

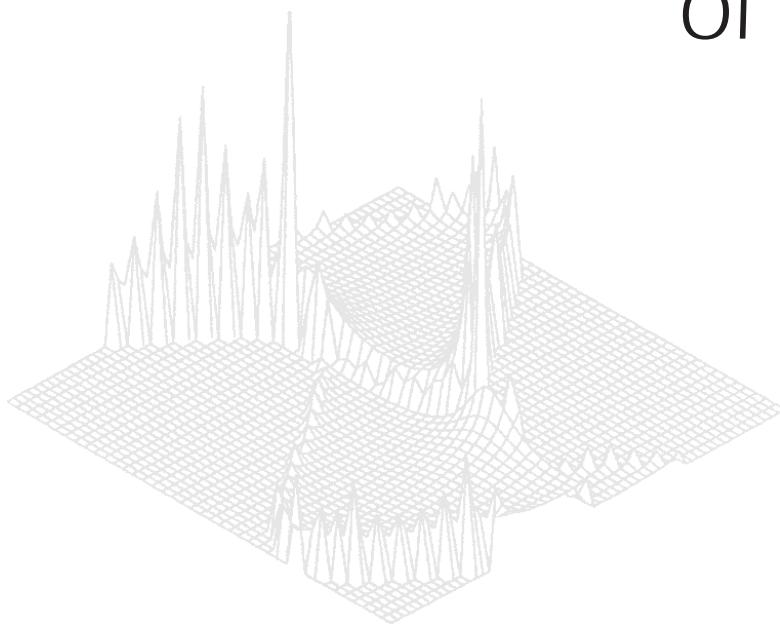
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# Australian Journal of Physics

Volume 51, 1998  
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## Implications of a Nonzero Cosmological Constant and Luminosity Selection Effects on Cosmological Tests

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### Abstract

This paper examines the implications of a nonzero cosmological constant  $\Lambda_0$  on the amount of linear size evolution and the luminosity selection effects usually required in the interpretation of the angular diameter–redshift ( $\theta$ – $z$ ) test. This is based on three typical cases chosen on various plausible assumptions which can be made concerning the contribution of  $\Lambda_0$  to the density of the universe (parametrised by  $\Omega_0$ ). The results show that a fairly strong linear size evolution will be required to interpret the  $\theta$ – $z$  data of extended steep spectrum quasars for all three cases, if luminosity effects are neglected. However, this evolution is significantly steeper in a matter-dominated universe with  $\Omega_M = \Omega_0 = 1$  than in both the flat universe with  $\Omega_A = 0.8$ ,  $\Omega_M = 0.2$  and an open universe with  $\Omega_M = 0.2$ ,  $\Omega_A = 0$ .

Furthermore, when the luminosity selection effects present in the sample are considered, a milder size evolution is obtained for the  $\Omega_M = 1$ ,  $\Omega_A = 0$  model while little or no size evolution is found for the other two cases. There is therefore no significant difference in our results for an open low density universe with  $\Omega_A = 0$  and a flat universe dominated by  $\Omega_A$  predicted by inflation. The present results therefore imply that an open low density universe with  $\Omega_M = 0.2$  and  $\Omega_A = 0$  is compatible with an inflationary model of the universe with  $\Omega_M = 0.2$  and  $\Omega_A = 0.8$ . This leads to a contradiction since the universe cannot be open and spatially closed at the same time (the existence of one should preclude the other).

### 1. Introduction

It has long been realised (e.g. Hoyle 1958) that extragalactic radio sources can be used, at least in principle, as test particles to probe the universe by determining the fundamental parameters which describe its geometry. Miley (1968, 1971) was the first to find an obvious correlation between the largest angular size and redshift for a sample of quasars. Unfortunately, the observed data show a steeper slope than one can expect in any uniform world model.

