Anomalous Anisotropic Diffusion of Electron Swarms in A.C. Electric Fields

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Abstract
A time-dependent multi-term solution of the Boltzmann equation is used to calculate the drift and diffusion coefficients of electron swarms in gases under the influence of a time varying electric field. Two model gases are considered and for a.c. electric fields results are presented for a wide range of applied frequencies. Of particular interest is the anomalous temporal behaviour of the longitudinal diffusion coefficient, which is discussed here for the first time.

1. Introduction
The phenomenon of anisotropic diffusion of electrons in gases in d.c. electric fields was first observed by Wagner et al. (1967) and subsequently explained theoretically by ‘two-term’ spherical harmonic solutions of the Boltzmann equation (Skullerud 1969; Parker and Lowke 1969; Lowke and Parker 1969; Francey 1969; Huxley 1972; Lowke 1973) and nonequilibrium thermodynamics (Robson 1972). The contrast between pronounced anisotropy in configuration space, as represented by the difference between the diffusion coefficients $D_L$ and $D_T$, longitudinal and transverse to the field $E$ respectively, and the near isotropy in velocity space, as indicated by the often rapid convergence of the expansion of the electron phase-space distribution function $f(r, c, t)$ in spherical harmonics,

$$f(r, c, t) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_{lm}(r, c, t)Y_{lm}(\hat{c}),$$

is quite striking, but is well understood nevertheless. Indeed, there have been many studies of ion and electron transport in d.c. fields over the last 50 years or so and in particular in the last two decades both theory and experiment have been developed to quite a high degree of sophistication (Huxley and Crompton 1974; Kumar et al. 1980; Robson and Ness 1986; Mason and McDaniel 1988; Viehland 1992; Schmidt 1993). In stark contrast, and in spite of the obvious significance for applications to modern day industry (e.g. plasma processing using r.f. discharges), the basic physics of charged particle transport in a.c. fields has not received anything like the same degree of attention, not even for simple, ‘benchmark’ models.
of interaction. With this motivation, we have developed what we believe is a comprehensive, fundamental theory of particles of charge $q$, mass $m$ in a.c. fields based upon solution of Boltzmann’s equation,

$$\left( \partial_t + c \cdot \nabla + qE/m \cos \omega t \cdot \partial_c + J \right) f(r, c, t) = 0$$

(2)

and the results will be reported in a series of papers elsewhere (White et al. 1996; Robson et al. 1996). The ultimate goal is to calculate all transport coefficients of interest for arbitrary $E/n_0$ (ratio of field amplitude to neutral gas density), arbitrary frequency-to-density ratios $\omega/n_0$ and for all types of interactions (elastic, inelastic, ionising, attaching) as incorporated in the collision operator $J$. If the density $n$ of charged particles is assumed so low compared to $n_0$ that mutual interactions between charged particles are negligible, then $J(f)$ is linear in $f$, representing charged particle–neutral molecule collisions only. The discussion ranges from fundamental physical reasoning and semiquantitative modeling (Robson et al. 1996) through to highly accurate (to 1% or better) solutions of (2), achieved by ‘multiterm’ representations of $f$, in which the upper limit of the summation over $l$ in (1) is set at whatever value necessary to achieve the desired accuracy (White et al. 1996). In the course of this investigation, we have observed many interesting phenomena which appear to be unique to a.c. fields and there is no more striking example of this than the anomalous nature of anisotropic diffusion, which we feel is of sufficient interest in its own right to warrant the immediate, separate discussion reported in the present article. Since this is the first time the phenomenon has been discussed, we choose to emphasise the fundamental physics, rather than implications for applications. However, the obvious need for a knowledge of both diffusion coefficients $D_L$ and $D_T$ in a.c. electric fields for the modeling of rf discharges is shown in the series of papers by Makabe and co-workers (Maeda and Makabe 1994a, b, c, 1995; Nakano et al. 1994a, b). Additional application of the present work could extend to analysing the effect of a finite chamber length in the original Cavalleri (1969) experiment. In summary this work removes the restrictions associated with the idealised assumptions of spatially uniform swarms which have dominated the literature to present (Ferreira and Loureiro 1983; Makabe and Goto 1988; Goto and Makabe 1990; Loureiro 1993).

