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Virtual Compton Scattering from the Proton and the Properties of Nucleon Excited States

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Abstract

We calculate the N^* contributions to the generalised polarisabilities of the proton in virtual Compton scattering. The following nucleon excitations are included: $N^*(1535)$, $N^*(1650)$, $N^*(1520)$, $N^*(1700)$, $\Delta(1232)$, $\Delta^*(1620)$ and $\Delta^*(1700)$. The relationship between nucleon structure parameters, N^* properties and the generalised polarisabilities of the proton is illustrated.

1. Introduction

The study of virtual Compton scattering (VCS), $e + p \longrightarrow e' + p' + \gamma$, at CEBAF and MAMI (Audit *et al.* 1993) could provide valuable information on the structure of the nucleon, complementing the information obtained from elastic form factors, real Compton scattering, and deep inelastic scattering. In this paper we concentrate on the kinematic region where the final photon has low energy—i.e. below the threshold for π^0 production. As shown by Guichon *et al.* (1995) the low-energy cross sections are parametrised by ten generalised polarisabilities (GP), functions of the virtual photon mass. Their evaluation requires a knowledge of the nucleon excited states. This sensibility to the nucleon spectrum can provide substantial insight into the non-perturbative aspects of the QCD Hamiltonian.

Guichon *et al.* (1995) made an initial evaluation of the GPs to provide an order of magnitude estimate of these new quantities and to illustrate their variation as a function of the virtual photon mass. In that calculation we neglected all recoil effects, that is, terms which vary like the velocity of the nucleon. As a result of that approximation only seven GPs were nonzero.

In this paper we extend the calculations to include the recoil corrections which turn out to contribute only when the final photon is magnetic. We also study the relationship of the nucleon excited states to the GPs. We use the nonrelativistic quark model (NRQM) to take advantage of its simplicity and the ready availability of its wavefunctions. Also, the separation of the centre-of-mass and internal motion greatly simplifies the calculation, making it analytically tractable. In principle, it is possible to use other wavefunctions but this can be prohibitively laborious and messy. The NRQM estimate should be a useful guide to the analysis of the forthcoming experimental data on the $p(e, e'p)\gamma$ reaction.

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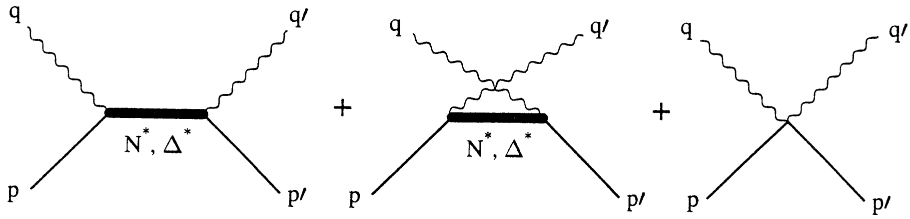

Fig. 1. Direct, cross and seagull terms of the hadronic tensor in the lowest-order QED perturbation theory.

2. General Forms of GPs in terms of Current Densities

We first briefly outline the formalism for the definition and the calculation of the GPs. We refer to Guichon *et al.* (1995) for a detailed account of the problem, as well as for the notation and conventions. The hadronic tensor (see Fig. 1) is defined by

$$H^{\mu\nu}_{NB}(\boldsymbol{q}'m'_s, \boldsymbol{q}m_s) =$$

$$\int d\boldsymbol{p}_X \sum_{X\neq N} \left[\langle N(\boldsymbol{p}')|J^\mu(0)|X(\boldsymbol{p}_X)\rangle \frac{\delta(\boldsymbol{p}_X - \boldsymbol{p}' - \boldsymbol{q}')}{E_N(\boldsymbol{q}') + q' - E_X(\boldsymbol{p}_X)} \langle X(\boldsymbol{p}_X)|J^\nu(0)|N(\boldsymbol{p})\rangle \right.$$

$$\left. + \langle N(\boldsymbol{p}')|J^\nu(0)|X(\boldsymbol{p}_X)\rangle \frac{\delta(\boldsymbol{p}_X - \boldsymbol{p} + \boldsymbol{q}')}{E_N(\boldsymbol{q}) - q' - E_X(\boldsymbol{p}_X)} \langle X(\boldsymbol{p}_X)|J^\mu(0)|N(\boldsymbol{p})\rangle \right] + H^{\mu\nu}_{seagull}, \quad (1)$$

where J^μ is the hadronic current, X the intermediate baryon excitations and $H^{\mu\nu}_{seagull}$ the contact term generally required by gauge invariance. The reduced multipoles are then defined according to

$$H^{(\rho'L',\rho L)S}_{NB}(q', q) = \frac{1}{2S+1} \sum_{m'_s m_s M' M} (-)^{\frac{1}{2}+m'_s+L+M} \langle \tfrac{1}{2}\, -m'_s\,, \tfrac{1}{2}\, m_s \,|\, S\, s\rangle$$

$$\times \langle L'\, M'\,, L\, M \,|\, S\, s\rangle H^{\rho'L'M',\rho LM}_{NB}(q'm'_s, qm_s), \quad (2)$$

with

$$H^{\rho'L'M',\rho LM}_{NB}(q'm'_s, qm_s) =$$

$$(4\pi)^{-1} \int d\hat{q}\, d\hat{q}'\, V^*_\mu(\rho'L'M', \hat{q}')\, H^{\mu\nu}_{NB}(\boldsymbol{q}'m'_s, \boldsymbol{q}m_s)\, V_\nu(\rho LM, \hat{q}), \quad (3)$$

where $V_\nu(\rho LM, \hat{q})$ are the charge ($\rho = 0$), magnetic ($\rho = 1$) and electric ($\rho = 2$) basis vectors defined by Guichon *et al.* (1995).

