

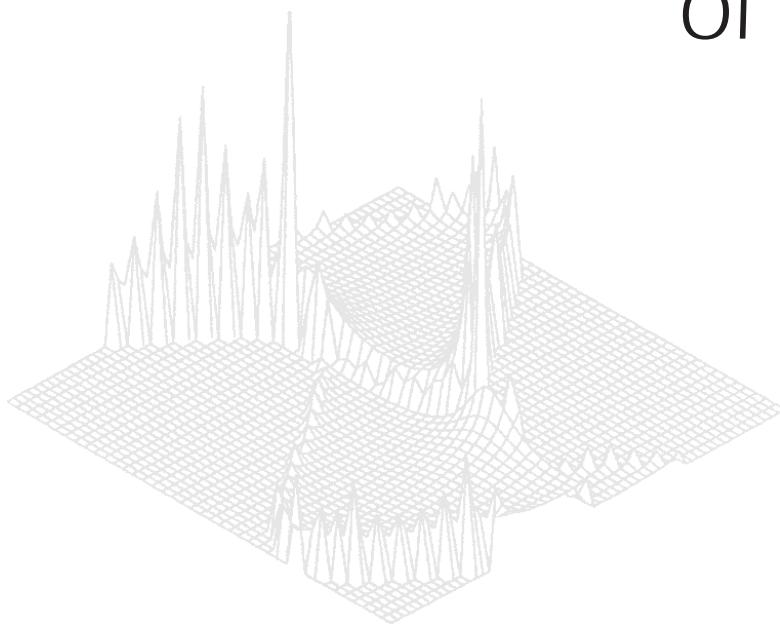
---

CSIRO PUBLISHING

---

# Australian Journal of Physics

Volume 51, 1998  
© CSIRO 1998



A journal for the publication of  
original research in all branches of physics

**[www.publish.csiro.au/journals/ajp](http://www.publish.csiro.au/journals/ajp)**

All enquiries and manuscripts should be directed to

*Australian Journal of Physics*

**CSIRO PUBLISHING**

PO Box 1139 (150 Oxford St)

Collingwood

Vic. 3066

Australia

Telephone: 61 3 9662 7626

Facsimile: 61 3 9662 7611

Email: [peter.robertson@publish.csiro.au](mailto:peter.robertson@publish.csiro.au)



Published by **CSIRO PUBLISHING**  
for CSIRO and the  
Australian Academy of Science



## Deep Convection in the Interior of Major Planets: A Review

*Jun-Ichi Yano*

CRC-Southern Hemisphere Meteorology, Monash University,  
Clayton, Vic. 3168, Australia.

### *Abstract*

Theories for deep convection in the interior of major planets are reviewed. The focus is on Busse's theory, whose problems are critically analysed. The importance of the analogy and differences with the Earth's mantle convection and oceanographic convection are emphasised.

### 1. Introduction

Two major factors come into consideration of the general circulation of the atmosphere of major planets. First, the atmospheres emit substantially more energy by long-wave radiation than the energy they absorb from the Sun. It means their atmospheres receive additional heat supply from the interior of the planets. The energy ratio, i.e. the ratio of the total radiative energy emitted by a planet to the total solar energy absorbed, is estimated to be 1.67 for Jupiter (Hanel *et al.* 1981), 1.78 for Saturn (Hanel *et al.* 1983), and more than 2.3 for Neptune (Pearl and Conrath 1991), but less than 1.14 for Uranus (Pearl *et al.* 1990). Hence, except for Uranus, the heat supply from the interior of the major planets is more than half of that from the Sun. The second important factor is the absence of the planet surface in these atmospheres. Standard models for the interior (e.g. Hubbard 1981; Stevenson 1982; Hubbard and Marley 1989; Chabrier *et al.* 1992) show that only a few tens of per cent of the planetary radius is occupied by a solid core, which is immediately surrounded by the liquid metallic hydrogen layer. The thickness of the atmospheric layer occupying immediately above this metallic hydrogen layer is estimated as 20% and 50% of the radius of Jupiter and Saturn, respectively.

Thermal convection was identified by earlier studies (Hubbard 1968) to be the main mechanism to transport heat in those deep atmospheres because the thermal diffusivity is too small for effective thermal diffusion and the opacity is predicted to be too low for effective radiative transfer. Consequently, the possibility of deep thermal convection comes into serious consideration for the circulation of Jovian atmospheres. Busse (1976) originally hypothesised that such deep thermal convection drives and maintains the dominant zonal winds (i.e. longitudinally-directed winds) observed at the cloud-level of the atmosphere, and

this hypothesis was later endorsed by Ingersoll and Pollard (1982), Yano (1987*a*, 1987*b*) and others. The recent Galileo probe's *in situ* observation, showing that the cloud-top level winds extend as deep as the 20 bar pressure-level (Atkinson *et al.* 1996), appears to add extra credibility to Busse's theory. Henceforth, the present review also focusses on Busse's hypothesis.

## 2. Pictures for Jovian Convection

Distinctively different pictures exist for thermal convection inside major planets. One extreme picture is that thermal convection is completely random, consisting of small-scale incoherent eddies. Hence, convection is best described by a diffusion equation. Such an approach was taken by Ingersoll and Porco (1978) to investigate the adjustment of the Jovian atmosphere by convection against solar differential heating. Williams (1978, 1979, 1985) took this picture to justify his shallow-dynamics approach. More recently, Smith and Gierasch (1995) used this approach to investigate the thermal structure inside the major planets.

The approach basically follows the classical view for the fully-nonlinear turbulent system, in which the motions are completely random and incoherent so that a statistical description (e.g. in terms of the diffusion) best suits this type of system. Certainly, the Jovian interior is in this regime, which can justify the diffusion description. However, such a classical picture for the turbulence is increasingly disputed by recent direct simulations of fully-developed turbulence (e.g. Métais and Lesieur 1989; McWilliams and Weiss 1994). These studies show that coherent structures are more commonly developed in fully-developed turbulence.

Busse's picture also assumes such a large-scale coherent structure for deep Jovian convection, but it is mostly based on linear analysis. We examine this picture more closely in the next three sections.

An alternative picture still exists, which is partially based on an analogy with oceanographic convection. Such convection arises when the system is strongly controlled by boundary forcings: cooling at the top of the atmosphere and/or heating at the bottom of the deep atmosphere. Then convection is likely to take an intermittent, plume-like structure rather than steady large-scale overturning. This possibility is addressed in Section 6.

