}{2}}(1 + u\cos\theta'/c)^{-1}\,, \tag{69}$$

which applies for both $u^2 < c^2$ and $u^2 > c^2$. Examining (69) shows that when $u^2 > c^2$, $\sin\theta$ is imaginary. This is because the tachyonic transverse axes are imaginary according to a bradyonic observer (see Paper I), and so the factor of $i$ is interpreted as being the conversion between an imaginary axis and a real axis in complex space–time. Inserting test values for $\theta'$ and $|u/c| > 1$ in (69) gives

$|\sin\theta| \leq 1$ in most cases, while using test values in limits such as $\theta' \to \pm\pi/2$ when $u \gg c$, or $\cos\theta' \to -c/u$, give $|\sin\theta| > 1$.

This is analogous to the question of 'absurd angles' in the treatment of Fresnel relations encountered in SR (Panofsky and Phillips 1962). Even though the transformations may give $|\sin\theta'|$ greater than 1 or $\cos\theta'$ as being imaginary for specific cases, the Fresnel relations still give meaningful results with regard to reflection and refraction of electromagnetic waves at a plane boundary. Similarly, in the present ER case it is the final result which matters, not the intermediate steps.

The problem of 'absurd angles' occurs for other trigonometric transformations in ER and is caused by combining the usual contraction and dilation effects of the tachyonic relative speed with the distortion due to the behaviour of imaginary and real axes in complex space–time, which simply does not happen in SR. Hence the concept of transforming trigonometric functions between bradyonic and tachyonic frames requires interpretation. The comment above about the Fresnel relations in SR is again relevant.

## 8. Electromagnetic Four-potential

### *(8a) Transformation of the Scalar and Vector Potentials*

Earlier sections in this paper have dealt with the electric and magnetic fields generated by a charged tachyon. Here the associated electromagnetic scalar and vector potentials produced by a charged tachyon will be studied.

The scalar and vector potentials in bradyonic frame $\Sigma$ are $\phi$ and $\mathbf{A}$ respectively and are related to the fields $\mathbf{E}$ and $\mathbf{B}$ via the following equations:

$$\mathbf{E} = -\nabla\phi - \partial\mathbf{A}/\partial t\,, \tag{70}$$

$$\mathbf{B} = \nabla \times \mathbf{A}\,. \tag{71}$$

In tachyonic frame $\Sigma'$ the potentials $\phi'$ and $\mathbf{A}'$ are related to the fields $\mathbf{E}'$ and $\mathbf{B}'$ via equations of the same form:

$$\mathbf{E}' = -\nabla'\phi' - \partial\mathbf{A}'/\partial t'\,, \tag{72}$$

$$\mathbf{B}' = \nabla' \times \mathbf{A}'\,. \tag{73}$$

Using the transformations of the partial derivatives (18) and the electromagnetic field transformations (21) and (22), it can be shown by substitution that for $u^2 > c^2$ the potentials transform as follows:

$$A_{x'} = i\gamma_u(A_x - u\phi/c^2)\,,\ A_{y'} = iA_y\,,\ A_{z'} = iA_z\,,\ \phi' = i\gamma_u(\phi - uA_x)\,. \tag{74}$$

The inverse transformations in ER are

$$A_x = -i\gamma_u(A_{x'} + u\phi'/c^2)\,,\ A_y = -iA_{y'}\,,\ A_z = -iA_{z'}\,,\ \phi = -i\gamma_u(\phi' + uA_{x'})\,. \tag{75}$$

Hence for $u^2 > c^2$ the potentials $\phi'$ and $\mathbf{A}'$ transform as a spacelike four-vector $(\mathbf{A}', i\phi'/c)$:

$$-i^2\phi'^2/c^2 - A_{x'}^2 - A_{y'}^2 - A_{z'}^2 = i^2\phi^2/c^2 + A_x^2 + A_y^2 + A_z^2\,. \tag{76}$$

The transformations of $\phi$ and $\mathbf{A}$ given here differ from those of Corben (1975) by a sign in the transverse components. If the upper sign is used in Corben's $\gamma$-factor then the longitudinal components $A_x$ and $\phi$ transform in the same way in the two formulations. The difference in the transverse components is due to the slightly different form of the two SLTs.