When $\rho,\ \rho'$ are equal to 0 or 1 the GPs are defined by

$$P^{(\rho' L', \rho L)S}(q) = \left[\frac{1}{{q'}^{L'} q^{L}} H_{NB}^{(\rho' L', \rho L)S}(q', q) \right]_{q'=0} . \tag{4}$$

In the case of a virtual electric photon the analogous definition does not yield a GP with the usual photon limit as $q \to 0$. As explained by Guichon *et al.* (1995), one must therefore introduce mixed GPs according to

$$\hat{H}_{NB}^{\rho' L' M', LM}(q' m_s', q m_s) =$$

$$(4\pi)^{-1} \int d\hat{q}\, d\hat{q}'\, V_\mu^*(\rho' L' M', \hat{q}') \sum_i H_{NB}^{\mu i}(\boldsymbol{q}' m_s', \boldsymbol{q} m_s) \left(\mathcal{Y}_M^{LL+1}(\hat{q})\right)^i , \tag{5}$$

$$\hat{P}^{(\rho' L', L)S}(q) = \left[\frac{1}{{q'}^{L'} q^{L+1}} \hat{H}_{NB}^{(\rho' L', L)S}(q', q) \right]_{q'=0} , \tag{6}$$

where $\mathcal{Y}_M^{Ll}(\hat{q})$ is the vector spherical harmonic. The ten independent GPs needed to describe the low-energy regime are then

$$\begin{array}{ccccc} P^{(11,00)1}, & P^{(11,02)1}, & P^{(11,11)0}, & P^{(11,11)1}, & \hat{P}^{(11,2)1}, \\ P^{(01,01)0}, & P^{(01,01)1}, & P^{(01,12)1}, & \hat{P}^{(01,1)0}, & \hat{P}^{(01,1)1}. \end{array} \tag{7}$$

In the low-energy regime the following excited states contribute: $N^*(\frac{1}{2}^-, 1535)$, $N^*(\frac{1}{2}^-, 1650)$, $N^*(\frac{3}{2}^-, 1520)$, $N^*(\frac{3}{2}^-, 1700)$, $\Delta(\frac{3}{2}^+, 1232)$, $\Delta^*(\frac{1}{2}^-, 1620)$ and $\Delta^*(\frac{3}{2}^-, 1700)$.

In the NRQM the current density in (1) takes the form

$$\langle X(\boldsymbol{p}_X) | J^0(0) | N(\boldsymbol{p}) \rangle = N_0\, \rho_X(\boldsymbol{p}_X - \boldsymbol{p}),$$

$$\langle X(\boldsymbol{p}_X) | \boldsymbol{J}(0) | N(\boldsymbol{p}) \rangle = N_0 \left[\frac{\boldsymbol{p}_X + \boldsymbol{p}}{6 m_q} \rho_X(\boldsymbol{p}_X - \boldsymbol{p}) + \boldsymbol{P}_X(\boldsymbol{p}_X - \boldsymbol{p}) \right.$$

$$\left. + \frac{i}{2 m_q} \boldsymbol{\Sigma}_X(\boldsymbol{p}_X - \boldsymbol{p}) \times (\boldsymbol{p}_X - \boldsymbol{p}) \right] . \tag{8}$$

Here $\rho, \boldsymbol{P}$ and $\boldsymbol{\Sigma}$ are overlap integrals of current operators. If one takes into account the factorisation of the c.m. and internal baryon wavefunctions, they can be written in the following forms:

$$\rho_X(\boldsymbol{p}_X-\boldsymbol{p}) =$$

$$\int d\boldsymbol{\rho}\, d\boldsymbol{\lambda}\, \mathrm{e}^{-i\sqrt{\frac{2}{3}}(\boldsymbol{p}_X-\boldsymbol{p})\cdot\boldsymbol{\lambda}}\, \phi_X^{\dagger}(\boldsymbol{\rho},\boldsymbol{\lambda},J_X,M_X,T_X,\tau_X)\,\hat{Q}\,\phi_N(\boldsymbol{\rho},\boldsymbol{\lambda},m_N,\tau_N), \quad (9)$$

$$\boldsymbol{P}_X(\boldsymbol{p}_X-\boldsymbol{p}) = \sqrt{\tfrac{2}{3}}\frac{1}{2m_q}\int d\boldsymbol{\rho}\, d\boldsymbol{\lambda}\, \mathrm{e}^{-i\sqrt{\frac{2}{3}}(\boldsymbol{p}_X-\boldsymbol{p})\cdot\boldsymbol{\lambda}},$$

$$\phi_X^{\dagger}(\boldsymbol{\rho},\boldsymbol{\lambda},J_X,M_X,T_X,\tau_X)\,(\boldsymbol{i}\,\overrightarrow{\nabla}_\lambda-\boldsymbol{i}\,\overleftarrow{\nabla}_\lambda)\hat{Q}\,\phi_N(\boldsymbol{\rho},\boldsymbol{\lambda},m_N,\tau_N), \quad (10)$$

$$\boldsymbol{\Sigma}_X(\boldsymbol{p}_X-\boldsymbol{p}) =$$

$$\int d\boldsymbol{\rho}\, d\boldsymbol{\lambda}\, \mathrm{e}^{-i\sqrt{\frac{2}{3}}(\boldsymbol{p}_X-\boldsymbol{p})\cdot\boldsymbol{\lambda}}\, \phi_X^{\dagger}(\boldsymbol{\rho},\boldsymbol{\lambda},J_X,M_X,T_X,\tau_X)\,\boldsymbol{\sigma}_3\hat{Q}\,\phi_N(\boldsymbol{\rho},\boldsymbol{\lambda},m_N,\tau_N), \quad (11)$$

where $\hat{Q}=(\frac{1}{6}+\tau_3^z/2)$ is the charge operator of the third quark and $\boldsymbol{\sigma}_3$ is twice the spin operator of the third quark.

With specific internal wavefunctions for the nucleon (ϕ_N) and the intermediate excitations (ϕ_X), one obtains explicit forms for the current density and hence the hadronic tensor.

3. Current Density and Hadronic Tensor in NRQM

To calculate the integrals $\rho(\boldsymbol{p}_X-\boldsymbol{p})$, $\boldsymbol{P}(\boldsymbol{p}_X-\boldsymbol{p})$ and $\boldsymbol{\Sigma}(\boldsymbol{p}_X-\boldsymbol{p})$ for the seven intermediate states X of $N^*(\frac{1}{2}^-,1535)$, $N^*(\frac{1}{2}^-,1650)$, $N^*(\frac{3}{2}^-,1520)$, $N^*(\frac{3}{2}^-,1700)$, $\Delta(\frac{3}{2}^+,1232)$, $\Delta^*(\frac{1}{2}^-,1620)$ and $\Delta^*(\frac{3}{2}^-,1700)$, we use the wavefunctions from Isgur and Karl (1978). The calculation is straightforward though fairly lengthy. We arrive at the following expressions:

$$\rho_{N(^28)}(\boldsymbol{q}) = -i\frac{\sqrt{8\pi}}{9}\frac{q}{\alpha}\mathrm{e}^{-q^2/6\alpha^2}\tau_N(-1)^{\frac{1}{2}+M_X}\sqrt{2J_X+1}\begin{pmatrix}1 & \frac{1}{2} & J_X\\ M_X-m_N & m_N & -M_X\end{pmatrix} \times Y^*_{1M_X-m_N}(\hat{q}), \quad (12)$$