The immediate interpretation of the angular size–redshift data is to invoke linear size evolution. However, the amount of size evolution required appears to be very sensitive to the assumed world model (see Kapahi 1987). In particular, Krauss and Schramm (1993) have observed that it has not been easy making distinctions between cosmological models if the cosmological constant is nonzero. A nonzero cosmological constant is predicted by the inflationary model of the universe, which has now been widely accepted because it is successful in overcoming the major drawbacks of the standard Friedmann–Robertson–Walker cosmologies (see Carroll *et al.* 1992 for a summary). In this paper we investigate the effect of

introducing a non-vanishing cosmological constant in the analyses of the angular size–redshift data using a large sample of lobe-dominated quasars.

## 2. Dependence of Linear Size on Radio Luminosity and Redshift and the Effect of Nonzero Cosmological Constant

It has now been generally accepted (e.g. Oort *et al.* 1987; Kapahi 1989; Singal 1993) that linear sizes of extragalactic radio sources in general appear to depend on both radio luminosity and redshift. This implies that the amount of linear size evolution occurring among extragalactic radio sources depends on radio luminosity for any given world model. The inflationary model predicts that the density parameter  $\Omega_0$  of the universe is exactly unity. However, most of the available dynamical estimates suggest a relatively very low value for  $\Omega_0$ . A universe dominated by the cosmological constant therefore supplies the ‘missing mass’ required to make  $\Omega_0 = 1$ . In quantum field theory,  $\Lambda_0$  is interpreted in terms of the invariant vacuum self-energy given by

$$\Lambda_0 = 8G\epsilon_v\pi/c^4, \quad (1)$$

where  $\epsilon_v$  is the energy density of the vacuum,  $G$  is the gravitational constant, and  $c$  the speed of light. Peebles (1984) has observed that in a spatially flat universe, low-density universes can be reconciled with inflation if

$$\Lambda_0 = 3(1 - \Omega_0)H_0^2. \quad (2)$$

In this section we compare the linear size–redshift correlation for different world models, depending on the various plausible assumptions which can be made concerning the contributions to the total density parameter  $\Omega_0$  of the universe from the cosmological constant term  $\Omega_\Lambda$  and that arising from its baryonic matter content  $\Omega_M$ . We consider three typical cases: (a) a matter dominated universe with  $\Omega_M = 1$ ,  $\Omega_\Lambda = 0$ ; (b) an open universe with  $\Omega_M = 0.2$ ,  $\Omega_\Lambda = 0$  and (c) an  $\Omega_\Lambda$  dominated universe with some matter, i.e.  $\Omega_M = 0.2$ ,  $\Omega_\Lambda = 0.8$ . These are summarised in Table 1. In the inflationary world model the density parameters are related by

$$\Omega_M + \Omega_\Lambda = \Omega_0 = 1. \quad (3)$$

**Table 1. The three typical cases considered in the text**

Case	$\Omega_M$	$\Omega_\Lambda$	$\Omega_0$
a	1.0	0	1.0
b	0.2	0	0.2
c	0.2	0.8	1.0

Actually, cases (b) and (c) are supported by most recent observational results (e.g. Fukugita *et al.* 1990; Efsthathiou *et al.* 1990; Olive *et al.* 1990; Kofman *et al.* 1993), while case (a) appears untenable because of its short expansion time-scale (see Ballinger *et al.* 1996) but is included for comparison because it provides the steepest evolution in a spatially flat universe (see Kapahi 1987).

For the present analyses we have used the extended (linear size  $D \geq 20$  kpc) steep spectrum (spectral index  $\alpha \geq 0.5$ ) quasars from the Nilsson *et al.* (1993) sample. Out of 273 quasars in the sample 29 did not meet our criteria and were excluded from the analyses. These 29 sources belong to either the flat spectrum or compact steep spectrum class whose evolutions are believed to differ from those of their extended steep spectrum counterparts being considered here. We adopted  $H_0 = 50$  km/s/Mpc. Linear sizes  $D$  and radio luminosities  $P$  were calculated from the observed angular sizes  $\theta$  and flux densities  $S_\nu$  at frequency  $\nu = 178$  MHz using the following relations:

$$D = \theta d_m (1+z)^{-1}, \quad (4)$$

$$P = 4\pi d_m^2 S (1+z)^{\alpha+1}, \quad (5)$$

where  $\alpha$  was given between 178 MHz and 5 GHz using  $S \propto \nu^{-\alpha}$ , and (cf. Carroll *et al.* 1992)

$$d_m = H_0^{-1} \int_0^z [(1+z)^2(1+\Omega_M z) - z(2+z)\Omega_\Lambda]^{-\frac{1}{2}} dz. \quad (6)$$