The transformations of $\phi$ and $\mathbf{A}$ given here disagree with the discussion of superluminal potentials by Mignani and Recami (1975). In their formulation the potentials are treated in a different manner by defining a complex four-potential and a complex four-current for tachyons. Mignani and Recami used this method to effectively add a term to the right-hand side of two of Maxwell's equations in order to symmetrise the set, with the result that superluminal charges behave like magnetic monopoles. Such a procedure is inappropriate in the present formulation as Maxwell's equations in free space are considered to be fundamental laws of physics and so, by postulate 1, must be valid in all reference frames regardless of whether the inertial reference frames or the charges are intrinsically bradyonic or tachyonic. This procedure is also in line with Corben's (1978) comment to the effect that the electromagnetic potentials, being four-vectors, must transform according to the same rules as (1).

In bradyonic frame $\Sigma$ the potentials $\phi$ and $\mathbf{A}$ satisfy the equations

$$\nabla.\mathbf{A}+\frac{1}{c^2}\frac{\partial\phi}{\partial t}=0\,, \tag{77}$$

$$\nabla^2\mathbf{A}-\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2}=-\mu_0\mathbf{j}\,, \tag{78}$$

$$\nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=-\rho/\epsilon_0\,, \tag{79}$$

while in tachyonic frame $\Sigma'$ the potentials $\phi'$ and $\mathbf{A}'$ satisfy identical equations with primed variables replacing unprimed variables. Each of these equations applies to potentials in free space: the modified forms for the potentials in a material medium will appear in the next paper in this series (Paper IV).

*(8b) The Four-potential of a Moving Tachyonic Charge*

The study of the electromagnetic field of a moving tachyonic charge showed distinct features such as the two source effect and the optic boom. Similar effects also occur for the scalar and vector potentials of a tachyonic charge and will be discussed below. The following derivation has been adapted from the corresponding relativistic case given by Rosser (1964).

Consider a tachyon carrying an electric charge $+q'$ as measured in its rest frame $\Sigma'$, which is moving with uniform velocity $\mathbf{u}$ through free space parallel to the common $x, x'$ axes relative to bradyonic frame $\Sigma$ as shown in Fig. 7. Due to the possibility of switching the apparent charge in frame $\Sigma$ is $q=\mathrm{sign}(\gamma_u)q'$. In tachyonic frame $\Sigma'$ the charge has position $(x', y', z')$ at a distance $r'$ relative to the point of observation $O'$. The origins $O$ and $O'$ of frames $\Sigma$ and $\Sigma'$ are

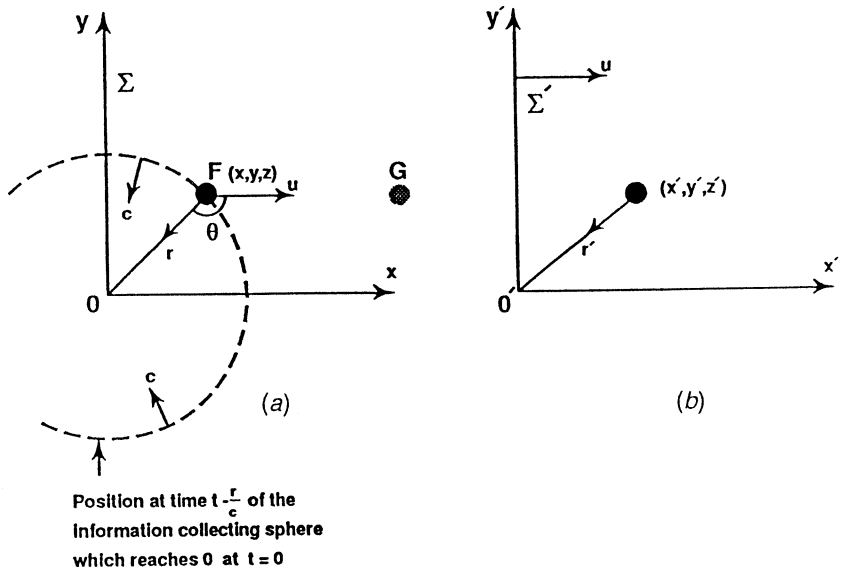


**Fig. 7.** Scalar and vector potentials of a tachyonic point charge moving with speed $u$ along the common $x, x'$ axes. In $(a)$ the information collecting sphere which carries the information from the charge to the origin to be calculated at time $t = 0$ leaves the retarded position F at a time $t - r/c$. During the time interval in which the information regarding the potentials travels from F to O the charge has moved from F to G. In $(b)$ the charge is at rest in frame $\Sigma'$, which has a uniform speed $u$ relative to $\Sigma$ along the common $x, x'$ axes. The origins of $\Sigma$ and $\Sigma'$ are coincident at $t = t' = 0$.

