

THE CHARACTERISTICS OF TRACKS IN NUCLEAR RESEARCH EMULSIONS

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[*Manuscript received September 27, 1954*]

Summary

Formulae are derived giving the dependence of the blob, grain, and gap densities, the total gap length, and the mean gap width in the tracks of charged particles as function of the probability of making a silver halide crystal developable. The growth of the grains during the development process is taken into account. The expressions found are suitable for use in discussions of the effects of changes in emulsion properties, and they show clearly the differences between the various methods of determining the rate of energy loss from the track characteristics. There is good agreement with experiment for both G5 and C2 emulsion.

I. INTRODUCTION

In the analysis of tracks of charged particles in nuclear research emulsions, the ionizing power of the particles is estimated from the characteristics of the tracks. Basically, the probability of making a silver halide crystal developable increases with the rate of energy loss by ionization of the particle, and any property of the track which depends on this probability can be used as a measure of the rate of energy loss. The earliest, and most obvious, of the properties thus used was the grain density, i.e. the number of developed grains per unit length of track. However, the measurement of grain density is highly subjective as soon as the grain density is so great that the grains occur in groups rather than as separate units. In addition, the grain density asymptotically approaches a maximum value, so that it is not a sensitive measure of the ionizing power when the track is a dense one.

A large number of techniques has been proposed and used by many workers in efforts to make the estimation of the rate of energy loss more accurate, more objective, and more reliable, and only a few can be mentioned here. For tracks of very low density, Voyvodic (1951) suggested the replacement of the grain density by the density of unresolved grain groups ("blobs"), and this technique has become generally accepted. The blob density is, of course, equal to the density of the gaps between the blobs. Hodgson (1950) first published the method in which the fraction of track occupied by gaps (that is the gap length per unit length of track) is measured. This method is particularly suitable for very dense tracks. The measurement of gap density (the number of gaps

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per unit length of track) has been discussed by O'Ceallaigh (1953) and Tennent (1953), and O'Ceallaigh (1953) has suggested the mean gap width as a measure of the ionizing power of the particle, whilst Tennant (1953) proposed the counting of gaps wider than a certain minimum.

Apart from these "visual" methods, a number of photometric ones have been developed and used (see, for example, von Friesen and Kristiansson 1951; Ceccarelli and Zorn 1952).

In general, all the track characteristics which are measured depend on the probability p that a specified silver halide crystal in the path of the particle be rendered developable. In the present paper, we give the relations describing the dependence on p of the various quantities observed in the visual methods. These relations allow one to discuss quantitatively the merits and ranges of usefulness of the different measuring techniques, and they can also be used to predict the effects of changes in emulsion properties.

In general, our approach is similar to that of Della Corte, Ramat, and Ronchi (1953), but it is more rigorous, and we have gone further in deriving the consequences of our model.

II. THE MODEL

In the derivation of the formulae a model is used in which all the fluctuations in the parameters have been neglected.

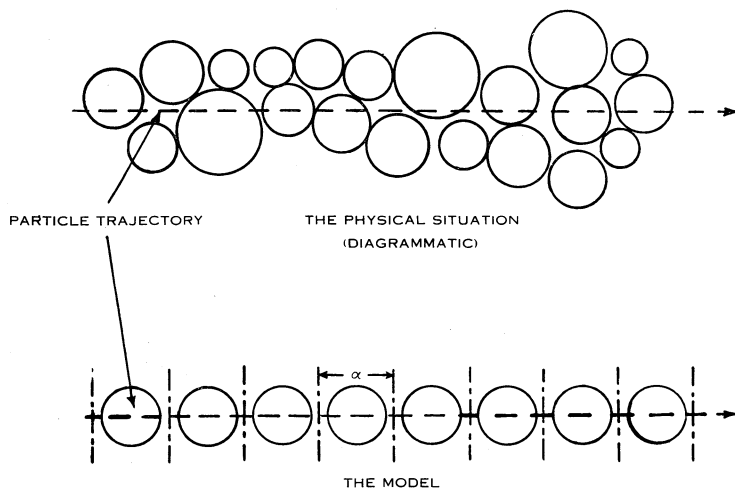


Fig. 1.—Comparison between the physical situation and the model.

It is assumed (see Fig. 1) that the track can be divided up into cells, all of equal length α , each containing one silver halide crystal. The probability that the crystal in any particular cell be developed is p . When the crystal is developed it grows to a size greater than α by a factor γ , so that the developed grain size (the diameter if the grain is spherical) is $\gamma\alpha$. Figure 2 shows the effect of this. It is clear that with this model blobs can never be less than $\gamma\alpha$ in diameter, but, depending on the value of γ , much smaller values of the gap width are possible.

III. DERIVATION OF THE FORMULAE

It is convenient to consider first the formation of the gaps in the tracks, for gaps are simple structures consisting only of a series of undeveloped cells with a developed one at each end.