$$\boldsymbol{P}_{N(^28)}(\boldsymbol{q}) = -i\frac{\sqrt{6}}{9}\frac{\alpha}{m_q}\mathrm{e}^{-q^2/6\alpha^2}\tau_N(-1)^{\frac{1}{2}+M_X}\sqrt{2J_X+1}\begin{pmatrix}1 & \frac{1}{2} & J_X\\ M_X-m_N & m_N & -M_X\end{pmatrix} \times \boldsymbol{e}^*_{M_X-m_N}, \quad (13)$$

$$\mathbf{\Sigma}_{N(^28)}(\boldsymbol{q}) =$$

$$-i\frac{\sqrt{\pi}}{18}\frac{q}{\alpha}\mathrm{e}^{-q^2/6\alpha^2}(1+4\tau_N)\sum_\mu(-1)^{\frac{1}{2}+M_X}\sqrt{2J_X+1}\begin{pmatrix} 1 & \frac{1}{2} & J_X \\ M_X-\mu & \mu & -M_X\end{pmatrix}$$

$$\times\ Y^*_{1M_X-\mu}(\hat{q})\,\langle\chi^\lambda_\mu|\boldsymbol{\sigma}_3|\chi^\lambda_{m_N}\rangle\,, \quad (14)$$

$$\mathbf{\Sigma}_{N(^48)}(\boldsymbol{q}) =$$

$$-i\frac{\sqrt{\pi}}{18}\frac{q}{\alpha}\mathrm{e}^{-q^2/6\alpha^2}(1-2\tau_N)\sum_\mu(-1)^{-\frac{1}{2}+M_X}\sqrt{2J_X+1}\begin{pmatrix} 1 & \frac{3}{2} & J_X \\ M_X-\mu & \mu & -M_X\end{pmatrix}$$

$$\times\ Y^*_{1M_X-\mu}(\hat{q})\,\langle\chi^{\frac{3}{2}}_\mu|\boldsymbol{\sigma}_3|\chi^\lambda_{m_N}\rangle\,, \quad (15)$$

$$\mathbf{\Sigma}_\Delta(\boldsymbol{q}) = \tfrac{1}{3}\mathrm{e}^{-q^2/6\alpha^2}\,\langle\chi^{\frac{3}{2}}_{m_\Delta}|\boldsymbol{\sigma}_3|\chi^\lambda_{m_N}\rangle\,, \quad (16)$$

$$\rho_{\Delta^*}(\boldsymbol{q}) = \frac{1}{2\tau_N}\rho_{N(^28)}(\boldsymbol{q}), \quad (17)$$

$$\boldsymbol{P}_{\Delta^*}(\boldsymbol{q}) = \frac{1}{2\tau_N}\boldsymbol{P}_{N(^28)}(\boldsymbol{q}), \quad (18)$$

$$\mathbf{\Sigma}_{\Delta^*}(\boldsymbol{q}) = \frac{-1}{1+4\tau_N}\mathbf{\Sigma}_{N(^28)}(\boldsymbol{q}), \quad (19)$$

where χ are spin wavefunctions, τ_N the isospin quantum number of the nucleon (i.e. $\pm\frac{1}{2}$ for $\binom{p}{n}$), and $\boldsymbol{e}_m$ is the mth component of the spherical basis vectors. Equations (12)–(19) are for intermediate states with the same isospin quantum number as the proton.

The main characteristics of these integrals are summarised as:

(a) The $\Delta(1232)$ and the ^{48} component of the N^* contribute only to $\mathbf{\Sigma}(\boldsymbol{p}_X-\boldsymbol{p})$.

(b) The ^{28} component of the N^* and the Δ^* contribute to all $\rho(\boldsymbol{p}_X-\boldsymbol{p})$, $\boldsymbol{P}(\boldsymbol{p}_X-\boldsymbol{p})$ and $\mathbf{\Sigma}(\boldsymbol{p}_X-\boldsymbol{p})$.

(c) For small $x=|\boldsymbol{p}_X-\boldsymbol{p}|$ the behaviour of the integrals are $\mathbf{\Sigma}_\Delta(\boldsymbol{x}), \boldsymbol{P}(\boldsymbol{x}) \propto O(1)$ and $\mathbf{\Sigma}_{N^*,\Delta^*}(\boldsymbol{x}), \rho(\boldsymbol{x}) \propto O(x)$.

The leading term and the recoil term of the hadronic tensor in (1) are separated in the following way. We work in the initial $N\gamma$ c.m. system so that $\boldsymbol{p}=-\boldsymbol{q}$. In the direct term of (1), $\boldsymbol{p}_X=0$ and $\boldsymbol{q}'=-\boldsymbol{p}'$. So, aside from the energy denominator, the direct amplitude factorises into a product of a $\boldsymbol{q}$-dependent current density $\boldsymbol{J}_{d,XN}(\boldsymbol{q})$ and a $\boldsymbol{q}'$-dependent current density $\boldsymbol{J}_{d,NX}(\boldsymbol{q}')$, where

$$\begin{aligned}
\boldsymbol{J}_{d,XN}(\boldsymbol{q}) &= \langle X(\boldsymbol{p}_X)|\boldsymbol{J}(0)|N(\boldsymbol{p})\rangle_{direct} \\
&= N_0\left[\frac{-\boldsymbol{q}}{6m_q}\rho_X(\boldsymbol{q}) + \boldsymbol{P}_X(\boldsymbol{q}) + \frac{i}{2m_q}\boldsymbol{\Sigma}_X(\boldsymbol{q})\times\boldsymbol{q}\right], \\
\boldsymbol{J}_{d,NX}(\boldsymbol{q}') &= \langle N(\boldsymbol{p}')|\boldsymbol{J}(0)|X(\boldsymbol{p}_X)\rangle_{direct} \\
&= N_0\left[\frac{-\boldsymbol{q}'}{6m_q}\rho^*_X(\boldsymbol{q}') + \boldsymbol{P}^*_X(\boldsymbol{q}') - \frac{i}{2m_q}\boldsymbol{\Sigma}^*_X(\boldsymbol{q}')\times\boldsymbol{q}'\right].
\end{aligned}$$