synchronised so that they coincide at the instant when $t' = t = 0$. In frame $\Sigma$ the point of observation is at $O$. Fig. 7 shows that if the tachyon produces potentials at position F, then the tachyon has moved to position G by the time the information collecting sphere, travelling with speed $c$, has reached the origin $O$ from position F. This sphere has speed $c$ due to the second postulate of ER, even though the source producing the potentials is a tachyon. Hence the position of the tachyon when it actually produced the scalar and vector potentials measured at $O$ is the 'retarded position' $(x, y, z)$ at a time $t - r/c$, as the information requires an elapsed time equal to $r/c$ to travel from F to $O$.

The potentials in frame $\Sigma'$ at the origin $O'$ are given by

$$\phi' = q'/4\pi\epsilon_0 r', \qquad \mathbf{A}' = 0\,, \tag{80}$$

where $r'^2 = x'^2 + y'^2 + z'^2$. The values of $\mathbf{A}$ and $\phi$ at the origin of $\Sigma$ at $t = 0$ for $u^2 > c^2$ can be found using (80) and the transformations (75):

$$A_x = -\mathrm{sign}(\gamma_u) i\mu_0 u\gamma_u q/(4\pi r')\,,\ A_y = A_z = 0\,,\ \phi = -\mathrm{sign}(\gamma_u) i\gamma_u q/4\pi\epsilon_0 r'\,. \tag{81}$$

The corresponding expressions for $u^2 < c^2$ can be obtained by removing the factor $-\mathrm{sign}(\gamma_u)i$. As these are electromagnetic potentials, in frame $\Sigma'$ the information travels with speed $c$ from the particle's stationary position to the point of observation at the origin $O'$. Therefore it takes an elapsed time equal to $r'/c$ for the information to go from position $(x', y', z')$ to $O'$ and hence the distance $r'$ is measured at a time $t' = -r'/c$. This means that the inverse SLTs lead to

$$x = -i\gamma_u(x' - ur'/c)\,,\ y = -iy'\,,\ z = -iz'\,,\ t = -i\gamma_u(-r' + ux'/c)/c\,.$$

Substituting these expressions into $r^2 - c^2t^2 = x^2 + y^2 + z^2 - c^2t^2$ gives $r^2 - c^2t^2 = r'^2 - x'^2 - y'^2 - z'^2 = 0$. Of the two possible solutions $r = \pm ct$, the one chosen is $r = -ct$ as it corresponds to the position of the charge at the time $t = -r/c$ in frame $\Sigma$ and time $t' = -r'/c$ in frame $\Sigma'$. This is the same result as in SR, once again showing how the effect of the imaginary factors disappears to yield the expected result. Note that the vector $\mathbf{r}$ measured at the time $t = -r/c$ is taken from the retarded position of the charge to the point of observation.

Using $t = -r/c$ in the ER transformation $t' = i\gamma_u(t - ux/c^2)$ gives $r' = -ct' = i\gamma_u(r + ux/c)$. As $\mathbf{u}.\mathbf{r} = ur\cos\theta = -ux$, where $\theta$ is the angle between $\mathbf{u}$ and $\mathbf{r}$ (see Fig. 7) then

$$r'/\gamma_u = i(r - \mathbf{u}\,.\,\mathbf{r}/c)\,. \tag{82}$$

The corresponding SR case can be obtained by deleting the factor of $i$. Using (82) in (81) with $t = 0$ gives the potentials measured in frame $\Sigma$ for $u^2 > c^2$:

$$A_x = -\frac{\mu_0}{4\pi}\left(\frac{\mathrm{sign}(\gamma_u)uq}{r - \mathbf{u}\,.\,\mathbf{r}/c}\right)\,,\ A_y = A_z = 0\,,\ \phi = -\frac{1}{4\pi\epsilon_0}\left(\frac{\mathrm{sign}(\gamma_u)q}{r - \mathbf{u}\,.\,\mathbf{r}/c}\right)\,. \tag{83}$$

The corresponding SR expressions can be obtained by deleting the factor of $-\mathrm{sign}(\gamma_u)$ in (83).