The variation of linear size with redshift and luminosity is given by (see Oort *et al.* 1987; Singal 1993)

$$D = P^q (1+z)^{-n}. \quad (7)$$

Also, in flux density-limited source samples, there is a strong correlation between luminosity and redshift which can be written in the form (e.g. Onuora and Okoye 1983)

$$P = P_0 (1+z)^\beta, \quad (8)$$

where  $P_0$  is a normalising luminosity. In flux density-limited samples  $\beta$  is generally attributed to selection effects due to the well-known Malmquist bias. Following Ubachukwu (1995), we combine equations (7) and (8) to obtain

$$D = D_0 (1+z)^{-x}, \quad (9)$$

where

$$x = n - q\beta \quad (10)$$

and  $D_0$  is a normalising linear size. The last equation thus relates the apparent linear size evolution parameter  $x$  to its true value  $n$  and to that which arises from luminosity selection effects present in the sample and is represented by the product  $q/\beta$ .

In order to obtain quantitative estimates of the parameters  $x$ ,  $q$  and  $\beta$  for the present data, we have carried out linear regression analyses of  $\log D$  on  $\log(1+z)$  and  $\log P$  on the one hand, and  $\log P$  on  $\log(1+z)$  on the other. The values of these parameters which provide good fits to the observed data for the

three cases considered, together with the corresponding values  $n$  calculated using equation (10), are shown in Table 2.

**Table 2.** Best-fit regression parameters for the three cases described in Table 1

Case	$x$	$q$	$\beta$	$n$
a	$1.75 \pm 0.20$	$-0.32 \pm 0.03$	$3.8 \pm 0.2$	$0.53 \pm 0.02$
b	$1.33 \pm 0.22$	$-0.28 \pm 0.02$	$4.1 \pm 0.2$	$0.18 \pm 0.08$
c	$1.21 \pm 0.22$	$-0.24 \pm 0.02$	$4.2 \pm 0.1$	$0.20 \pm 0.11$

### 3. Discussion

We have carried out quantitative analyses of the implications of a nonzero cosmological constant  $\Lambda_0$  in the amount of linear size evolution occurring in extended steep spectrum quasars based on three typical cases contrived on observational as well as theoretical prejudices. The results are summarised in Table 2. The correlation coefficients are highly significant ( $r > 0.7$  in each case).

It can be seen from Table 2 that, in the absence of any luminosity effects, a steeper size evolution ( $x \sim 1.8$ ) is obtained for the matter-dominated universe of case (a) than for the other two cases. These show no significant difference in the evolution parameter, i.e.  $x \sim 1.3$  for case (b) and  $\sim 1.2$  for case (c). The same trend was also found when luminosity effects were considered. However, luminosity effects appear to account for a very highly significant fraction of the observed size evolution in all the three cases considered. More specifically, only a mild intrinsic evolution ( $n \sim 0.5$ ) was obtained for case (a), while little or no evolution ( $n \sim 0.2$ ) was found for the other two cases.

Comparison of  $x$  and  $n$  obtained for cases (a) and (c), which are both consistent with a spatially flat universe predicted by inflation, shows that the amount of size evolution is in general significantly reduced by the introduction of a high value of  $\Omega_A$ . This reduction appeared higher ( $>60\%$ ) for  $n$  (i.e. when the luminosity effects were included) than ( $\simeq 30\%$ ) for  $x$  obtained in the absence of any luminosity effects. This indicates that the angular size-redshift slope for an  $\Omega_A$  dominated universe is steeper than for a universe dominated by  $\Omega_M$ . On the other hand, both cases (b) and (c) appear to give similar values for both  $x$  and  $n$  implying that an open universe with a low baryon density is compatible with inflation. This leads to a contradiction because the universe cannot be open and spatially closed at the same time. It is already very well established that an open universe with low density cannot be reconciled with the present age of the universe based on the recent estimates of  $H_0$  (e.g. Tully 1988; Leonard and Lake 1995). Alternatively, if we assume  $\Omega_0 = 1$ ,  $\Omega_A = 0$  and inflation we run into problems with the results of the anisotropy measurements of the cosmic microwave background (Bond *et al.* 1991; Vittorio *et al.* 1991). Furthermore, recent calculations based on cosmic nucleosynthesis together with the presently available observational estimates on the abundances of light elements (see Gerhard and Silk 1996 and references therein) have put limits on  $\Omega_M$  within the range  $0.01 < \Omega_M h^2 < 0.02$  ( $h$  is the Hubble constant in units of 100 km/s/Mpc). This immediately implies  $H_0 < 15$  km/s/Mpc which is a factor of more than 3 less than the current lower limit of  $H_0 \sim 50$  km/s/Mpc. Bucher *et al.* (1995) have