## 3. Busse's Model: Linear Theory

Busse's model for thermal convection in major planets follows a line of thought in classical studies of Rayleigh–Benard convection under the Boussinesq approximation (cf. Chandrasekhar 1961). Along this line, the first goal is to define the Rayleigh number for the onset of convection by a linear stability analysis. This problem is symbolically stated as

$$\left(\frac{\partial}{\partial t} + \mathbf{L}\right)\phi = 0,$$

where  $\mathbf{L}$  is a linear operator,  $\phi$  is a vector representing a set of dependent

variables (velocity, temperature). The time derivative  $\partial/\partial t$  can be, furthermore, replaced by an eigenvalue  $\sigma$ , so that the problem reduces to

$$(\sigma + \mathbf{L})\phi = 0. \quad (1)$$

The onset of convection is defined by

$$\text{Re}(\sigma) = 0.$$

In this formulation, we look for the minimum Rayleigh number (critical Rayleigh number)  $R$  to satisfy this condition, and an accompanying eigensolution (i.e. marginal convection).

The physically relevant asymptotic limit for Jovian convection is the limit of rapid rotation, i.e.  $\Omega \rightarrow +\infty$ , and low viscosity, i.e.  $\nu \rightarrow 0$ , where  $\Omega$  is the planetary rotation rate and  $\nu$  the viscosity. In terms of the nondimensional parameters, this means taking an asymptotic limit  $E \rightarrow +0$ , where  $E \equiv \nu/2\Omega r_0^2$  is the Ekman number and  $r_0$  is the planetary radius.

The steady motion in this limit satisfies the geostrophic balance

$$2\mathbf{\Omega} \times \mathbf{v} = -\nabla\pi$$

in the absence of buoyancy, where  $\mathbf{v}$  is the velocity, and the right-hand side represents the pressure force.

By applying  $\nabla \times$  to the above, we obtain the Taylor–Proudman theorem (cf. Pedlosky 1987, Sect. 2.7)

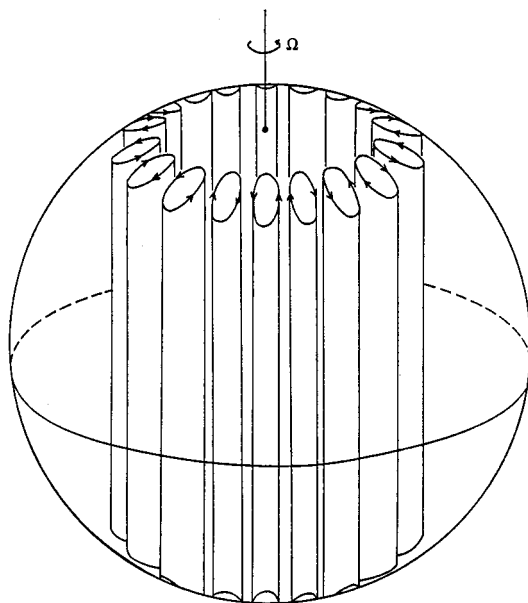
$$(\mathbf{\Omega} \cdot \nabla)\mathbf{v} = 0,$$

or

$$\frac{\partial}{\partial z}\mathbf{v} = 0$$

by taking the  $z$ -coordinates in the direction of the axis of rotation. This theorem implies that any motion in this limit is of columnar structure in the direction of the axis of rotation, in which the fluid moves in alignment (i.e. the Taylor column). This imposes a strong constraint to thermal convection in a rotating sphere, because no motion away from the axis of rotation is possible by this theorem for a spherical geometry.

The inclusion of buoyancy does not resolve this constraint (cf. Hirsching and Yano 1994). Hence, in order to set thermal convection, this constraint must be perturbed by either the viscosity  $\nu$ , thermal diffusivity  $\kappa$  or unsteadiness of the motion. Roberts (1968) showed by a systematic scale analysis that the system overcomes this constraint by forming an internal viscous boundary layer of the scale  $r_0 E^{\frac{1}{3}}$  in the direction perpendicular to the axis of rotation, centred at a certain distance (i.e. critical distance), say,  $s = s_0$  from the axis of rotation. Hence, marginal convection takes a form of Taylor columns aligned to the inside of this internal boundary layer confined to the cylindrical surface  $s = s_0$  (Busse 1970: see Fig. 1).



**Fig. 1.** Busse's schematic picture for Jovian thermal convection.  
[Reproduced from Fig. 1 of Busse 1970.]

This configuration of marginal convection justifies seeking a WKBJ solution confined to a cylindrical surface  $s = s_0$ , i.e.

$$\phi \sim \exp[ik(s - s_0) + im\varphi + i\omega t], \quad (2)$$

where  $k$  is the local radial wavenumber,  $m$  the azimuthal wavenumber,  $\varphi$  the coordinate in azimuthal direction and  $\omega$  the frequency ( $\omega \equiv \text{Im}\sigma$ ).

By substitution of (2) into the linear equation (1) and by taking a determinant, we obtain the complex dispersion relationship

$$\mathcal{L}(s_0, k, m, \omega, R) = 0, \quad (3)$$

or by solving it in terms of the Rayleigh number

$$R = R(s_0, k, m, \omega).$$

The critical Rayleigh number is obtained by minimising the Rayleigh number against  $s_0$ ,  $k$  and  $m$ . In particular, from the condition

$$\partial R / \partial s_0 = 0,$$

we obtain,  $s_0/r_0 \simeq 0.5$ , i.e. convective Taylor columns are formed at approximately half the planetary radius from the axis of rotation.

However, this WKBJ solution contains a very odd feature in that the critical radial wavenumber  $k$  defined by  $\partial R / \partial k = 0$  becomes  $k = 0$ . Hence, no radial

structure is defined to the leading order of the problem. Consequently, we have to move to a higher order modulation equation to define the radial structure of marginal convection. Soward (1977) showed that this modulation equation leads to a radial structure whose amplitude exponentially increases outward from the critical distance  $s = s_0$ . This singular behaviour of the solution is evidently not consistent with the original assumption of convection confined to a cylindrical surface  $s = s_0$ . Hence we have to conclude that the critical Rayleigh number and critical convection defined by this method are incorrect.