The current density in the cross term has a more complicated $\boldsymbol{q}$ and $\boldsymbol{q}'$ dependence. Because $\boldsymbol{p}_X = -\boldsymbol{q}-\boldsymbol{q}'$, $\boldsymbol{p}+\boldsymbol{p}_X = -\boldsymbol{q}'-2\boldsymbol{q}$ and $\boldsymbol{p}+\boldsymbol{p}'_X = -\boldsymbol{q}-2\boldsymbol{q}'$, it involves terms depending upon both $\boldsymbol{q}$ and $\boldsymbol{q}'$. Let

$$\begin{aligned}
\langle X(\boldsymbol{p}_X)|\boldsymbol{J}(0)|N(\boldsymbol{p})\rangle_{cross} &= \boldsymbol{J}_{c,XN}(\boldsymbol{q}') + \delta\boldsymbol{J}_{XN}(\boldsymbol{q},\boldsymbol{q}'), \\
\langle N(\boldsymbol{p}')|\boldsymbol{J}(0)|X(\boldsymbol{p}_X)\rangle_{cross} &= \boldsymbol{J}_{c,NX}(\boldsymbol{q}) + +\delta\boldsymbol{J}_{NX}(\boldsymbol{q},\boldsymbol{q}'),
\end{aligned}$$

then $\boldsymbol{J}_{c,XN}(\boldsymbol{q}')$, $\boldsymbol{J}_{c,NX}(\boldsymbol{q})$, $\delta\boldsymbol{J}_{XN}(\boldsymbol{q},\boldsymbol{q}')$ and $\delta\boldsymbol{J}_{NX}(\boldsymbol{q},\boldsymbol{q}')$ are given by the following expressions:

$$\boldsymbol{J}_{c,XN}(\boldsymbol{q}') = N_0\left[\frac{-\boldsymbol{q}'}{6m_q}\rho_X(-\boldsymbol{q}') + \boldsymbol{P}_X(-\boldsymbol{q}') + \frac{i}{2m_q}\boldsymbol{\Sigma}_X(-\boldsymbol{q}')\times(-\boldsymbol{q}')\right],$$

$$\boldsymbol{J}_{c,NX}(\boldsymbol{q}) = N_0\left[\frac{-\boldsymbol{q}}{6m_q}\rho^*_X(-\boldsymbol{q}) + \boldsymbol{P}^*_X(-\boldsymbol{q}) - \frac{i}{2m_q}\boldsymbol{\Sigma}^*_X(-\boldsymbol{q})\times(-\boldsymbol{q})\right],$$

$$\delta\boldsymbol{J}_{XN}(\boldsymbol{q},\boldsymbol{q}') = N_0\frac{-\boldsymbol{q}}{3m_q}\rho_X(-\boldsymbol{q}'),$$

$$\delta\boldsymbol{J}_{NX}(\boldsymbol{q},\boldsymbol{q}') = N_0\frac{-\boldsymbol{q}'}{3m_q}\rho^*_X(-\boldsymbol{q}).$$

From (8) we see that J^0 depends only on $\boldsymbol{q}$ or $\boldsymbol{q}'$, so that $\delta J^0 = 0$. We define the *leading term* of the hadronic tensor $H^{\mu\nu}_{NB}(\boldsymbol{q}'m'_s,\boldsymbol{q}m_s)$ by neglecting the terms depending on both $\boldsymbol{q}$ and $\boldsymbol{q}'$ in the cross term, and the $\boldsymbol{q}'$ dependence of $E_X(\boldsymbol{q}+\boldsymbol{q}')$ in the energy denominator of the cross amplitude, i.e.

$$H^{\mu\nu}_{NB-leading}(\boldsymbol{q}'m'_s,\boldsymbol{q}m_s) =$$

$$\sum_{X\neq N}\left[\frac{J^\mu_{d,NX}(\boldsymbol{q}')\,J^\nu_{d,XN}(\boldsymbol{q})}{M-M_X} + \frac{J^\mu_{c,XN}(\boldsymbol{q}')\,J^\nu_{c,NX}(\boldsymbol{q})}{E(q)-E_X(q)}\right] + H^{\mu\nu}_{seagull}. \quad (20)$$

The *recoil term* of $H^{\mu\nu}_{NB}(\boldsymbol{q}'m'_s, \boldsymbol{q}m_s)$ includes all the effects coming from terms depending on both $\boldsymbol{q}$ and $\boldsymbol{q}'$, and those arising from the expansion of the energy denominator of the cross term to order q'. The recoil contribution contains terms to order q' and higher, and can be written as

$$H^{\mu\nu}_{NB-recoil}(\boldsymbol{q}'m'_s, \boldsymbol{q}m_s) = \sum_{X\neq N}\left[\frac{J^{\mu}_{c,XN}(\boldsymbol{q}')\,\delta J^{\nu}_{NX}(\boldsymbol{q},\boldsymbol{q}') + \delta J^{\mu}_{XN}(\boldsymbol{q},\boldsymbol{q}')\,J^{\nu}_{c,NX}(\boldsymbol{q})}{E(q) - E_X(q)}\right.$$
$$\left. + \frac{1}{[E(q) - E_X(q)]^2}\left(q' + \frac{\boldsymbol{q}'\cdot\boldsymbol{q}}{E_X(q)}\right) J^{\mu}_{c,XN}(\boldsymbol{q}')\,J^{\nu}_{c,NX}(\boldsymbol{q})\right]. \tag{21}$$

To get the GP we need only keep terms to order q'. Thus the recoil effects contribute only to GPs with a magnetic final photon, $\mu \neq 0$.

With (20) and (21) for the *leading* and *recoil terms*, one can carry out the partial wave projection using the definitions of Section 2. The partial wave projection poses no particular difficulty except the need for careful book-keeping. We therefore skip the details of the partial wave decomposition and give the final expressions for the GPs in Section 4.

4. Generalised Polarisabilities in the NRQM

Here we give the final results for the ten GPs in the NRQM. We also study the properties of the GPs in relation to the parameters determining the nucleon structure in the NRQM. Our aim is to find those properties of the nucleon structure to which the GPs are most sensitive. This should help identify the most useful aspects of VCS in studying nucleon structure.

The analytic expressions for the ten GPs can be written in the following form, where the leading contributions are the same as in Guichon *et al.* (1995). [Note, however, that the curve for $P^{(01,1)S}$ in Guichon *et al.* has an error where the cross term was a factor of 2 too large. We correct it here. The overall sign for $P^{(01,1)S}$ is also changed to conform with the definition for the electric virtual photon case.]

The GPs are plotted in Fig. 2.