The above derivation is of course incomplete as the two source effect is once again applicable, just as it was for the discussion of the fields produced by a charged tachyon. Fig. 8 shows how the two source effect applies to the retarded scalar and vector potentials. The information collecting sphere travelling with speed $c$ intersects the path of the tachyon at two points, instead of at a single point had the charged particle been a bradyon. Fig. 8 clearly shows there are two retarded positions for the tachyon, given by $(r_1, \theta_1)$ and $(r_2, \theta_2)$. The derivation given above applies to the point whose coordinates are $(x_1, y_1, z_1)$ so that $t_1 = -r_1/c$. The particle is stationary in tachyonic frame $\Sigma'$ so that $t' = -r'/c$ as before. The earlier retarded position has coordinates $(x_2, y_2, z_2)$ so that $t_2 = -r_2/c$. The derivation above also applies to this point, so that $r' = -ct' = i\gamma_u(r_2 + ux_2/c)$ for the earlier retarded position, with $r' = -ct' = i\gamma_u(r_1 + ux_1/c)$ for the later retarded position. Earlier equations involving quantities measured in frame $\Sigma$ can also be written as two separate equations with subscripts '1' or '2' depending upon the retarded position being discussed.

For the earlier position $\mathbf{u}.\mathbf{r}_2 = ur_2\cos\theta_2 = +ux_2$ (see Fig. 8) so that $r'/\gamma_u = i(r_2 + \mathbf{u}.\mathbf{r}_2/c)$. Hence the net scalar and vector potentials measured at

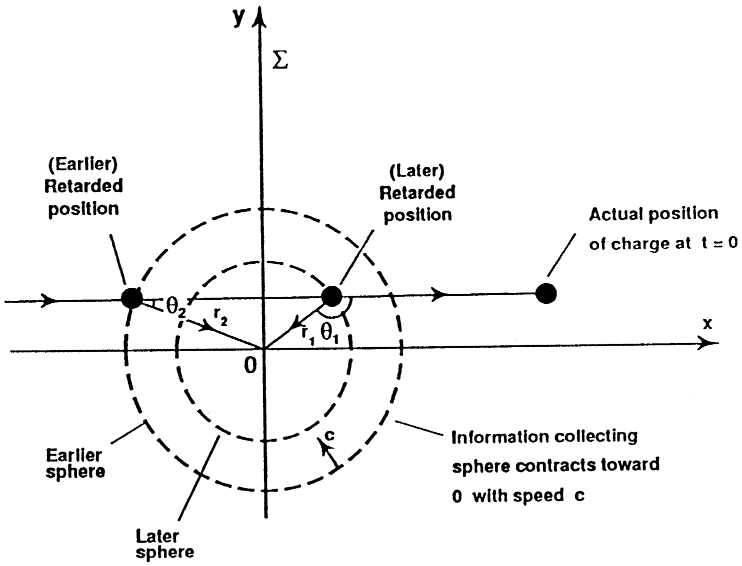


**Fig. 8.** Origin of the two source effect for scalar and vector potentials generated by a charged tachyon. The tachyon intersects the information collecting sphere at one point, and in the time it takes for the sphere to partially contract towards O at the speed of light the tachyon has crossed the sphere and intersected it on the other side. Therefore at the instant the potentials are measured at the origin at time $t = 0$ there are two contributions, one from each of the retarded positions, while the tachyon itself has exited the sphere.

$O$ in bradyonic frame $\Sigma$ due to the superposition of the potentials produced at each of the retarded positions is

$$A_{x(total)} = -\frac{\mathrm{sign}(\gamma_u)\mu_0 u q}{4\pi}\left(\frac{1}{r_1 - \mathbf{u}\,.\,\mathbf{r}_1/c} + \frac{1}{r_2 + \mathbf{u}\,.\,\mathbf{r}_2/c}\right),$$

$$A_{y(total)} = A_{z(total)} = 0\,,$$

$$\phi_{(total)} = -\frac{\mathrm{sign}(\gamma_u) q}{4\pi\epsilon_0}\left(\frac{1}{r_1 - \mathbf{u}\,.\,\mathbf{r}_1/c} + \frac{1}{r_2 + \mathbf{u}\,.\,\mathbf{r}_2/c}\right). \tag{84}$$