2. Theory

In the time-dependent hydrodynamic regime, the space dependence of the phase-space distribution distribution $f(r, c, t)$ and all its moments may be assumed to be carried by the electron number density $n(r, t)$, the functional relationship usually (but not necessarily) being represented in the form of a density gradient expansion with time-dependent expansion coefficients (Kumar et al. 1980; Kumar 1981). In particular we define the ‘flux’ drift velocity $W(t)$, and the ‘flux’ diffusion tensor $D(t)$, in the conventional way, as the proportionality constants of $n(r, t)$ and $\nabla n(r, t)$ respectively in the gradient expansion of the swarm particle flux $\Gamma(r, t)$:

$$\Gamma(r, t) = n(c) = W(t)n - D(t)\nabla n(r, t) + \cdots ,$$

(3)
where
\[ D(t) = D_T(t)I + (D_L(t) - D_T(t)) \hat{a} \hat{a} \]

and \( D_L(t) \) and \( D_T(t) \) represent the longitudinal and transverse diffusion coefficients. In (3) \( \langle c \rangle \) is the local instantaneous average velocity and \( \mathbf{W}(t) \) can be thought of as the spatial average of this. In addition the gradient expansion of the mean energy is given by
\[ \varepsilon(r,t) = \langle \frac{1}{2}mc^2 \rangle = \varepsilon(t) + \gamma(t) \cdot \frac{1}{n} \nabla n(r,t) + \ldots, \]

where \( \varepsilon(t) \) is the spatial average of the mean energy \( \varepsilon(r,t) \) and \( \gamma(t) = \gamma(t) \hat{a} \) is a quantity, especially important to the present discussion, which we shall refer to as the ‘gradient energy’ parameter. Other notation is that \( \langle \ldots \rangle \) represents a velocity average with \( f(r,c,t) \), \( I \) is the unit tensor and \( \hat{a} \) is a unit vector in the direction of \( qE \). If reactive collisions (e.g. ionisation, attachment) were significant, it would be necessary to go to second order in the density gradient to calculate corrections to both \( \mathbf{W}(t) \) and \( \mathbf{D}(t) \) but, for the purpose of clarity, we exclude these effects in the present discussion and leave it to a subsequent paper (White et al. 1996). Although much of what we say is applicable to charged particles of arbitrary mass \( m \), the following discussion is framed in terms of electrons or light ions, for which the mass \( m \ll m_0 \), the mass of the neutral particle.

Solution of (2) is made by extending the moment method of Robson and Ness (1986) and Ness and Robson (1986) to include a time-dependent field. This method of solution has been thoroughly tested for electrons and ions in DC electric and magnetic fields for all types of interaction processes (Ness and Robson 1986, 1994; Ness 1994). In summary \( f \) is expanded in terms of normalised Burnett functions about a Maxwellian at an arbitrary time-dependent temperature \( T_b(t) \). As described previously the spatial dependence is expressed by an expansion in terms of the gradient of the swarm number density \( n(r,t) \), resulting in the following expansion of the phase space distribution function:
\[ f(r,c,t) = \sum_{\nu=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=-1}^{1} \sum_{s=0}^{\infty} \sum_{\lambda=0}^{\infty} \mathcal{F}(\nu lm|s\lambda; \alpha(t),t) \phi_m^{(\nu l)}(\alpha(t)c)G_m^{(s\lambda)}n(r,t), \]

where the normalised Burnett functions \( \phi_m^{(\nu l)}(\alpha(t)c) \) and spherical form of the gradient tensor \( G_m^{(s\lambda)} \) are defined in Robson and Ness (1986) and Ness and Robson (1986). The quantity \( \bar{w}(\alpha(t),c) \), with \( \alpha^2(t) \equiv m/kT_b(t) \), represents a Maxwellian distribution at a temperature \( T_b(t) \) which serves as a single free parameter used to optimise convergence.

The generalised continuity equation in irreducible tensor notation is given by
\[ \frac{\partial n}{\partial t} = \sum_{s=0}^{\infty} \sum_{\lambda=0}^{\infty} \omega(s\lambda;t)G_0^{(s\lambda)}n(r,t), \]
Fig. 1. Temporal variation of the electron transport properties over a range of reduced angular frequencies $\omega/\alpha_0$ for the constant cross section model $\sigma_m = 5 \, \text{Å}^2$, zero gas temperature and neutral atom mass 4 amu. Applied frequencies $\omega/\alpha_0$ (rad m$^3$s$^{-1}$) are (a) $1 \times 10^{-19}$, (b) $1 \times 10^{-18}$, (c) $5 \times 10^{-18}$, (d) $1 \times 10^{-17}$ and (e) $1 \times 10^{-16}$.
Fig. 2. Temporal variation of the electron transport properties over a range of reduced angular frequencies $\omega/\omega_0$ for the Reid (1979) ramp model. Applied frequencies $\omega/\omega_0$ (rad m s$^{-1}$) are (a) 1.777×10$^{-14}$, (b) 1.777×10$^{-15}$, (c) 1.777×10$^{-16}$, (d) 1.777×10$^{-17}$, and (e) 1.777×10$^{-18}$. 