(4a) *Leading Contributions to the Generalised Polarisabilities*

$$P^{(0101)S} = \frac{1}{2S+1}\frac{1}{18}\frac{1}{\alpha^2}e^{-q^2/6\alpha^2}\sum_{X=N^*,\Delta^*} a_X^2\left(\frac{Z_d^{S,J_X}}{M - M_X} + \frac{Z_c^{S,J_X}}{E(q) - E_X(q)}\right), \tag{22}$$

$$P^{(0112)1} =$$

$$\frac{1}{108}\sqrt{\frac{3}{5}}\frac{1}{m_q\alpha^2}e^{-q^2/6\alpha^2}\sum_{X=N^*,\Delta^*} a_X^2\frac{(-1)^{I_x-1/2}}{2I_x}\left(\frac{Z_{ad}^{2,S,J_X}}{M - M_X} - \frac{Z_{ac}^{2,S,J_X}}{E(q) - E_X(q)}\right), \tag{23}$$

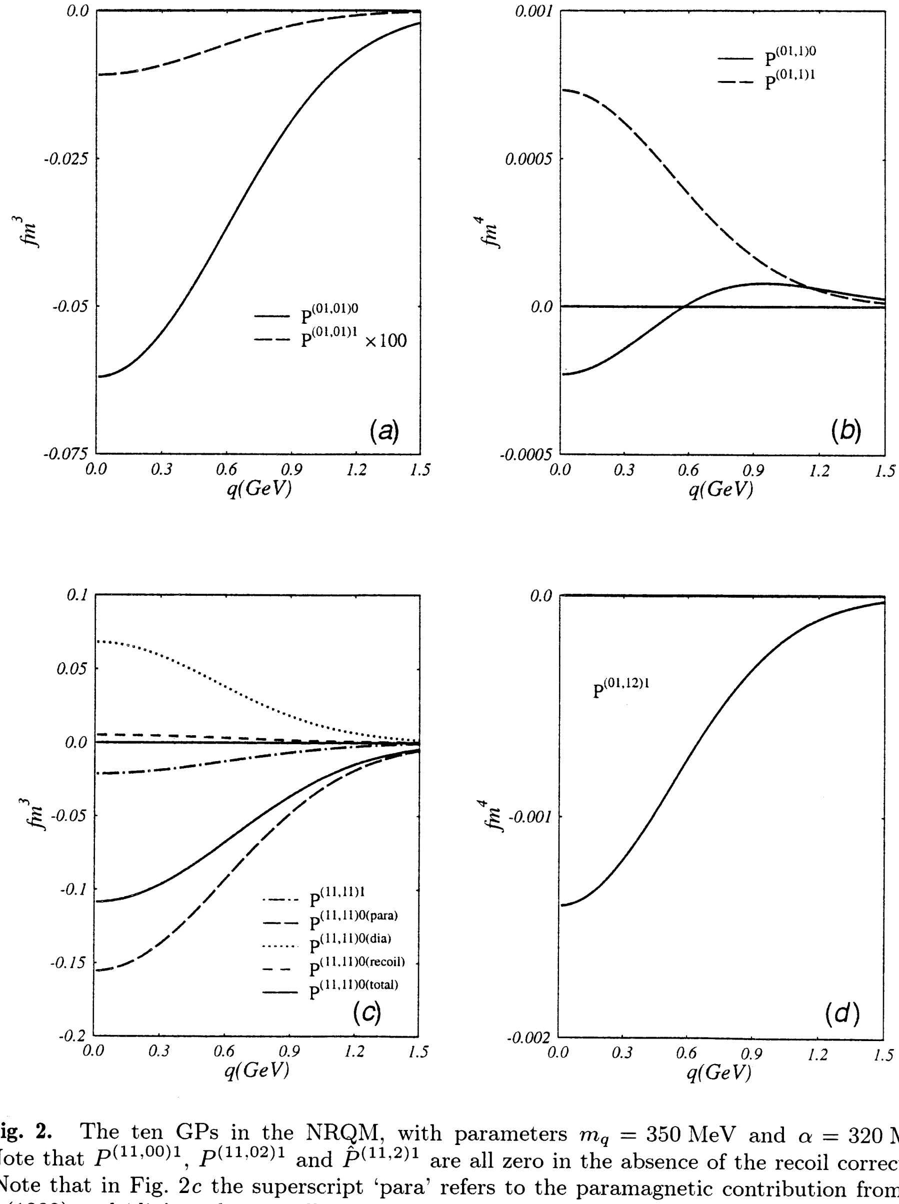

Fig. 2. The ten GPs in the NRQM, with parameters $m_q = 350\,\text{MeV}$ and $\alpha = 320\,\text{MeV}$. Note that $P^{(11,00)1}$, $P^{(11,02)1}$ and $\hat{P}^{(11,2)1}$ are all zero in the absence of the recoil correction. [Note that in Fig. 2c the superscript 'para' refers to the paramagnetic contribution from the $\Delta(1232)$ and 'dia' to the seagull contribution which has the opposite sign.]

$$P_{para}^{(1111)S} = \frac{1}{2S+1}\frac{4}{27}\frac{1}{m_q^2}e^{-q^2/6\alpha^2}\left(\frac{Z_\Delta^S}{M-M_\Delta} + \frac{Z_\Delta^S}{E(q)-E_\Delta(q)}\right), \tag{24}$$

$$P_{dia}^{(1111)S} = \delta_{S0}\frac{7\sqrt{6}}{54}\frac{1}{m_q\alpha^2}e^{-q^2/6\alpha^2}, \tag{25}$$