These expressions can be generalised to give the tachyonic Liénard–Wiechert potentials:

$$\mathbf{A}_{(total)} = -\frac{\mu_0 q}{4\pi}\left(\frac{\mathrm{sign}(\gamma_{u_1})\mathbf{u}_1}{r_1 - \mathbf{u}_1\,.\,\mathbf{r}_1/c} + \frac{\mathrm{sign}(\gamma_{u_2})\mathbf{u}_2}{r_2 + \mathbf{u}_2\,.\,\mathbf{r}_2/c}\right),$$

$$\phi_{(total)} = -\frac{1}{4\pi\epsilon_0}\left(\frac{\mathrm{sign}(\gamma_{u_1}) q}{r_1 - \mathbf{u}_1\,.\,\mathbf{r}_1/c} + \frac{\mathrm{sign}(\gamma_{u_2}) q}{r_2 + \mathbf{u}_2\,.\,\mathbf{r}_2/c}\right). \tag{85}$$

Each set of values $(\mathbf{r}_1, \mathbf{u}_1)$ and $(\mathbf{r}_2, \mathbf{u}_2)$ refer to the retarded positions of the tachyonic charge. The different values $\mathbf{u}_1$ and $\mathbf{u}_2$ refer to the tachyon's velocity at each of the retarded positions. Note that if $u_1^2 > c^2$ then $u_2^2 > c^2$, as a tachyon in one bradyonic frame appears to be a tachyon in any other bradyonic frame.

## 9. Generalised Four-vector Transformations in ER

The inherent similarity in the form of the transformations of four-vectors in SR and ER indicates that they can be expressed as a more generalised matrix equation. As ER transformations only differ from the corresponding SR cases in factors of $+i$ or $-i$ and in having a different range of allowable relative speeds for $u$, it is expected that the generalised matrix equation which applies for $u^2 > c^2$ will be only slightly changed from the equation for $u^2 < c^2$.

In SR the transformation of any four-vector $B_\lambda$ can be expressed as (Rosser 1964; Lawden 1975)

$$B'_\lambda = \sum_{\nu=1}^{4} L_{\lambda\nu} B_\nu \,, \tag{86}$$

where $L_{\lambda\nu}$ is a $4 \times 4$ matrix such that

$$L_{\lambda\nu} = \begin{bmatrix} \gamma_u & 0 & 0 & iu\gamma_u/c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -iu\gamma_u/c & 0 & 0 & \gamma_u \end{bmatrix} \tag{87}$$

The corresponding inverse transformation is

$$B_\nu = \sum_{\lambda=1}^{4} L'_{\nu\lambda} B'_\lambda \,, \tag{88}$$

where the matrix $L'_{\nu\lambda}$ is the inverse of $L_{\lambda\nu}$ and is given by

$$L'_{\nu\lambda} = \begin{bmatrix} \gamma_u & 0 & 0 & -iu\gamma_u/c \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ iu\gamma_u/c & 0 & 0 & \gamma_u \end{bmatrix} \tag{89}$$

Hence $L_{\lambda\nu} L'_{\nu\lambda} = I_4$ where $I_4$ is the $4 \times 4$ identity matrix. Examples of four-vectors which obey (86) and (88) in SR include

- space–time position $X_\lambda = (\mathbf{x}, ict)$,
- energy–momentum $P_\lambda = (\mathbf{p}, iE/c)$,
- charge and current density $J_\lambda = (\mathbf{j}, ic\rho)$,
- electromagnetic potential $A_\lambda = (\mathbf{A}, i\phi/c)$,
- partial derivatives $D_\lambda = (\partial/\partial\mathbf{x},\ \partial/ic\partial t)$.

For tachyonic transformations it is necessary to include the factor of either $+i$ or $-i$ which appears in each term, so that the four-vectors transform according to

$$B'_\lambda = \pm i \sum_{\nu=1}^{4} L_{\lambda\nu} B_\nu\,, \tag{90}$$

with the inverse transformation being

$$B_\nu = \mp i \sum_{\lambda=1}^{4} L'_{\nu\lambda} B'_\lambda\,. \tag{91}$$

It must of course be remembered that frame $\Sigma'$ is now a tachyonic frame while frame $\Sigma$ remains a bradyonic frame. The matrices $L_{\lambda\nu}$ and $L'_{\nu\lambda}$ are still defined by (87) and (89), even though the generalised transformations given by (90) and (91) apply for $u^2 > c^2$. The upper sign in (90) and (91) applies to the four-vectors $X_\lambda$, $P_\lambda$ and $A_\lambda$, while the lower sign applies to the four-vectors $J_\lambda$ and $D_\lambda$.