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where in the absence of reactions the coefficients \( \omega(s\lambda; t) \) are related to the time-dependent flux transport coefficients through equations (8)–(10). Use of expansion (6), (7) and an implicit finite difference scheme in time, transform (2) into a hierarchy of doubly infinite coupled matrix equations. On truncation of (6), solution for the moments \( F^n_{\alpha_n}(\nu l m|s\lambda) \) yields the time-dependent transport coefficients at the \( n \)th time step:

\[
W(t_n) = i\omega_n(11) = i\alpha_n F^n_{\alpha_n}(010|00), \tag{8}
\]

\[
D_L(t_n) = \frac{1}{\sqrt{3}}[\omega_n(20) - \sqrt{2}\omega_n(22)], \tag{9}
\]

\[
D_T(t_n) = \frac{1}{\sqrt{3}}[\omega_n(20) + \frac{1}{\sqrt{2}}\omega_n(22)], \tag{10}
\]

where

\[
\omega_n(20) = \frac{-1}{\alpha_n \sqrt{3}}[F^n_{\alpha_n}(010|11) + 2F^n_{\alpha_n}(011|11)],
\]

\[
\omega_n(22) = \frac{\sqrt{2}}{\alpha_n \sqrt{3}}[F^n_{\alpha_n}(010|11) - F^n_{\alpha_n}(011|11)], \tag{11}
\]

\[
F^n_{\alpha_n}(\nu l m|s\lambda) = F(\nu l m|s\lambda; \alpha(t_n), t_n).
\]

A detailed description of the theory used including reactive effects will be given in a subsequent paper (White et al. 1996).

Our only assumption in dealing with the space dependence in the problem is that \( f(r, c, t) \) and all its moments are expressible in terms of a gradient expansion in the number density. In general this assumption is valid for weak gradients in the number density. Even for initial conditions with steep density gradients (e.g. \( n(r, 0) = \delta(r) \)) we expect diffusion processes will act to validate this assumption after a certain period of time (Kumar 1981; Robson and Makabe 1994).

3. Results and Discussion

Figs 1 and 2 show electron a.c. transport properties obtained through a time-dependent multi-term solution of (2) over a wide range of applied reduced angular frequencies \( \omega/n_0 \) for a constant elastic cross section and the Reid (1979) ramp interaction models, respectively. The amplitude of the field is 1 Td for the constant cross section model and 12\( \sqrt{2} \) Td for the Reid ramp model. We shall focus in particular on the rather peculiar structure of \( D_L(t) \) as shown in Figs 1 and 2.

At low \( \omega/n_0 \), a small but pronounced spike appears near the phase where \( E \) and \( W \) are small. Here we observe the anomalous situation where \( D_L \) is larger than \( D_T \), in contrast to the d.c. case, where always \( D_L < D_T \) for these models at all values of \( E/n_0 \). As \( \omega/n_0 \) increases this spike grows in height and duration and eventually becomes the dominant feature of \( D_L(t) \). Note that
there is a transition from anisotropic diffusion at low frequencies to isotropic diffusion (in a cycle-averaged sense) at high frequencies. A full understanding of this phenomenon requires the complete picture of all the transport coefficients and we note in particular the phase shifts between $\varepsilon$, $\gamma$ and the applied field $E$. The behaviour of $\varepsilon$ and $W$ in a.c. electric fields is in general well understood in terms of the relations between the energy and momentum transfer collision frequencies, $\nu_e(\varepsilon)$ and $\nu_m(\varepsilon)$ respectively, and the applied frequency $\omega$ (Ferreira and Loureiro 1983; Makabe and Goto 1988; Goto and Makabe 1990; Loureiro 1993). The temporal behaviour of $\gamma$ in an a.c. system has never been reported and is fundamental to an understanding of the origin of anomalous anisotropic diffusion.