A closer inspection of the modulation equation can show that the problem stems from the fact that  $\partial R/\partial s_0$  does not completely vanish at the critical point defined by the above method. More precisely, although the extremum for the Rayleigh number is defined along the real axis  $s_r = \text{Re}(s_0)$  of the critical distance, i.e.  $\partial R/\partial s_r = 0$ , or  $\text{Re}[\partial R/\partial s_0] = 0$ , its extremum is not yet taken in the imaginary direction  $s_i = \text{Im}(s_0)$  of the critical distance  $s_0$ . Hence, it still remains  $\partial R/\partial s_i \neq 0$ , or  $\text{Im}[\partial R/\partial s_0] \neq 0$ . In order to obtain a well-behaved localised solution,  $\partial R/\partial s_i = 0$  must be also satisfied as well as  $\partial R/\partial s_r = 0$ . In other words, we have to satisfy the condition  $\partial R/\partial s_0 = 0$  on the complex plane of  $s_0$ . Hence, the critical distance  $s_0$  becomes a complex number for this problem. Yano (1992) sought such a complex critical distance, which is defined as the saddle point of critical Rayleigh number (i.e. already minimised against  $k$  and  $m$ ) on the complex  $s_0$  surface. Consequently, the critical Rayleigh number defined by this new method becomes higher than the original one.

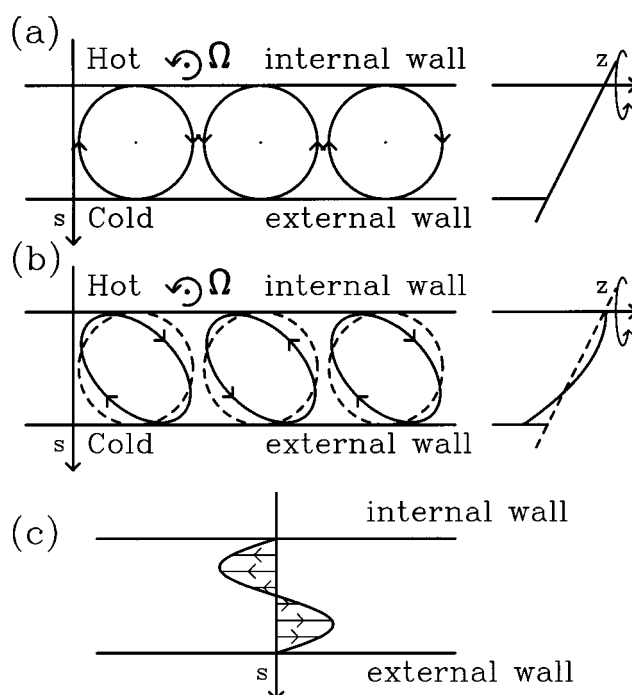
Finally, the radial structure of critical convection is obtained by substituting the critical Rayleigh number in the dispersion relation (3) and solving it for the radial wavenumber  $k$  along the real axis, i.e.  $\text{Im}(s_0) = 0$ . As a result, we obtain a complex radial wavenumber, which ensures an exponential confinement of Taylor columns. The deviation of the complex critical distance from the real axis increases with a decrease of the Prandtl number  $P = \nu/\kappa$ , and hence, a change of critical Rayleigh number from the original estimate. This increasing tendency for the complex critical distance results in a larger deformation of the Taylor columns in the radial direction (spiralling mode: see Zhang 1992)

However, this columnar-spiralling mode becomes a less efficient mechanism to break the Taylor–Proudman constraint with a further decrease of the Prandtl number by increase of the critical Rayleigh number. Eventually, this is taken over by another mode (inertial mode) which uses the unsteadiness to break the Taylor–Proudman constraint. This mode is named the wall-attached mode, because its Taylor columns are attached to the equatorial outer wall (Zhang 1992). Physical interpretations for the interplay of those marginal modes (columnar-spiralling, wall-attached) were developed by Hirsching and Yano (1994).

#### 4. Busse's Zonal Flow Theory

By facing the afore-mentioned difficulty in determining the radial structure of convection, Busse virtually gave up the effort to solve this problem properly, and this attitude still remains the same with him (cf. Busse 1994). In order to simplify the problem and to make it solvable, instead, Busse placed the side walls, say, at  $s = s_0 \pm D/2$  separated by a distance  $D$ . This enables the Taylor columns to be isolated in the radial direction artificially. The mathematical treatment

can be further simplified by taking the limit  $D/r_0 \ll 1$ , because the cylindrical curvature no longer becomes a primary concern. The zonal flow theory for major planets by Busse (1983) was constructed under this framework (Fig. 2).



**Fig. 2.** Schematics for the construction of Busse's zonal flow theory for major planets. (a) The leading order approximation, and (b) a correction to a higher order. The left frame shows the top view (with the longitudinal coordinate shown horizontally), and the side view with the rotation axis ( $z$ -axis) placed horizontally to the right side. (c) The resulting zonal winds (top view): the westerly jet to the external side, and the easterly to the inside.

To first approximation, the curvature effect of the spherical boundary at the top and bottom of the cylindrical channel is neglected and is replaced by a constant slope (Fig. 2a). Circular Taylor columns drifting with a constant phase velocity are obtained as a result. The curvature effect of the boundaries is considered as a correction term to a higher order (Fig. 2b), which is mathematically accomplished by adding higher harmonics to the leading order solution. This leads to eastward-tilted columns in the direction away from the axis of rotation, which results in an outward transportation of the westerly\* momentum. The generated zonal flow is computed by assuming a balance of its viscous dissipation with the eddy momentum flux assuming a no slip boundary condition at the side walls. This results in a pair of jets, westerly to the external wall side and easterly to the internal wall side (Fig. 2c).