The square of all the four-vectors listed above is

$$\sum_{\lambda=1}^{4} B'^2_\lambda = \pm \sum_{\lambda=1}^{4} B^2_\lambda\,, \tag{92}$$

where the upper (+) sign applies for $u^2 < c^2$ and the lower (−) sign applies for $u^2 > c^2$.

There is a third class of transformations in SR and ER which applies to quantities which normally only make up three-vectors, such as velocity and force. These quantities have the common properties that their transformations are exactly the same for $-\infty < u < \infty$ and that transformations of the components perpendicular to the boost contain factors of $\gamma$. As these quantities have the same transformation in SR and ER, then the square of their four-vector $B_\lambda$ is

$$\sum_{\lambda=1}^{4} B'^2_\lambda = \sum_{\lambda=1}^{4} B^2_\lambda\,. \tag{93}$$

One example is the velocity four-vector $U_\lambda = (\gamma_u \mathbf{u}, ic\gamma_u)$ for which $U_\lambda U_\lambda = -c^2$ for $-\infty < u < \infty$, while a second example is the force four-vector $F_\lambda = (\gamma_u \mathbf{F},\, i\gamma_u \mathbf{F}.\mathbf{u}/c)$.

Specific four-vector transformations derived in this formulation do not have extra double signs for $u^2 > c^2$, as is the case for similar transformations derived by Olkhovsky and Recami (1971), Recami and Mignani (1972), Mignani *et al.* (1972), Antippa (1972) and Corben (1975). Mignani *et al.* (1972) pointed out that double signs are necessary in order to allow generalised Lorentz transformations to have an inverse transformation. This is consistent with the choice made by Recami and Mignani (1974) in choosing to develop a generalised law which applies to bradyons and tachyons in a different manner, as discussed in Section 3. The present authors have found that by electing to develop ER using the simpler choice of the possible approaches following Corben (1975, 1976, 1978), in which tachyons and bradyons obey the same fundamental laws of physics, there is no longer any sign ambiguity in the ER transformations.

## 10. Electromagnetic Four-tensors

Having developed the tachyonic transformations of various four-vectors and electromagnetic quantities, it is now possible to discuss the ER transformations of some electromagnetic four-tensors. The first such tensor is the electromagnetic field tensor $F_{\alpha\beta}$ given by

$$F_{\alpha\beta} = \begin{bmatrix} 0 & B_z & -B_y & -iE_x/c \\ -B_z & 0 & B_x & -iE_y/c \\ B_y & -B_x & 0 & -iE_z/c \\ iE_x/c & iE_y/c & iE_z/c & 0 \end{bmatrix} . \tag{94}$$

The following discussion is adapted from the SR case given by Lawden (1975).

The four-vector potential $A_\lambda$ in inertial frame $\Sigma$ is defined to be $A_\lambda = (\mathbf{A}, i\phi/c)$ and so the equations describing the relations between the vector and scalar potentials, (77), (78) and (79), are then equivalent to

$$\Box^2 A_\lambda = -\mu_0 J_\lambda \,, \tag{95}$$

where

$$\Box^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \tag{96}$$

and $J_\lambda = (\mathbf{j}, ic\rho)$ is the four-current density. The corresponding equations in tachyonic frame $\Sigma'$ are

$$\Box'^2 A'_\lambda = -\mu_0 J'_\lambda \,, \tag{97}$$

where

$$\Box'^2 = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2} \tag{98}$$

and $J'_\lambda = (\mathbf{j}', ic\rho')$. By using $A_\lambda$ with (70) and (71) it can be shown that the field tensor components may be written as

$$F_{\alpha\beta} = \partial A_\beta/\partial X_\alpha - \partial A_\alpha/\partial X_\beta \,. \tag{99}$$

As the individual components of $F_{\alpha\beta}$ obey the same set of transformations for $u^2 < c^2$ and $u^2 > c^2$ then the field tensor in frame $\Sigma'$ can be written as

$$F'_{\alpha\beta} = \begin{bmatrix} 0 & B_{z'} & -B_{y'} & -iE_{x'}/c \\ -B_{z'} & 0 & B_{x'} & -iE_{y'}/c \\ B_{y'} & -B_{x'} & 0 & -iE_{z'}/c \\ iE_{x'}/c & iE_{y'}/c & iE_{z'}/c & 0 \end{bmatrix} \tag{100}$$