That such a complicated time-dependence for $D_L$ should emerge from such a simple model of constant elastic cross section is striking and must be explained before proceeding to the next level of degree of difficulty, the Reid ramp model (Fig. 2). The explanation is by no means trivial and we must return to basics in order to appreciate the origin of the detailed structure in Fig. 1. Before doing that, there is one point concerning anisotropy which should be emphasised at the start: For the constant elastic cross section model, the electron phase-space distribution function is very nearly isotropic in velocity-space and expansion (1) converges very rapidly (only the terms for which $l = 0, 1$ are required to achieve four figure accuracy in transport coefficients), with the result that the temperature tensor is effectively a scalar, i.e.

$$kT = m((e - \langle e \rangle)(e - \langle e \rangle))$$

$$= kT_T I + (kT_L - kT_T) \hat{a} \hat{a} \approx \frac{2 \varepsilon}{3} I.$$

That is, random energies $kT_L$, $kT_T$ associated with motion longitudinal and transverse to the field respectively are almost the same, with a value effectively equal to two-thirds of the mean energy. Thus, for this model, anisotropic diffusion is not attributable to anisotropy in thermal motion. For the Reid ramp model however the effect of thermal anisotropy on diffusion is more significant. We now briefly review the phenomenon of anisotropic diffusion in a d.c. field.

Consider a pulse of electrons drifting and diffusing in a gas under the influence of a d.c. electric field $E$. The situation is shown in Fig. 3. Higher energy electrons are located at the leading edge of the pulse (Skullerud 1969), and thus the product $W \gamma < 0$. If the collision frequency $\nu_m(\varepsilon)$ increases with $\varepsilon$ (as it does for the constant cross section model, $\nu_m \sim \varepsilon^{1/2}$), then electrons at the leading edge suffer more collisions relative to those at the trailing edge, and the local instantaneous average velocity at the trailing edge exceeds that at the leading edge. (We will refer to this as the ‘differential velocity’ effect.) Therefore the combined effect of $W \gamma < 0$ and $\nu_m/\nu_e > 0$ is to inhibit the diffusion of the swarm longitudinally. Of course, there is an additional, general tendency due to thermal motion, for electrons to disperse in all directions, and this is the only contributing factor in transverse diffusion. However, while the thermal motion contributes positively for $D_L$, there is also the extra inhibiting differential velocity
effect described above, and hence \( D_L \leq D_T \) for \( d\nu_m/d\epsilon > 0 \) in d.c. fields. The converse is true for \( d\nu_m/d\epsilon < 0 \), i.e., the differential velocity effect enhances longitudinal diffusion and \( D_L \geq D_T \). These remarks are true for d.c. fields of any strength (equality holds for zero field) and one may expect they would carry over to time-varying fields, \( E(t) \), i.e., that \( D_L (E(t)/n_0) \) would always be less (greater) than \( D_T (E(t)/n_0) \) at all times \( t \) for \( d\nu_m/d\epsilon > 0 \) (\( < 0 \)). This, however, is not the case.

The transient behaviour of \( D_L \) in response to a step-function time-varying field provides the springboard from which the anomalous behaviour of \( D_L \) in a.c. fields can be understood. Fig. 4 shows the response of the transport coefficients (again for the constant elastic cross section model) to an instantaneous reversal in the field from 0.4 to \(-0.4\) Td. In any discussion on electron transport in time-varying fields, key factors are the representative values for the relaxation times \( \tau_m \sim \nu_m^{-1} \) and \( \tau_e = m_0 \tau_m/2m \gg \tau_m \) for momentum and energy transfer, respectively (Makabe and Goto 1988; Goto and Makabe 1990; Ferreira and Loureiro 1990; Loureiro 1993). The drift velocity responds almost instantaneously, in a time \( \sim \tau_m \), to a reversal of \( E \) and reverses its direction while retaining the same magnitude. Note that the magnitude of \( E \) remains constant and that both \( \epsilon \) and \( D_T \) effectively remain constant also. (There are actually small dips in both \( \epsilon \) and \( D_T \) at the instant \( E \) reverses, but these are too small to be seen on the scale of the plots in Fig. 4). However, both \( \gamma \) and \( D_L \) show a much slower response to the field reversal.