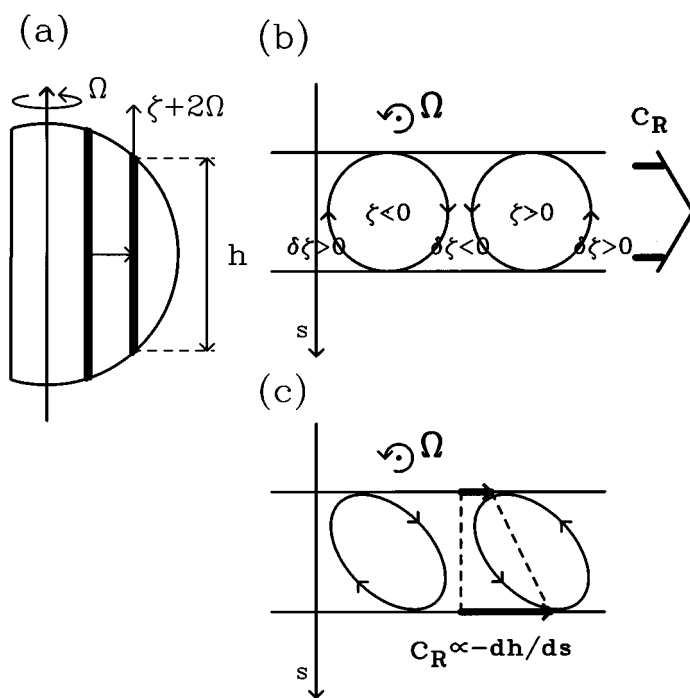
\* By meteorological convention, westerly means the winds blow from the west, and easterly those from the east.

However, it is not quite clear how this calculation leads to a schematic of multiple jets in Jovian atmospheres with a multiple cylindrical layer of Taylor columns in Fig. 2 of Busse (1983) and the subsequent plots of the so-called theoretical prediction of the zonal flows for Jupiter and Saturn in his Fig. 3. The schematic obviously does not agree with the structure of theoretically predicted critical convection. The latter only predicts a single cylindrical layer of Taylor columns. However, Busse appears to believe that once the supercriticality is increased, both the outer and inner sides of the Taylor-column cylindrical layer are gradually filled by another Taylor-column cylindrical layer with accompanying pair jets. Certainly, such a speculation is plausible but not substantiated.

Even the physical mechanism to obtain a pair of zonal jets with the cylindrical channel configuration is not at all explained by Busse. This mechanism may be understood in terms of the potential vorticity, i.e.

$$\frac{\zeta + 2\Omega}{h},$$

which is conserved along a movement of a parcel column, where  $\zeta$  is the relative vorticity, and where  $h$  is the total ‘depth’ (i.e. height) of the parcel column in



**Fig. 3.** Physical interpretation of Busse’s zonal flow theory for major planets. (a) A side view of the planet depicting the conservation of the potential vorticity  $(\zeta + 2\Omega)/h$  along the movement of the Taylor column. (b) The eastward propagation of the vortices due to the  $\beta$  effect (top view). (c) The differential propagation of the vortices due to the increase of  $\beta \propto -dh/ds$  in the outward direction from the axis of rotation, which induces the tilt of the vortices (top view).



the direction of the axis of rotation, in which direction the motion of fluid is almost aligned by the Taylor–Proudman theorem (Fig. 3*a*). As a result, the relative vorticity decreases when a parcel is shifted outward (hence  $h$  decreases) and vice versa. This leads to generation of a new vorticity anomaly to the east of a vorticity anomaly and, hence, leads to an eastward drift of vorticity columns (Fig. 3*b*). It also transpires that the drift speed is proportional to the slope  $dh/ds$  of the boundary, which can be further interpreted as a topographic  $\beta$  effect but with a negative sign (Pedlosky 1987, Sect. 3.17). Because the magnitude of the slope  $|dh/ds|$  increases with the radial distance  $s$ , the outer part of a convection column tends to move faster eastwards than its inner part (Fig. 3*c*). Such a differential drifting of a Taylor column leads to an eastward tilting of columns outward and generation of a westerly jet to the equatorial side. This whole process may be contrasted with eddy dynamics on a rotating sphere. A similar argument follows for the eddy momentum flux in this case as well. However, due to the positive sign of the  $\beta$  effect, it leads to the generation of an equatorial easterly as observed in the Earth’s atmosphere. This point is re-addressed in the next section.

### 5. Problems with Busse’s Theory

Various problems can be pointed out in Busse’s approach and model. At the most technical level, the radial structure of convection is not properly determined in Busse’s analysis. The problem and its remedy were already discussed in Section 3. From a more practical point of view, Busse’s linear convection predicts only a single cylindrical layer of Taylor columns and, hence, only a single pair of jets is generated. It is not immediately clear how multiple jets in the major planets are explained from this theory.

Nevertheless, the most serious defect of Busse’s theory is that it only deals with the linear problem. In the fully-nonlinear regime, a completely different convection mode can be developed. Or does Busse’s theory still qualitatively apply in a fully nonlinear regime? The most recent laboratory (Manneville and Olson 1996) and numerical (Sun *et al.* 1993) experiments at high supercriticalities (700 and 50 times the critical Rayleigh numbers, respectively) appear to support the second view. Both experiments show that the Taylor-column type structure of convection is fairly well preserved in these highly nonlinear regimes, albeit these columns are highly transient and seldom survive more than one turnover time scale individually. Unlike the linear solution, these Taylor columns tend to form multiple cylindrical layers (band structure on the surface) accompanied by multiple jets. In the laboratory experiment (Manneville and Olson), the number of bands appears to increase with supercriticality (see Figs 4 and 5). These results as a whole suggest that the constraint by the Taylor–Proudman theorem in the rapidly rotating convective system is so robust that quasi-two-dimensionality of the flow is maintained even in the fully nonlinear regime. It further suggests that deep convection in major planets may be idealised as two-dimensional turbulence confined *in* a rotating sphere.

Numerical experiments by Cho and Polvani (1996) are very suggestive in this respect: they performed two-dimensional turbulence simulations *on* the sphere with the parameters for four major planets by using the shallow-water model system. Surprisingly, they obtained mean zonal flows fairly similar to the observations for Jupiter and Saturn, apart from a single defect that the flow is

completely reversed in sign. Arguably, this is due to the fact that they used the wrong sign for the  $\beta$  effect; it was pointed out at the end of the last section that the sign of the  $\beta$  effect is reversed by considering the deep two-dimensional flow confined *inside* the rotating sphere. On the other hand, remarkably, the results for Uranus and Neptune agree qualitatively well including the sign of the zonal flow. This implies shallow dynamics for these planets.

A further scepticism arises from Busse's oversimplification of adopting the Boussinesq approximation. Most distinctively, it neglects the strong density stratification inside the planets (see e.g. Fig. 6 of Guillot *et al.* 1994a). Obviously, it is hard to believe that a Taylor-column like motion is formed crossing many density scale heights.

