Aust. J. Phys., 1996, **49**, 899–903

Brans–Dicke Cosmology with Causal Viscous Fluid

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Abstract

Exact solutions for the spatially flat ($k = 0$) Robertson–Walker cosmological model in Brans–Dicke scalar tensor theory have been obtained in the presence of a causal viscous fluid. It is found that if the scale factor is a power function of the scalar field, then solutions can be obtained in the full causal theory but not in the truncated theory of non-equilibrium thermodynamics.

1. Introduction

Recently there has been a lot of renewed interest in the Brans–Dicke scalar tensor theory (Brans and Dicke 1961) mainly because of its possible role in producing what is known as an extended inflationary scenario (Mathiazhagan and Johri 1984; La and Steinhardt 1989). In this scenario, the expansion rate of the universe is slowed down by the scalar field from exponential to polynomial so that there is enough time for the universe to complete the phase transition or 'roll over' from the inflationary phase to the radiation dominated phase. On the other hand, dissipative effects such as viscosity are of enormous importance in the early stages of the evolution of the universe particularly before the time of nucleosynthesis (see Gron 1990 and references therein). Moreover, the existence of bulk viscosity itself can lead to inflationary-like solutions (Padmanabhan and Chitre 1987). In this connection, Johri and Sudharsan (1989) and Beesham (1994) obtained some exact solutions in Friedmann–Robertson–Walker (FRW) cosmological models with a Brans–Dicke scalar field and a viscous fluid together. But these investigations considered only the first order deviation from equilibrium. Viscous fluids with this property lead to a violation of causality. To preserve causality, second order deviations from equilibrium need to be considered. But these second order theories, although being causal, may lead also to pathological behaviour of the cosmological model. It was pointed out by Hiscock and Lindblom (1983) that this undesirable feature is due to the fact that certain divergence terms had been dropped in these theories (see also Hiscock and Salmonson 1991). For this reason, the second order theories without the additional divergence terms are referred to as 'truncated' causal theories. For a full causal thermodynamic

0004-9506/96/050899$05.00

model, the bulk viscous stress should involve both the first and second order deviations from equilibrium, as well as the divergence terms. For an excellent review, we refer to the recent work of Maartens (1995).

In the present work, a spatially flat Robertson–Walker model is examined in Brans–Dicke theory with a bulk viscous fluid with full causal non-equilibrium thermodynamics. Following Johri and Sudharsan (1989), we assume a relationship between the scale factor R and the scalar field ϕ in the form $R \sim \phi^\alpha$ and investigate the possibility of finding a power law solution for the scale factor R.

2. Field Equations and Their Solutions

The gravitational field equations in Brans–Dicke (BD) theory can be written as

$$G_{\mu\nu} + \frac{\omega}{\phi^2}\left(\phi_{,\mu}\phi_{,\nu} - \tfrac{1}{2}\phi'^{\alpha}\phi_{,\alpha}\,g_{\mu\nu}\right) + \frac{1}{\phi}[\phi_{,\mu;\nu} - \Box\phi g_{\mu\nu}] = \frac{1}{\phi}T_{\mu\nu}\,, \tag{1}$$

in units where $8\pi G$ and c are both equal to unity. The wave equation for the scalar field will be

$$\Box\phi = \frac{1}{2\omega+3}T\,. \tag{2}$$

Here ϕ is the scalar field, the constant ω is the BD parameter and T is the trace of the energy momentum tensor $T_{\mu\nu}$. For a perfect fluid distribution in a spatially flat ($k=0$) FRW spacetime given by the metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + R^2(t)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)\,, \tag{3}$$

the field equations and the wave equation take the form

$$3\frac{\dot{R}^2}{R^2} + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 = \frac{\rho}{\phi}\,, \tag{4}$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{\ddot{\phi}}{\phi} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} = -\frac{p}{\phi}\,, \tag{5}$$

$$\ddot{\phi} + 3\frac{\dot{R}}{R}\dot{\phi} = \frac{1}{2\omega+3}(\rho - 3p)\,, \tag{6}$$

ρ and p being the density and pressure of the fluid. But if the effect of viscosity has to be included, the perfect fluid pressure should be replaced by an effective pressure p_{eff}, which is given by

$$p_{\mathrm{eff}} = p + \pi\,. \tag{7}$$

Here p is the perfect fluid contribution and π is the bulk viscous stress. For the full causal theory of non-equilibrium thermodynamics, π is given by the equation

$$\pi + \tau\dot{\pi} = -3\xi H - \frac{\epsilon}{2}\tau\pi\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right), \tag{8}$$

where ξ is the coefficient of bulk viscosity, τ is the relaxation time for bulk viscous effects, T is the temperature and H is the Hubble parameter $\dot{R}/R$. Here ξ, τ and T are all positive. The parameter ϵ can take the values 0 and 1 for the truncated and full causal theories. For $\tau = 0$, the equation reduces to that for the noncausal theory. We shall also assume an equation of state connecting the perfect fluid pressure p and the density ρ in the form

$$p = \gamma\rho, \tag{9}$$

where γ ($0 \leq \gamma \leq 1$) is a constant. With equations (7) and (9), equations (4)–(6) now become

$$3\frac{\dot{R}^2}{R^2} + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 = \frac{\rho}{\phi}, \tag{10}$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{\ddot{\phi}}{\phi} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} = -\frac{\gamma\rho + \pi}{\phi}, \tag{11}$$

$$\ddot{\phi} + 3\frac{\dot{R}}{R}\dot{\phi} = \frac{1}{2\omega + 3}(\rho - 3\gamma\rho - 3\pi). \tag{12}$$

Now one has the three equations (10)–(12), but four unknown quantities, namely the scale factor R, the scalar field ϕ, the density ρ and the bulk viscous stress π; hence the system of equations cannot be solved. We shall therefore assume a relationship between ϕ and R of the form

$$\phi = AR^{\alpha}, \tag{13}$$

and investigate the nature of the solution for the scale factor R.