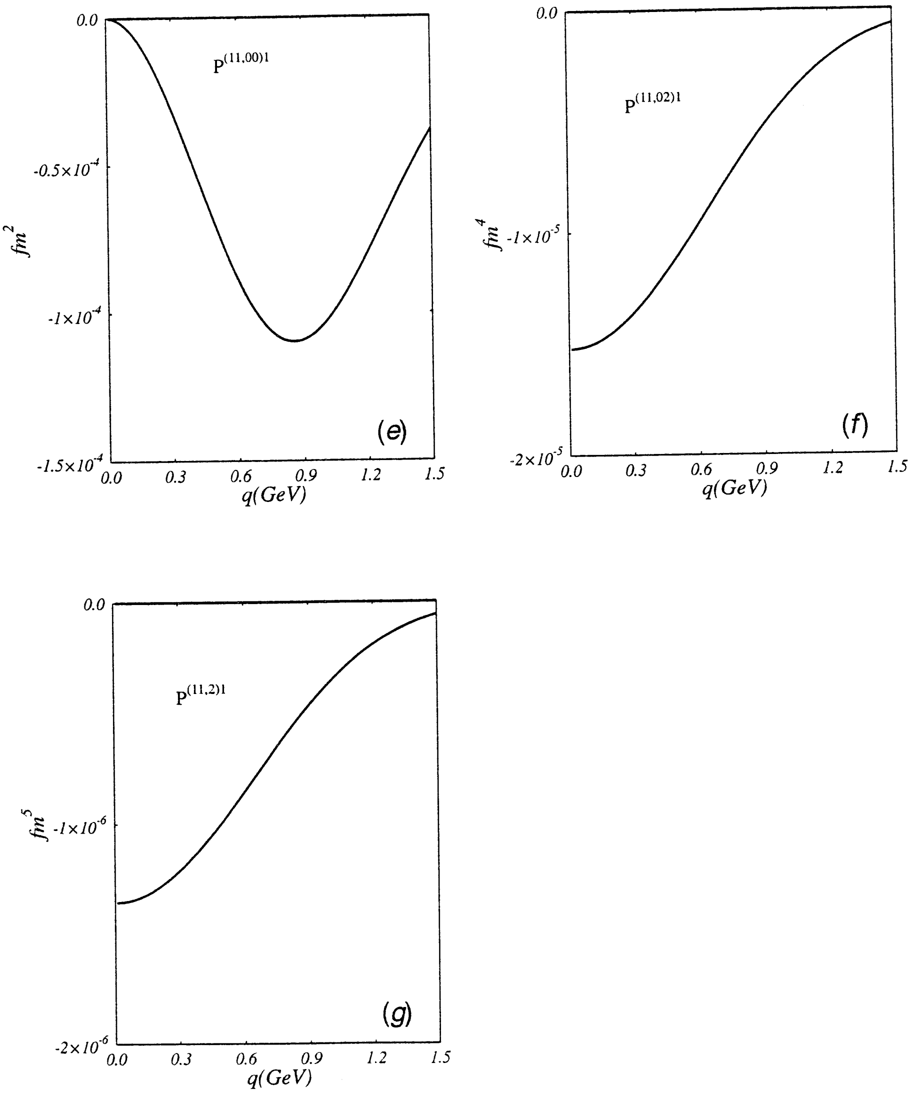

Fig. 2 (*Continued*)

The mixed GP is the sum of two terms:

$$\hat{P}^{(01,1)S} = \hat{P}_F^{(01,1)S} + \hat{P}_S^{(01,1)S}, \tag{26}$$

$$\hat{P}_F^{(01,1)S} = \frac{1}{2S+1}\frac{1}{108}\frac{1}{m_q\alpha^2}\mathrm{e}^{-q^2/6\alpha^2}\sum_{X=N^*,\Delta^*} a_X^2\left(\frac{Z_d^{S,J_X}}{M-M_X} - \frac{Z_c^{S,J_X}}{E(q)-E_X(q)}\right), \tag{27}$$

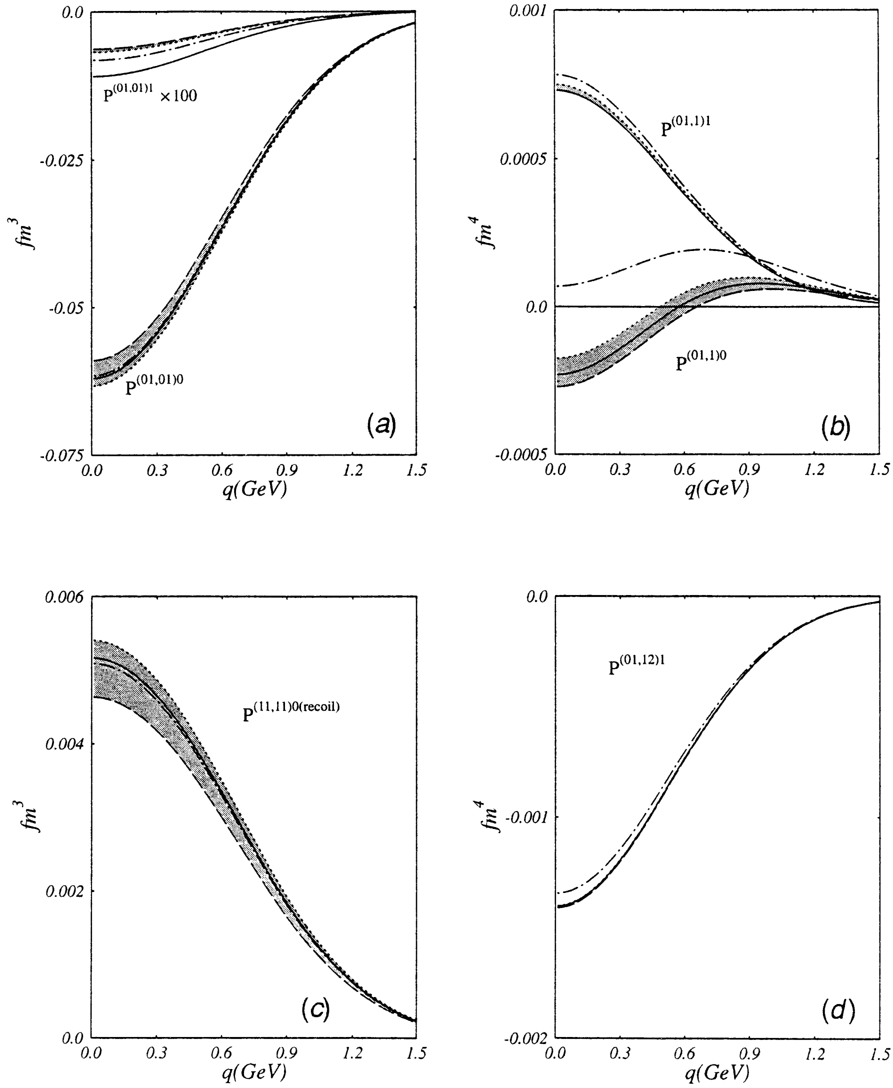

Fig. 3. Effects of the masses of nucleon (Δ) excitations on the GPs. Four groups of different masses for P-wave excitations are used: dotted curve—using the lower limit of the masses from the Particle Data Tables (1994): $N^*(1520)$ $N^*(1640)$ $N^*(1515)$ $N^*(1650)$ $\Delta^*(1615)$ $\Delta^*(1670)$; dashed curve—using the upper limit of the masses from the Particle Data Tables (1994): $N^*(1555)$ $N^*(1680)$ $N^*(1530)$ $N^*(1750)$ $\Delta^*(1675)$ $\Delta^*(1770)$; dot–dash curve—using the theoretical masses given in Isgur and Karl (1978): $N^*(1490)$ $N^*(1655)$ $N^*(1535)$ $N^*(1745)$ $\Delta^*(1685)$ $\Delta^*(1685)$; and solid curve—same as Fig. 2, using the average masses from the Particle Data Tables (1994): $N^*(1535)$ $N^*(1650)$ $N^*(1520)$ $N^*(1700)$ $\Delta^*(1620)$ $\Delta^*(1700)$.