The McWilliams *et al.* (1994) experiment may be illuminating to partially answer this objection. They performed a full three-dimensional simulation of the quasi-geostrophic system (e.g. Pedlosky 1987) with a constant density stratification initialised by a highly random initial condition. They found that this fully nonlinear system settles into a Taylor-column like structure in its final stage. This leads to a heuristic speculation that such a Taylor-column like structure is relatively robust even under a fairly strong density stratification.

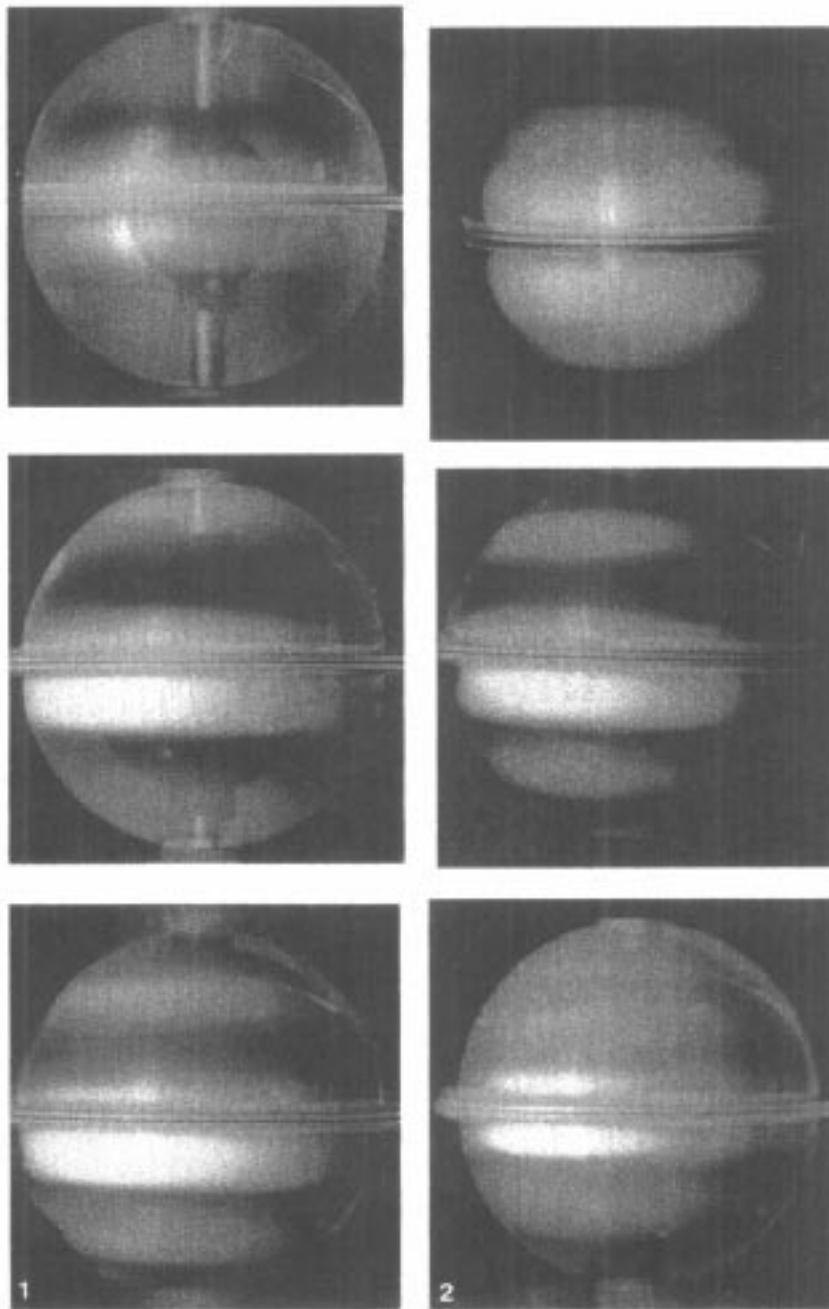
The ultimate limitation of Busse's theory applying to the major planets is the complexity of their internal structure, which has been more clearly realised recently. Stevenson (1985) was probably the first to point out the possibility for more heterogeneity in the internal structure of the major planets than previously thought. He speculated that the Jovian atmospheres may not constitute a single well-mixed layer as traditionally assumed, but instead might consist of multiple mutually-nonpenetrable layers. As a result, convective mixing and convective heat transfer are much more suppressed than previously thought.

Guillot *et al.* (1994a, 1994b) made a careful re-analysis of the opacity of the Jovian gas and found that the Jovian atmosphere is likely to be transparent to infrared radiation at the level of 1–42 kbar, which indicates that the vertical stratification of the upper level of the molecular-hydrogen layer (or atmospheric layer) is substantially different from the adiabatic lapse rate. Furthermore, Ioannou and Lindzen (1993) concluded from their estimate of the tidal dissipation rate for Jupiter that its interior must be mostly stably stratified.