It can also be shown by using $A'_\lambda$ with (72) and (73) that the electromagnetic field tensor $F'_{\alpha\beta}$ in tachyonic frame $\Sigma'$ is

$$F'_{\alpha\beta} = \partial A'_\beta/\partial X'_\alpha - \partial A'_\alpha/\partial X'_\beta \,. \tag{101}$$

By substituting the tachyonic transformations for the partial derivatives (18) and electromagnetic potentials (74) into (101) it can be shown that

$$F'_{\alpha\beta} = \begin{bmatrix} 0 & \gamma_u(B_z - uE_y/c^2) & -\gamma_u(B_y + uE_z/c^2) & -iE_x/c \\ -\gamma_u(B_z - uE_y/c^2) & 0 & B_x & -i\gamma_u(E_y - uB_z)/c \\ \gamma_u(B_y + uE_z/c^2) & -B_x & 0 & -i\gamma_u(E_z + uB_y)/c \\ iE_x/c & i\gamma_u(E_y - uB_z)/c & i\gamma_u(E_z + uB_y)/c & 0 \end{bmatrix}.$$

This same result could have been obtained simply by transforming the individual tensor components according to (21) and (22), which are valid for $-\infty < u < \infty$. Hence for $u^2 > c^2$ the electromagnetic field tensor transforms according to

$$F'_{\alpha\beta} = \sum_{\mu=1}^{4} \sum_{\nu=1}^{4} L_{\alpha\mu} F_{\mu\nu} L'_{\nu\beta} \,. \tag{102}$$

As this is exactly the same transformation as for $u^2 < c^2$, then (102) is valid for $-\infty < u < \infty$. The tachyonic four-vectors involved in this transformation are $A_\lambda$ and $D_\lambda$ which transform according to the upper and lower signs respectively in (90), so the factors of $+i$ and $-i$ in the ER transformations of these quantities will combine to give $+1$ for the field tensor. This again demonstrates how the imaginary factors in this formulation cancel out when appropriate to produce reasonable and consistent results.

The components of the electromagnetic stress–energy tensor $T_{\alpha\beta}$ are given by (Muirhead 1973; Jackson 1975; Lawden 1975)

$$T_{ij} = \delta_{ij}(\epsilon_0 E^2 + B^2/\mu_0)/2 - (\epsilon_0 E_i E_j + B_i B_j/\mu_0)\,, \tag{103}$$

$$T_{j4} = T_{4j} = i(\mathbf{E} \times \mathbf{B})_j/c\mu_0 = i\mathbf{S}_j/c\,, \tag{104}$$

$$T_{44} = -(\epsilon_0 E^2 + B^2/\mu_0)/2 = -U_{em}\,, \tag{105}$$

where $\mathbf{S}$ is Poynting's vector and $U_{em}$ is the energy density of the electromagnetic field. As the components of $T_{\alpha\beta}$ undergo the same transformations for $u^2 > c^2$

as they do for $u^2 < c^2$, then $T'_{\mu\nu}$ must be related to $T_{\alpha\beta}$ by

$$T'_{\mu\nu} = \sum_{\alpha=1}^{4} \sum_{\beta=1}^{4} L_{\mu\alpha} T_{\alpha\beta} L'_{\beta\nu} \tag{106}$$

for $-\infty < u < \infty$. Further electromagnetic four-tensors involving electric displacement and magnetic field strength will be discussed in the next paper in this series (Paper IV), which will examine further aspects of electrodynamics for tachyons.

## 11. Conclusion

It has now been shown that charged tachyons can be incorporated into the theory of electromagnetism in a logical and consistent manner. Tachyons obey Maxwell's equations in free space, as required by the first postulate of ER in which 'the laws of physics are the same in all inertial systems'. The imaginary factors and high relative speeds appearing in the theory of tachyons do not change the transformations of $\mathbf{E}$ and $\mathbf{B}$, the form of the Lorentz force law or the transformations of the electromagnetic field tensor or stress–energy tensor.

The total electric charge in any given inertial reference frame is always conserved, but the apparent charge is no longer the same when measured by different observers due to some of the tachyons appearing to be switched in some frames. This does not present a problem, as it is always possible to transform to another reference frame in which all of the tachyons appear to be unswitched. Then the total charge is the same as if all the particles were bradyons instead.