$$\hat{P}_S^{(01,1)S} =$$

$$\frac{1}{2S+1}\frac{1}{36\sqrt{3}}\frac{1}{m_q\alpha^2}e^{-q^2/6\alpha^2}\sum_{X=N^*,\Delta^*}a_X^2\frac{(-1)^{I_x-1/2}}{2I_x}\left(\frac{Z_{ad}^{1,S,J_X}}{M-M_X}-\frac{Z_{ac}^{1,S,J_X}}{E(q)-E_X(q)}\right), \tag{28}$$

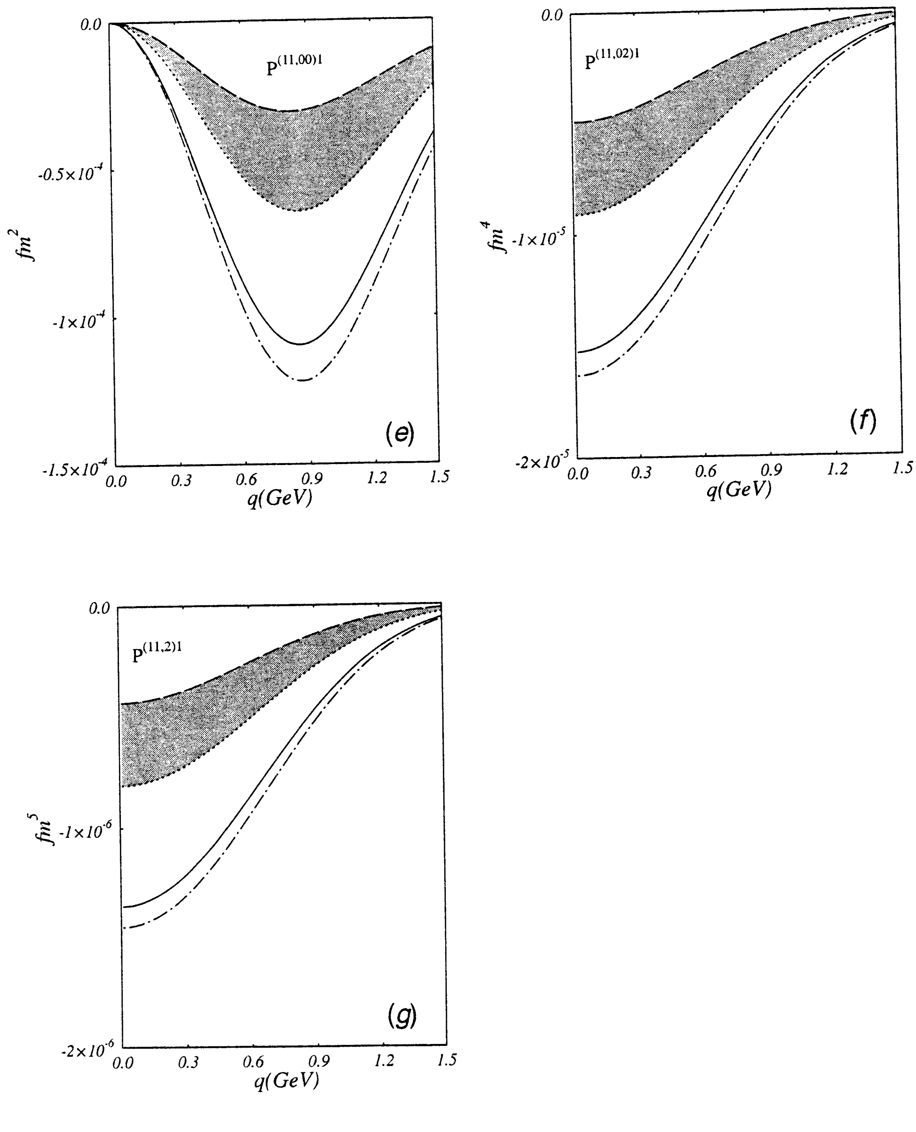

Fig. 3 (*Continued*)

Angular functions Z^{S,J_X}, Z^{L,S,J_X} and $Z^S_{\Delta_{1232}}$ in the above summation are given by:

L	S	J_X	Z_d^{S,J_X}	Z_c^{S,J_X}	Z_{ad}^{L,S,J_X}	Z_{ac}^{L,S,J_X}	$Z^S_{\Delta_{1232}}$
1	0	1/2	$\sqrt{2/3}$	$\sqrt{2/3}$	$-2/\sqrt{3}$	$2/\sqrt{3}$	
1	0	3/2	$2\sqrt{2/3}$	$2\sqrt{2/3}$	$2\sqrt{3}$	$-2\sqrt{3}$	$\sqrt{6}$
1	1	1/2	2	$-2/3$	$-2\sqrt{2}$	$-2\sqrt{2/3}$	
1	1	3/2	-2	2/3	$-\sqrt{2}$	$-\sqrt{2/3}$	1
2	1	3/2			$\sqrt{30}$	$\sqrt{30/3}$	

(4b) Recoil Contributions to Generalised Polarisabilities

$$P^{(1100)1}_{recoil} =$$

$$-\frac{1}{3\sqrt{3}}\frac{q^2}{m_q}\,e^{-q^2/6\alpha^2}\sum_{X=N^*,\Delta^*}\frac{a_X^2 Z^{J_X}_{1100}}{E_X(q)[E(q)-E_X(q)]^2}\left[1-\frac{E_X(q)[E(q)-E_X(q)]}{3\alpha^2}\right], \tag{29}$$

$$P^{(1102)1}_{recoil} =$$

$$-\frac{1}{3\sqrt{3}}\frac{1}{m_q}\,e^{-q^2/6\alpha^2}\sum_{X=N^*,\Delta^*}\frac{a_X^2 Z^{J_X}_{1102}}{E_X(q)[E(q)-E_X(q)]^2}\left[1-\frac{E_X(q)[E(q)-E_X(q)]}{3\alpha^2}\right], \tag{30}$$