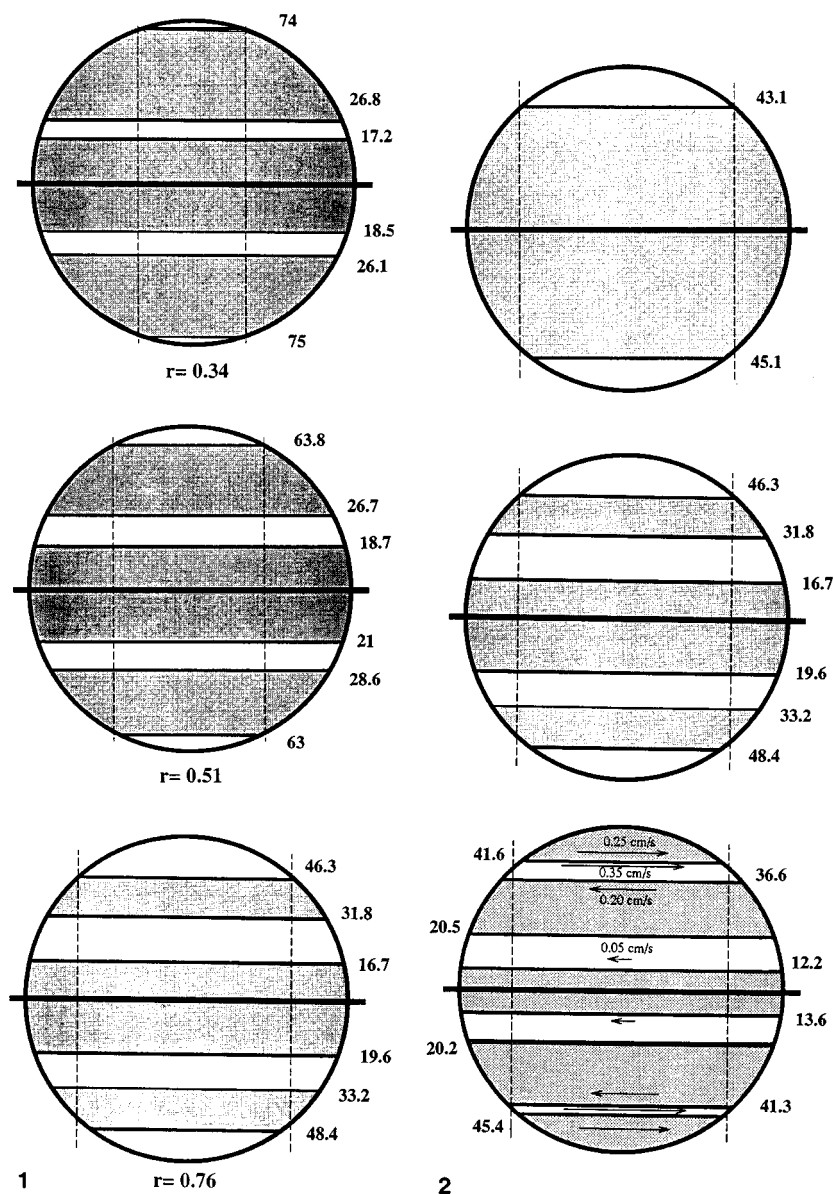
## 6. Mantle Convection and Oceanographic Analogy

In considering the future direction for studies of Jovian convection, some lessons may be learned from studies of the Earth's mantle convection (cf. Davies and Richards 1992). A vast difference between the two types of convection is certainly evident. Earth's mantle convection is in a highly viscous regime (and also in the high Prandtl number limit): the planetary rotation (Coriolis effect) is of no importance, and even the inertial term drops in the momentum equation. This is strikingly contrasted with Jovian convection, where the viscosity is low (and the Prandtl number is of order unity or less), and both the Coriolis effect and the inertial effects must be fully taken into account in the momentum equation.

However, the complexity of the interior of major planets can be of similar order as the Earth's mantle, and we can certainly learn more from Earth mantle studies in this respect. The viscosity is a strong function of temperature and



**Fig. 4.** Laboratory experiments by Manneville and Olson (1996). Time lapse photos in UV light show the band structure of thermal convection in rotating spherical shells. The left side shows various thicknesses from bottom to top, with the inner/outer radius  $r = 0.76, 0.51, 0.34$ . The Rayleigh number is about 100 times the critical value. The right side shows multiplication of bands with increasing Rayleigh number with  $r = 0.76$ : from top to bottom, the Rayleigh numbers are  $-87, 102$  and  $775$  times the critical value. [Reproduced from Fig. 1 of Manneville and Olson 1996.]



**Fig. 5.** The same as Fig. 4 but showing interpretative sketches. [Reproduced from Fig. 2 of Manneville and Olson 1996.]

pressure in the Earth's mantle. It is estimated to change by a factor of  $10^2$ – $10^3$  from the top to the bottom of mantle, and it is further speculated to change by a similar order of magnitude within the thin low viscosity zone just below the lithosphere. The thermal diffusivity and the thermal expansion coefficient may also strongly depend on mantle temperatures and pressures. Similar high temperature–pressure dependences are likely to be equally important in Jovian convection.

Furthermore, the rheology of mantle material is poorly constrained and likely to be highly non-Newtonian, and hence, the Boussinesq approximation is only marginally useful. The deep fluid interior of the Jovian atmospheres may be closer to Newtonian, however, a non-Boussinesq treatment would certainly be equally desirable.

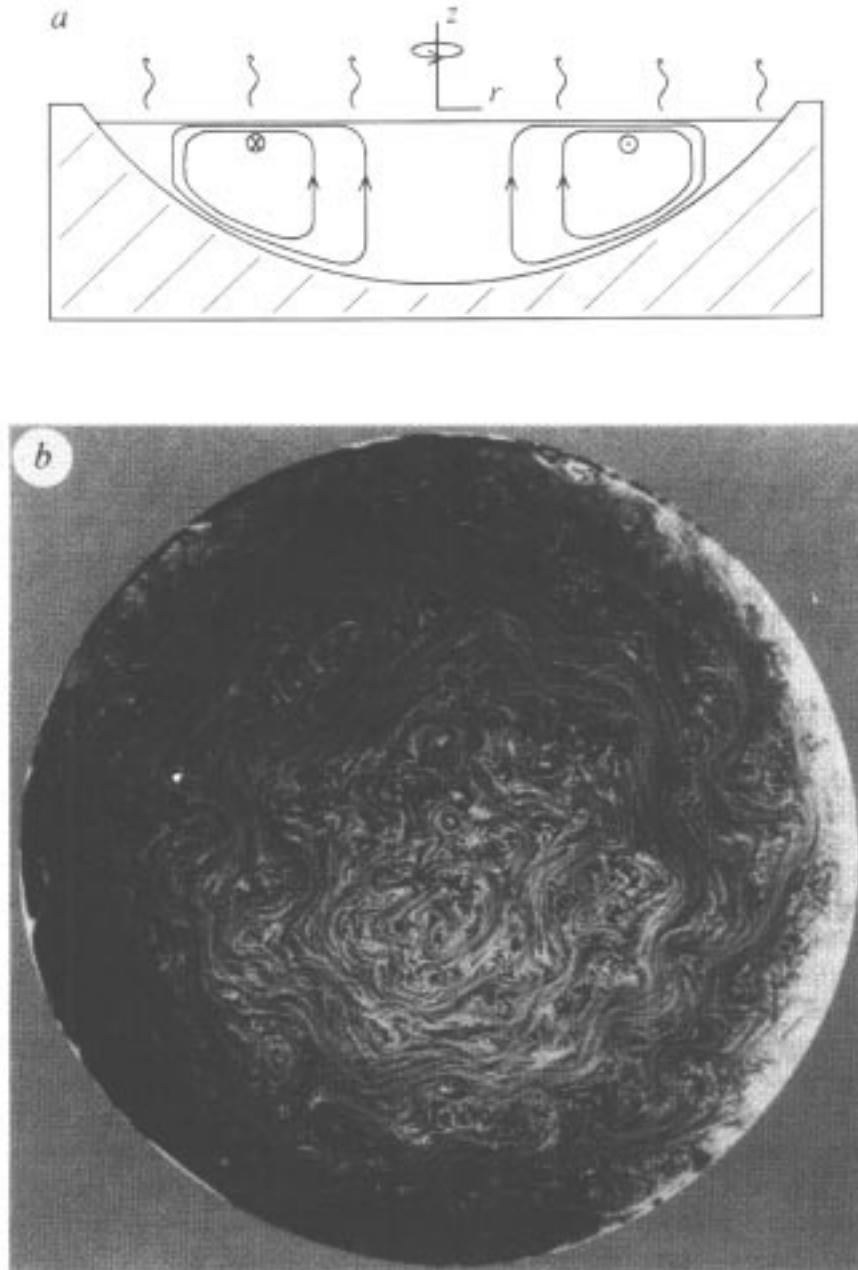
In the Earth's mantle convection, two types of heating contribute to the two distinctively different convection modes. Internal heating by the decay of remnant radioactive isotopes, which accounts for 80–90% of total heating, drives a large-scale 'laminar' overturning-type convection. On the other hand, the surface flux from the core–mantle boundary which accounts for the remaining 10–20% of the heating (boundary forcing) generates plume-type convection. The latter type of convection is expected to be intermittent in time. In contrast, no substantial radioactive heating exists in Jovian atmospheres. It is rather mostly in a simple cooling process from a primordial hot state of the planet, although some mass differentiation such as helium precipitation may contribute to internal heating. Hence, Jovian convection is mostly controlled by boundary forcing and it is expected to be dominated by intermittent plume-type convection, rather than a large-scale overturning.