By eliminating ρ and π from equation (12) with the help of (10) and (11) one obtains

$$2\omega\frac{\ddot{\phi}}{\phi} + 6\omega H\frac{\dot{\phi}}{\phi} - \omega\frac{\dot{\phi}^2}{\phi^2} = 6\dot{H} + 12H^2, \tag{14}$$

where $H = \dot{R}/R$. Using equation (13) it is now easy to obtain from (14) the following equation for the scale factor:

$$\dot{H} + \beta H^2 = 0, \tag{15}$$

where β is a constant given by

$$\beta = \frac{\omega\alpha^2 + 6\omega\alpha - 12}{2(\omega\alpha - 3)}. \tag{16}$$

Equation (15) can be easily integrated twice to yield

$$R = at^{1/\beta}, \tag{17}$$

where a is a constant of integration and the second constant of integration, with the choice of the initial condition $R(t = 0) = 0$, can be put equal to zero.

With the help of equations (13) and (17), expressions for ρ and π as functions of time can be obtained from (10) and (11) as

$$\rho = \rho_0 t^b, \tag{18}$$

$$\pi = \pi_0 t^b, \tag{19}$$

where $b = \alpha/\beta - 2$, ρ_0 is a constant and

$$\pi_0 = Aa^\alpha \frac{[2\alpha\beta + 4\beta - 6 - 4\alpha - (2+\omega)\alpha^2]}{2\beta^2} - \gamma\rho_0 = \text{constant}.$$

Both in the truncated and the full version of causal thermodynamics, the coefficient of bulk viscosity ξ and the relaxation time τ are assumed to be simple power functions of ρ in most investigations. In what follows, we shall determine the temporal behaviour of the absolute temperature T and also show that the solution that we have obtained for R is incompatible with the truncated theory. If we assume the simple relations

$$\xi = \xi_0 \rho^q \quad \text{and} \quad \tau = \xi/\rho, \tag{20}$$

where ξ_0 is a constant ($\xi_0 > 0$), we can integrate (8) with the help of (19) to get

$$a_1 t^{b-bq+1} + a_2 \ln t + D = \frac{\epsilon\pi_0}{2} \ln T, \tag{21}$$

where

$$a_1 = \frac{\pi_0}{\xi_0 \rho_0^{q-1}(b - bq + 1)},$$

$$a_2 = \pi_0 b + \frac{3\rho_0}{\beta} + \frac{3\epsilon\pi_0}{2\beta} + \frac{\epsilon\pi_0 b}{2},$$

$$D = \frac{\epsilon}{2}\pi_0(\ln \rho_0 - \ln a_3),$$

and a_3 is a constant of integration.

For $\epsilon = 0$, one gets the truncated theory but equation (21) is clearly inconsistent for $\epsilon = 0$. For the full theory ($\epsilon = 1$), however, the equation is consistent and one can find the temperature as a function of time as

$$T = T_0\, \mathrm{e}^{(2a_1/\epsilon\pi_0)t^{b-bq+1}}\, t^{2a_2/\epsilon\pi_0}$$

$$= T_0\, \mathrm{e}^{a_4 t^{b-bq+1}}\, t^{a_5}\,,$$

where $\epsilon = 1$, $a_4 = 2a_1/\pi_0$, $a_5 = 2a_2/\pi_0$ and T_0 is a constant.

3. Discussion

With the assumption $\phi \sim R^{\alpha}$, we have found the exact solution for the spatially flat Robertson–Walker cosmological model with the Brans–Dicke scalar field and a causal viscous fluid in a full theory of non-equilibrium thermodynamics. It is interesting to note that our solutions derived here do not have corresponding analogues in general relativity without a scalar field. Indeed, if we try to put $\phi =$ constant to generate the general relativity solution, we find that $R =$ constant.

Acknowledgment

One of the authors (N.B.) would like to thank the University of Zululand for hospitality and the FRD (South Africa) for financial support.

References

Beesham, A. (1994). Proc. 5th Canadian Conf. on General Relativity and Relativistic Astrophysics (Eds R. B. Mann and R. G. McLenaghan), pp. 64–8 (World Scientific: Singapore).

Brans, C., and Dicke, R. H. (1961). *Phys. Rev.* **124**, 925.

Gron, O. (1990). *Astrophys. Space Sci.* **173**, 191.

Hiscock, W. A., and Lindblom, L. (1983). *Ann. Phys. (NY)* **151**, 466.

Hiscock, W. A., and Salmonson, J. (1991). *Phys. Rev.* D **43**, 3249.

Johri, V. B., and Sudharsan, R. (1989). *Aust. J. Phys.* **42**, 215.

La, D., and Steinhardt, P. J. (1989). *Phys. Rev. Lett.* **62**, 376.

Maartens, R. (1995). *Class. Quant. Grav.* **12**, 1455.

Mathiazhagan, C., and Johri, V. BG. (1984). *Class. Quant. Grav.* **1**, L29.

Padmanabhan, T., and Chitre, S. M. (1987). *Phys. Lett.* A **120**, 433.

Manuscript received 18 July 1995, accepted 16 March 1996