At times prior to the field reversal the d.c. situation prevails where electrons at the leading edge of the swarm have higher energies than those at the trailing
edge, thus $W \gamma < 0$ and so $D_L < D_T$, since the differential velocity effect acts to inhibit $D_L$ as explained before. When the field reverses, the directional reversal of $W$ and indeed all the local instantaneous drift velocities along the direction of motion of the swarm, occurs in time $\sim \tau_m$, while $\gamma$ remains virtually constant during this time. In this brief time we have the unusual, anomalous situation of the electrons now at the leading edge of the swarm having lower energy than those at the trailing edge. This situation is shown in Fig. 5. Thus $W \gamma > 0$ and $D_L$ rises rapidly to exceed $D_T$ as the differential velocity effect has suddenly changed from inhibiting $D_L$ and to actually enhancing $D_L$! As time progresses, $\gamma$ passes through zero and eventually the faster electrons are again found at the leading edge and $\gamma$ finally reaches a value equal in magnitude but opposite in sign to that which it had before the field reversal. This process occurs over a time period of approximately $\tau_e$. In response to the changing $\gamma$ and differential velocity effect, $D_L$ starts to decrease and drops below $D_T$ as $\gamma$ changes sign; $D_L$ then regains the value it had before the field reversal in a time $\sim \tau_e$. The two points at which diffusion is isotropic ($D_L = D_T$) are significant. The first occurs as $D_L$ is rising to its maximum and corresponds to the point where $E$ and $W$ vanish. At this point there is no preferred direction and diffusion must be isotropic. The second point occurs as $D_L$ relaxes from its maximum and
Fig. 5. Schematic representation of a pulse of charged particles in response to a change in the field direction where \( W \) and all local instantaneous drift velocities along the pulse have changed sign before \( \gamma \). This represents the anomalous diffusion.

corresponds to \( \gamma = 0 \). Here there is no differential velocity effect and diffusion must again be isotropic. Between these two points we have the sharp peak where \( D_L > D_T \). This is what we refer to as 'anomalous anisotropic diffusion'. The above process repeats itself when the field is again reversed later, as shown in Fig. 4. One point to be emphasised is that the field must actually change sign for the anomalous effect to occur. (It is convenient, but not necessary in the above discussion, for the magnitude of the field to remain the same when the direction is reversed, in order to create the anomalous situation.) A simple change in field from one value to another of the same sign produces a peak in \( D_L \) but still \( D_L < D_T \). The prescription for anomalous anisotropic diffusion is therefore a large separation in the time scales \( \tau_m \) and \( \tau_e \) and a field which changes sign fast on a time scale of order \( \tau_m \). For cases where \( m \approx m_0 \) for which \( \tau_m \approx \tau_e \) (e.g. ions) or for slowly varying fields we would expect anomalous anisotropic diffusion to be suppressed.

We now extend these ideas to alternating fields. In Fig. 6a we portray electron transport properties \( W(t), D_L(t), D_T(t), \varepsilon(t) \) and \( \gamma(t) \) schematically for a low frequency a.c. field, for which \( \omega \sim \nu_e(\bar{\varepsilon}) \) (\( \bar{\varepsilon} \) is the cycle-averaged energy) and collisions are governed by a collision frequency which increases with energy, \( \nu_m' > 0 \). In regions [1] and [7], the field \( E(t) \) and therefore energy of the electrons are sufficiently large such that \( \omega \ll \nu_e(\varepsilon(t)) \) and all properties change in phase with the field, i.e. there is quasi-d.c. behaviour. Thus \( D_L \) and \( D_T \) both rise or fall monotonically as \( E(t) \) increases or decreases respectively, and \( D_L/D_T \) is maintained virtually constant at its d.c. value. The physical picture here corresponds to Fig. 3. Regions [2]–[6] are the most interesting since here \( \varepsilon(t) \) is such that \( \omega \lesssim \nu_e(\varepsilon(t)) \) and \( E(t) \) can therefore be considered to be
changing significantly on the time scale of $\tau_v$. Thus, although $\omega \ll \nu_v(\epsilon(t))$ for the entire cycle [1]–[7] and $W(t)$ changes almost exactly in phase with $E(t)$, the other quantities $\epsilon(t)$ and $\gamma(t)$ lag behind, due to their longer relaxation times $\sim [\nu_v(\epsilon(t))]^{-1}$. That is we have a separation of relaxation time scales for $W(t)$ and $\epsilon(t), \gamma(t)$ and a field which rapidly reverses sign. Therefore, we expect qualitatively similar behaviour to the step-function field situation as shown in Fig. 4 with anomalous anisotropic diffusion $D_L > D_T$ in a narrow region near where $E(t)$ and $W(t)$ are near zero. Thus in Fig. 6a at [2], $D_L$ begins to rise due to a weakening of the differential velocity effect in response to the falling $E$ and $W$. At [3] $E$ and $W \approx 0$ and $D_L = D_T$. Beyond [3], as $E$ and $W$ assume negative values, $D_L$ rises, as the differential velocity effect actually enhances longitudinal diffusion: The situation is similar to Fig. 5. However, as $\gamma(t)$ declines, this enhancement also falls, and at [4], $D_L$ begins to fall and becomes equal to $D_T$ once more at [5] where $\gamma = 0$. As $\gamma$ becomes negative, and both $\epsilon(t)$ and $E(t)$ increase, the differential velocity effect works once again as the conventional inhibiting factor, causing $D_L$ to fall below $D_T$. At [6], the thermal energy has picked up sufficiently to counteract a further fall in $D_L$, and by [7] the quasi-d.c. situation has been restored. The anomalous anisotropic diffusion regime is [3]–[5], where $D_L > D_T$. Notice that the minima at points [2] and [6] are not symmetric with respect to the maximum. This is basically due to the asymmetry introduced by the phase lag in $\epsilon(t)$ and $\gamma(t)$ with respect to the field.