$$P^{(1111)0}_{recoil} = \frac{1}{3}\frac{\alpha^2}{m_q^2}\,e^{-q^2/6\alpha^2}\sum_{X=N^*,\Delta^*}\frac{a_X^2 Z^{J_X}_{1111}}{E_X(q)[E(q)-E_X(q)]^2}, \tag{31}$$

$$\hat{P}^{(112)1}_{recoil} =$$

$$-\frac{1}{6\sqrt{15}}\frac{1}{m_q^2}\,e^{-q^2/6\alpha^2}\sum_{X=N^*,\Delta^*}\frac{a_X^2 Z^{J_X}_{1102}}{E_X(q)[E(q)-E_X(q)]^2}\left[1-\frac{E_X(q)[E(q)-E_X(q)]}{3\alpha^2}\right], \tag{32}$$

where

J_X	$Z^{J_X}_{1100}$	$Z^{J_X}_{1102}$	$Z^{J_X}_{1111}$
1/2	2/27	$\sqrt{2}/27$	$3\sqrt{6}/27$
3/2	−2/27	$-\sqrt{2}/27$	$6\sqrt{6}/27$

The $^4 8$ component of the excited state wavefunctions contribute nothing to the proton GPs, because of the isospin factor $(1-2\tau)$. However, they do contribute in the neutron case, which we do not study here. The parameters $m_q = 350$ MeV and $\alpha = 320$ MeV are taken from Isgur and Karl (1978). The P-wave intermediate states are ordered as in the following table, together with their representation mixing parameters a_X for the $^2 8$ representation:

X	$N^*(\frac{1}{2}^-, 1535)$	$N^*(\frac{1}{2}^-, 1650)$	$N^*(\frac{3}{2}^-, 1520)$	$N^*(\frac{1}{2}^-, 1700)$	$\Delta^*(\frac{1}{2}^-, 1620)$	$\Delta^*(\frac{3}{2}^-, 1700)$
a_X	0·85	−0·53	0·99	0·11	1·0	1·0

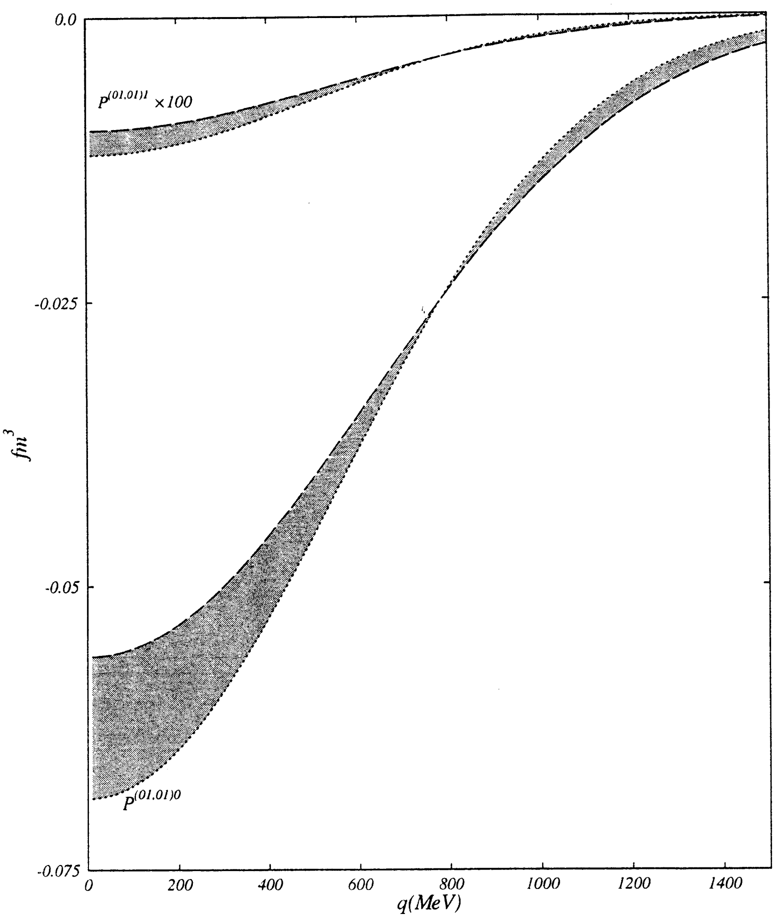

Fig. 4. Effects of a $\pm 5\%$ variation (304 and 336 MeV) in α from its normal value of 320 MeV. Other parameters remain the same as in Fig. 2. The dotted curve is for $\alpha = 304$ MeV, the dashed curve for $\alpha = 336$ MeV, and the solid curve for $\alpha = 320$ MeV, as in Fig. 2.

5. N^* Properties and the GPs

The NRQM parameters are well determined by fitting the static properties of baryons (Isgur and Karl 1978). Nonetheless, there are phenomenological models other than the NRQM, such as the bag models. Different models may not necessarily give exactly the same properties for the nucleon and its excitations. We do not intend to survey all the different nucleon models in this paper, but just investigate those nucleon and N^* properties which exert the most important influence on the behaviour of the GPs in the NRQM. Two main factors are studied here: the mass spectrum of the nucleon and the size parameter α. The results are displayed in Figs 2, 3 and 4.

The GPs have a strong dependence on the mass and energy spectrum of the excited states of the nucleon and the Δ. In Fig. 2 we used the average masses of the N^* and Δ^* from the Particle Data Tables (1994). However, these masses are all determined within a range and may be different from the predictions of the NRQM. We study the effects of the N^* mass spectrum by comparing the GPs calculated with the lower and upper limits of the masses from the Particle Data Tables (1994) and also with those predicted in the NRQM of Isgur and Karl (1978). Fig. 3 shows that some GPs are very sensitive to the N^* masses, particularly at small q. The effect on $\hat{P}^{(01,1)0}$ is especially large as compared with the theoretical masses of Isgur and Karl (1978). The changes in $P^{(11,00)1}$, $P^{(11,02)1}$ and $\hat{P}^{(11,2)1}$ are quite drastic due to a more complicated factor for the mass and energy differences.

The effects of the hadron size parameter α are illustrated in Fig. 4, where the plot is for a variation of $\pm 5\%$ in α from its normal value of 320 MeV. The main influences are again seen to be in the low-q region.

In conclusion, we have presented a calculation of the N^* contribution to the generalised polarisabilities for virtual Compton scattering on the proton. The dependence of these GPs on the N^* properties has been studied. We hope that there will soon be experimental data with which these estimates can be compared.

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