Fundamental constants such as the permittivity and permeability of free space have been shown to be the same regardless of whether the observer's inertial reference frame is bradyonic or tachyonic. This is a consequence of the two postulates of ER, but the rigorous derivation of this result serves to demonstrate the internal consistency of this formulation of tachyon theory. This point is significant for other branches of physics, not just for electromagnetism. In quantum mechanics it can be safely assumed that Planck's constant is invariant for bradyons and tachyons, thus simplifying some of the derivations for quantum tachyons which have been adapted from the corresponding relativistic cases (Dawe 1990). Dawe *et al.* (1989) have used an invariant Boltzmann constant when developing statistical mechanics and thermodynamics for tachyons.

The electric and magnetic fields (or alternatively the scalar and vector potentials) produced by a charged tachyon travelling through a vacuum at constant velocity are real and in principle detectable inside a Mach cone having semivertex angle $|c/u| = |\sin\theta|$, in agreement with Barut *et al.* (1982) and Recami (1986) and references therein. As the field is real and moves with constant speed $c$ regardless of the source speed, any point lying outside the cone corresponds to a position where the field is purely imaginary and is therefore undetectable. At the instant of contact with the cone describing the propagation of the field, any detection instruments would register a sudden jump called an 'optic boom', in analogy with the 'sonic boom' generated by supersonic aircraft (Barut *et al.* 1982). After the instant of initial contact, the image of the tachyon splits into two images travelling in opposite directions, an effect called the 'two source effect' (Recami *et al.* 1986; Recami 1986 and references therein). One of the images represents the tachyon as it travels forwards, while the second image which appears to go backwards is due purely to the time delay associated with electromagnetic effects having a fixed and finite speed, even though this is exceeded by the source speed. As neither the optic boom or the two source effect can be produced by individual particles which appear to be bradyons relative to the observer, then these effects would constitute definitive evidence of the existence of tachyons should they be detected in the laboratory.

The various tachyonic transformations were also shown to be consistent with the expected Doppler effects for relative approach (blueshift) and recession (redshift), including predicting the order in which signals from the tachyon would be received. The transverse Doppler effect for tachyons is a blueshift for $u^2 > 2c^2$ and a redshift for $u^2 < 2c^2$.

The overall result of the work presented in this paper is to demonstrate that charged tachyons, if they exist, can obey Maxwell's equations in a vacuum. In analogy with ordinary relativistic particles, tachyons have been shown to possess real and in principle detectable attributes such as an electromagnetic field and a Doppler effect. Moreover, in their own inertial reference frame, tachyons behave like bradyons and a comoving observer would consider them to be travelling more slowly than the propagation speed of electromagnetic radiation in a vacuum.

In the light of the above results for tachyonic electrodynamics in vacuo, further developments become possible and will be investigated in the next paper of the present series. The extension of the present work to cover tachyonic electrodynamics in a medium will be undertaken with the consideration of the electric displacement and polarisation vectors, together with the magnetic field intensity and magnetisation vectors. The treatment of these topics will be followed up by discussions of the electric dipole moment of a tachyonic current loop, constitutive equations and the velocity of light in a tachyonic medium. In order to provide a comprehensive picture of the behaviour of tachyons at the classical relativistic level as well as preparing the ground for tachyonic quantum mechanics, Paper IV will also contain a discussion of Lagrange's and Hamilton's equations for charged tachyons. There will also be an explanation of why tachyons can be considered to be effectively localised particles.

As foreshadowed in the Conclusion to Paper II, Paper IV will also consider the question of radiation emission by charged tachyons. This is a topic which is to be regarded as of critical importance in determining the existence or otherwise of tachyons. As pointed out by Treumann (1992), if tachyons can emit bremsstrahlung in collisions with other particles, regardless of whether the other particles are bradyonic or tachyonic, and also emit synchrotron radiation in interaction with a magnetic field, then this radiation should ultimately be detected by astrophysical observation. If such radiation is not detected then a choice must be made between the following alternatives: (i) tachyons do not exist, (ii) tachyons may exist but must be uncharged, or (iii) charged tachyons exist but radiate with a spectrum which is undetectable: this last option seems unphysical. Thus the study of electrodynamics for tachyons could lead to astrophysical observations to determine if any of these three options is appropriate, or if tachyons do in fact exist. In Paper IV a beginning is made with a treatment of Cerenkov radiation for tachyonic particles in tachyonic media.

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