This leads to a close analogy between Jovian convection and the Earth's oceanographic convection. Oceanographic convection, which often occurs at higher latitudes, is known to be controlled by strong cooling due to the upward infrared heat radiation at the top of the ocean. Both laboratory and numerical experiments have been performed to understand this process. For example, Maxworthy and Narimousa (1994) examined the evolution of a cold water mass initially placed at the top of a rotating water tank. With a substantially slow rotation rate, the intrusion of the cold mass into the lower layer creates a strongly turbulent interface (their Fig. 2*a*). With an increase of the rotation rate, the turbulent interface gradually turns into a more coherent, vertically aligned, filamental structure (their Figs. 2*b*–2*d*). From the top view (their Fig. 3), it is seen that those filamental structures in the vertical direction are accompanied by concentrated vortical structures under the geostrophic balance. The vortical structures become more and more compact on a horizontal plane with an increase of the rotation rate. We call these filamental-vortical structures the Taylor plumes.

The detailed structure of these Taylor plumes may be more easily seen from the numerical experiments by Julien *et al.* (1996). They investigated thermal convection induced by a strong cooling by heat flux at the top of a rotating system. As in the case of Maxworthy and Narimousa (1994), the horizontal scale of Taylor plumes decreases with an increase of the rotation rate. A close association of the plume–cold anomalies with the vorticity field is established, implying a loose geostrophy (or more generally, cyclostrophic balance) of the system.

This type of system was extended to the case with a topographic  $\beta$  effect (a topographic analogy to reproduce the effect of the change of the Coriolis parameter with latitude) by Condie and Rhines (1994) (see Fig. 6). Hot water in a rotating bowl is rapidly cooled at the top free surface exposed to room temperature, which induces plume-type convection (Fig. 6*a*). With the help of the topographic  $\beta$  effect, or as a result of the secondary Hadley-type cell, multiple meandering jets are generated (Fig. 6*b*). The authors claim that this laboratory system is a good

analogue of the Jovian atmospheric circulation driven by thermal convection. Such an oceanographic analogy to Jovian convection may eventually turn out to be a more realistic picture than Busse's theory.



**Fig. 6.** (a) Schematic side view of the experimental set up by Condie and Rhines (1994). (b) An example of the surface flow in a rotating bowl cooled from above and insulated at the sides. The flow is visualised by a streak photograph of floating aluminium powder. A number of meandering jets travelling clockwise around the bowl are recognised. [Reproduced from Fig. 1 of Condie and Rhines 1994.]

## Acknowledgments

The discussion with Fred Pribac on the Earth's mantle convection and communication with Sonya Legg on her oceanographic convection study are acknowledged. Editorial comments by Mikhail Nezlin have improved the presentation of the paper. An earlier version of this paper was presented at the Workshop on Two-dimensional Turbulence in Plasmas and Fluids held at the Australian National University from 16 June to 11 July 1997. The author is supported by the Australian Government Cooperative Research Centres Program.