suggested that a bubble nucleation within inflation is capable of producing an open universe. This may provide a way out of this dilemma.

### Acknowledgments

I wish to acknowledge an IAU Commission 38 Travel Grant and the hospitality of Hartebeesthoek Radio Astronomy Observatory where this work was completed. I am also grateful to Mike Gaylard for useful comments.

### References

- Ballinger, W. E., Peacock, J. A., and Heavens, A. F. (1996). *Mon. Not. R. Astron. Soc.* **282**, 877.
- Bond, J. R., Efstathiou, G., Lubin, P. M., and Meinhold, P. R. (1991). *Phys. Rev. Lett.* **17**, 2179.
- Bucher, M., Goldhaber, A. S., and Turok, N. (1995). *Phys. Rev.* **52**, 3314.
- Carroll, S. M., Press, W. H., and Turner, E. L. (1992). *Annu. Rev. Astron. Astrophys.* **30**, 499.
- Efstathiou, G., Sutherland, W. J., and Maddox, J. J. (1990). *Nature* **348**, 705.
- Fukugita, M., Takahara, F., Yamashita, K., and Yoshii, Y. (1990). *Astrophys. J. Lett.* **361**, L1.
- Gerherd, O., and Silk, J. (1990). *Astrophys. J.* **472**, 34.
- Hoyle, F. (1958). IAU Symp. on Radio Astronomy (Ed. R. N. Bracewell), p. 520 (Stanford Univ. Press).
- Kapahi, V. K. (1987). In 'Observational Cosmology', IAU Symp. 124 (Eds A. Hewitt *et al.*), p. 251 (Reidel: Dordrecht).
- Kapahi, V. K. (1989). *Astrophys. J.* **97**, 1.
- Kofman, L. A., Green, N. Y., and Bahcall, N. A. (1993). *Astrophys. J.* **431**, 1.
- Krauss, L. M., and Schramm, D. N. (1993). *Astrophys. J. Lett.* **405**, L43.
- Leonard, S., and Lake, K. (1995). *Astrophys. J. Lett.* **441**, L55.
- Miley, G. K. (1968). *Nature* **212**, 933.
- Miley, G. K. (1971). *Mon. Not. R. Astron. Soc.* **152**, 447.
- Nilsson, K., Valtonen, M. J., Kotilaine, J., and Jaakkola, T. (1993). *Astrophys. J.* **413**, 453.
- Olive, K. A., Schramm, D. N., Steigman, J., and Walker, T. P. (1990). *Phys. Lett. B* **236**, 454.
- Onuora, L. I., and Okoye, S. E. (1983). *Astrophys. J.* **270**, 360.
- Oort, M. J., Katgert, P., Steeman, F. W., and Windhorst, R. A. (1987). *Astron. Astrophys.* **179**, 41.
- Peebles, P. J. E. (1984). *Astron. J.* **84**, 730.
- Singal, A. K. (1993). *Mon. Not. R. Astron. Soc.* **363**, 209.
- Tully, R. B. (1988). *Nature* **334**, 209.
- Ubachukwu, A. A. (1995). *Astrophys. Sp. Sci.* **232**, 23.
- Vittorio, N., Meinhold, P. R., Muciaccia, P. E., Lubin, P., and Silk, J. (1991). *Astrophys. J. Lett.* **372**, L1.