At higher frequencies $\omega \gtrsim \nu_v(\epsilon)$ the phase lags referred to above increases, and the peak broadens to become the dominant feature in $D_L$. Since $D_T(t)$ essentially

Fig. 6. Schematic representation of $D_L, D_T, \epsilon$ and $\gamma$ over half a cycle of an a.c. field at low frequencies for (a) an elastic cross section with $d\nu_m/d\epsilon > 0$ and (b) the Reid (1979) ramp model.
follows $\varepsilon(t)$, this means that $D_L$ and $D_T$ oscillate effectively in antiphase. At much higher frequencies $\omega \gg \nu_c(\varepsilon)$ the response of $\varepsilon$ and $\gamma$ to changes in $E(t)$ is slow, and their modulation is correspondingly reduced. The same can be said for $D_L$ and $D_T$, which exhibit small antiphase oscillations. However, in a cycle-averaged sense, diffusion is isotropic.

Inelastic collisions characterised by a threshold can complicate the picture in two ways: Firstly, the variation in mean energy through the swarm may be suppressed, and $\gamma$ correspondingly reduced, for energies above threshold and secondly, $\tau_\varepsilon$ may be substantially lowered, and the separation between $\tau_m$ and $\tau_\varepsilon$ is similarly reduced. A case in point is the well known Reid (1979) ramp model (constant elastic cross section, inelastic cross section increasing linearly with energy above a threshold of 0.2 eV) for which the d.c. results available in the literature (Ness and Robson 1986) can be used to explain much of the low $\omega/n_0$ a.c. situation portrayed in Fig. 2. Thus since $\nu'_m > 0$, we have $D_L < D_T$, for all times except the anomalous ‘window’, where $|E/n_0|$ and $\varepsilon(t)$ are small, the latter being below threshold, and collisions are mostly governed by a constant elastic cross section. In accordance with the previous discussion for the latter model, we expect an anomalous region to develop with $D_L \geq D_T$, and this is portrayed schematically in amplified form in Fig. 6b. The anomalous spike is much sharper than for the simple constant cross section model, however, since the range of energies and times for which there is good separation between $\tau_m$ and $\tau_\varepsilon$ is considerably reduced by the onset of strong inelastic processes. The sharp spikes in $\gamma$ also originate from the pronounced transition from an elastic collision-controlled regime to one dominated by strong inelastic processes. At higher $\omega/n_0$, just as for the simple constant elastic cross section model, the spike grows into the dominant feature of the profile. However, the simple characterisation of the anomalous region by $W\gamma > 0$ is not applicable over the entire range of frequencies, since thermal energies $kT_L$, $kT_T$ may differ significantly in both magnitude and phase. A more comprehensive discussion can be found in White et al. (1996).

4. Conclusion

In summary, we have presented calculations of all transport coefficients for electrons in an a.c. field for the simple constant cross section and the Reid ramp model over a wide range of $\omega/n_0$. In particular for $D_L(t)$ we have identified and explained a region of anomalous behaviour, originating from the phase shifts in the gradient energy parameter $\gamma(t)$ of equation (4), and inexplicable through any d.c. results. A more detailed discussion of all transport properties is to be given in a series of companion papers (Robson et al. 1996; White et al. 1996).

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References


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