## References

- Atkinson, D. H., Pollack, J. B., and Seiff, A. (1996). Galileo Doppler measurements of the deep zonal winds at Jupiter. *Science*, **272**, 842–3.
- Busse, F. H. (1970). Thermal instabilities in rapidly rotating systems. *J. Fluid Mech.*, **44**, 441–60.
- Busse, F. H. (1976). A simple model of convection in the Jovian atmosphere. *Icarus*, **29**, 255–60.
- Busse, F. H. (1983). A model of mean zonal flows in the major planets. *Geophys. Astrophys. Fluid Dyn.*, **23**, 153–74.
- Busse, F. H. (1994). Convection driven zonal flows and vortices in the major planets. *Chaos*, **4**, 123–34.
- Chabrier, G., Saumon, D., Hubbard, W. B., and Lunine, J. I. (1992). The molecular-metallic transition of hydrogen and the structure of Jupiter and Saturn. *Astrophys. J.*, **391**, 817–26.
- Chandrasekhar, S. (1961). 'Hydrodynamic and Hydromagnetic Instability' (Clarendon Press: Oxford).
- Cho, J. Y.-K., and Polvani, L. M. (1996). The morphogenesis of bands and zonal winds in the atmospheres on the giant outer planets. *Science*, **273**, 335–7.
- Condie, S. A., and Rhines, P. B. (1994). A convective model for the zonal jets in the atmospheres of Jupiter and Saturn. *Nature*, **367**, 711–13.
- Davies, G. F., and Richards, M. A. (1992). Mantle convection. *J. Geology*, **100**, 151–206.
- Guillot, T., Chabrier, G., Morel, P., and Gautier, D. (1994a). Nonadiabatic models of Jupiter and Saturn. *Icarus*, **112**, 354–67.
- Guillot, T., Gautier, D., Chabrier, G., and Mosser, B. (1994b). Are the giant planets fully convective? *Icarus*, **112**, 337–53.
- Hanel, R. A., Conrath, B., Herath, L. W., Kunde, V. G., and Pirraglia, J. A. (1981). Albedo, internal heat, and energy balance of Jupiter: Preliminary results of the Voyager infrared investigation. *J. Geophys. Res.*, **86**, 8705–12.
- Hanel, R. A., Conrath, B., Kunde, V. G., Pearl, J. C., and Pirraglia, J. A. (1983). Albedo, internal heat flux, and energy balance of Saturn. *Icarus*, **53**, 262–85.
- Hirsching, W. R., and Yano, J.-I. (1994). Metamorphosis of marginal thermal convection in rapidly rotating self-gravitating spherical shells. *Geophys. Astrophys. Fluid Dyn.*, **74**, 143–79.
- Hubbard, W. B. (1968). Thermal structure of Jupiter. *Astrophys. J.*, **152**, 745–53.
- Hubbard, W. B. (1981). Constraints on the origin and interior structure of the major planets. *Phil. Trans. R. Soc. London*, **A303**, 315–26.
- Hubbard, W. B., and Marley, M. S. (1989). Optimized Jupiter, Saturn, and Uranus interior models. *Icarus*, **78**, 102–18.
- Ingersoll, A. P., and Pollard, D. (1982). Motion in the interior and atmospheres of Jupiter and Saturn: Scale analysis, inelastic equations, barotropic instability criterion. *Icarus*, **52**, 62–80.
- Ingersoll, A. P., and Porco, C. C. (1978). Solar heating and internal heat flow on Jupiter. *Icarus*, **35**, 27–43.
- Ioannou, P. J., and Lindzen, R. S. (1993). Gravitational tides on Jupiter. Part II. Interior calculations and estimation of the tidal dissipation factor. *Astrophys. J.*, **406**, 266–78.

- Julien, K., Legg, S., McWilliams, J., and Werne, J. (1996). Penetrative convection in rapidly rotating flows: Preliminary results from numerical simulations. *Dyn. Atmos. Ocean*, **24**, 237–49.
- McWilliams, J. C., and Weiss, J. B. (1994). Anisotropic geophysical vortices. *Chaos*, **4**, 305–11.
- McWilliams, J. C., Weiss, J. B., and Yavneh, I. (1994). Anisotropy and coherent structures in planetary turbulence. *Science*, **264**, 410–13.
- Manneville, J.-B., and Olson, P. (1996). Banded convection in rotating fluid spheres and the circulation of the Jovian atmosphere. *Icarus*, **122**, 242–350.
- Maxworthy, T., and Narimousa, S. (1994). Unsteady, turbulent convection into a homogeneous, rotating fluid, with oceanographic applications. *J. Phys. Oceanogr.*, **24**, 865–87.
- Métais, O., and Lesieur, M. (Eds) (1989). ‘Turbulence and Coherent Structures’ (Kluwer: Dordrecht).
- Pearl, J. C., and Conrath, B. J. (1991). The albedo, effective temperature, and energy balance of Neptune, as determined from Voyager data. *J. Geophys. Res.*, **96**, 18,921–30.
- Pearl, J. C., Conrath, B. J., Hanel, R. A., and Pirraglia, J. A. (1990). The albedo, effective temperature, and energy balance of Uranus, as determined from Voyager IRIS data. *Icarus*, **84**, 12–28.
- Pedlosky, J. (1987). ‘Geophysical Fluid Dynamics’, 2nd edn (Springer: Berlin).
- Roberts, P. H. (1968). On the thermal instability of a rotating fluid sphere containing heat sources. *Phil. Trans. R. Soc. London A* **263**, 93–117.
- Smith, M. D., and Gierasch, P. J. (1995). Convection in the outer planets atmospheres including ortho-para hydrogen convection. *Icarus*, **116**, 159–79.
- Soward, A. M. (1977). On the finite amplitude thermal instability of a rapidly rotating fluid sphere. *Geophys. Astrophys. Fluid Dyn.*, **9**, 19–74.
- Stevenson, D. J. (1982). Interiors of the giant planets. *Ann. Rev. Earth Planet. Sci.*, **10**, 257–95.
- Stevenson, D. J. (1985). Cosmochemistry and structure of the giant planets. *Icarus*, **62**, 4–15.
- Sun, Z. P., Schubert, G., and Glatzmaier, G. A. (1993). Banded surface flow maintained by convection in a model of the rapidly rotating giant planets. *Science*, **260**, 661–4.
- Williams, G. P. (1978). Planetary circulations: I. Barotropic representation of Jovian and terrestrial turbulence. *J. Atmos. Sci.*, **35**, 1339–426.
- Williams, G. P. (1979). Planetary circulations: II. The Jovian quasi-geostrophic regime. *J. Atmos. Sci.*, **36**, 932–68.
- Williams, G. P. (1985). Jovian and comparative atmospheric modeling. *Adv. Geophys. A* **28**, 381–429.
- Yano, J.-I. (1987a). Rudimentary considerations of the dynamics of Jovian atmospheres: Part I. The depth of motions and energetics. *J. Meteor. Soc. Japan*, **65**, 313–27.
- Yano, J.-I. (1987b). Rudimentary considerations of the dynamics of Jovian atmospheres: Part II. Dynamics of the atmospheric layer. *J. Meteor. Soc. Japan*, **65**, 329–40.
- Yano, J.-I. (1992). Asymptotic theory of thermal convection in rapidly rotating systems. *J. Fluid Mech.*, **243**, 103–31.
- Zhang, K. K. (1992). Spiralling columnar convection in rapidly rotating spherical fluid shells. *J. Fluid Mech.*, **236**, 535–56.