To start with we find the width of the (in theory) smallest possible gap. From Figure 2 it is seen that the width of this "first order" gap is $\alpha(\Gamma+1-\gamma)$, where Γ denotes the integral part of γ , that is, the nearest integer less than or equal to γ . The width of the next bigger gap, which we call a gap of order $i=2$, is (see Fig. 2) $\alpha(\Gamma+2-\gamma)$, and similarly, the width of an i th order gap will be

$$w_i = \alpha(\Gamma + i - \gamma). \quad \dots\dots\dots (1)$$

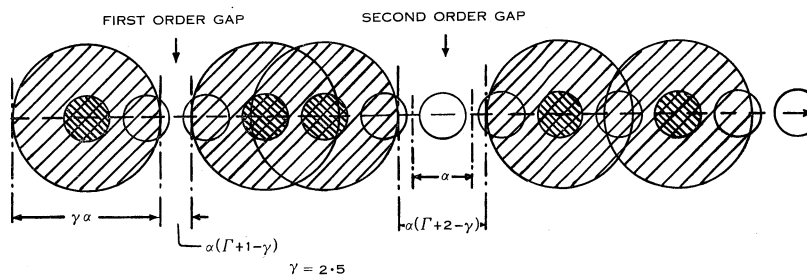


Fig. 2.—Details of the model. Small shaded circles represent developable crystals, large ones are developed grains.

The probability of finding an i th order gap starting in a specified cell of the track is the probability of getting two developable cells with $(\Gamma+i-1)$ undevelopable ones in between: $p^2(1-p)^{\Gamma+i-1}$. Hence, if the number of cells in track length t is T , the expected density of i th order gaps will be

$$\bar{n}_i(t) = Tp^2(1-p)^{\Gamma+i-1}, \quad \dots\dots\dots (2)$$

as long as $w_i \ll t$.

The gap density and the blob density are equal, and they are obtained by summing \bar{n}_i over all values of i :

$$\bar{n}_b = Tp(1-p)^\Gamma. \quad \dots\dots\dots (3)$$

The total length of gap in track length t is given by

$$\bar{x} = \sum_{i=1}^{\infty} w_i \bar{n}_i = t(1-p)^\Gamma \{1 - p(\gamma - \Gamma)\}. \quad \dots\dots\dots (4)$$

The total length of the blobs in length t will be

$$L_b = t - \bar{x} = t[1 - (1-p)^\Gamma \{1 - p(\gamma - \Gamma)\}], \quad \dots\dots\dots (5)$$

and, dividing this by the size of the developed grains, $\gamma\alpha$, we get the grain density as customarily defined:

$$\frac{L_b}{\gamma\alpha} = \frac{T}{\gamma} [1 - (1-p)^\Gamma \{1 - p(\gamma - \Gamma)\}]. \quad \dots\dots\dots (6)$$

A further important quantity is the expected value of the gap width, given by the particularly simple expression

$$\bar{w} = \alpha \left(\frac{1}{p} + \Gamma - \gamma \right). \quad \dots\dots\dots (7)$$

Finally, if only gaps of order greater than or equal to s (i.e. of width $\geq \alpha(\Gamma + s - \gamma)$) are counted, we find their density to be

$$\bar{n}_{i \geq s} = Tp(1-p)^{\Gamma+s-1}, \quad \dots\dots\dots (8)$$

and the expected value of their width becomes

$$\bar{w}_{i \geq s} = \alpha \left(\frac{1+p(s-1)}{p} + \Gamma - \gamma \right) = \bar{w} + \alpha(s-1). \quad \dots\dots (9)$$

IV. EXPERIMENTAL TESTING OF THE RELATIONS

Of the adjustable parameters, the size of the developed grains, $\gamma\alpha$, is found by plotting a distribution of blob sizes for tracks with a blob density near the minimum value. In such tracks, most of the grains will be single, that is, grown from single silver halide crystals, and they give a large peak in the distribution. We make the reasonable assumption that the distribution of single-grain sizes is symmetrical, so that its mean is equal to the mode of the total distribution. We thus take the mode as our value of $\gamma\alpha$.

TABLE I
REPRESENTATIVE RESULTS FOR TRACKS IN G5 EMULSION
 $\alpha = 0.30 \mu$, $\gamma = 2.00$, $t = 100 \mu$

Track No.	Observed			Calculated		
	\bar{w} (μ)	\bar{n} (100)	L_G (μ)	p	\bar{n} (100)	L_G (μ)
1	2.01	36.0	72.5	0.149	36.0	72.4
2	1.64	42.3	69.4	0.164	40.7	66.8
3	1.52	42.8	64.6	0.197	42.3	64.5
4	1.33	46.7	62.0	0.226	45.1	59.9
5	1.19	47.5	57.0	0.250	46.9	56.2
6	0.64	46.2	29.4	0.469	44.1	28.2
7	0.60	39.8	23.7	0.503	41.4	24.7

The cell size α has to be found by trial and error, using a dense track, as the characteristics of dense tracks are most sensitive to changes in the parameters. It is known from electron-microscope measurements that the size of the undeveloped crystals in Ilford G5 emulsion is of the order of 0.3μ (Baroni and Castagnoli 1950; George, personal communication 1952; Pickup 1953; Bradley 1954) and the trial values of α are therefore taken in this region.

Once a value of α has been found which gives the correct characteristics for a dense track in a given plate, it is also satisfactory for all the other tracks in that plate. The values of p are in all cases determined from the observed mean gap width \bar{w} and equation 7). Table 1 gives a representative set of

results for tracks observed in a G5 plate; similar agreement has been obtained in other plates, both of types G5 and C2, but, of course, with different values of γ and α . That the values of α and γ are critical can be seen from the fact that if we assume that $\alpha=0.29 \mu$ or $\alpha=0.31 \mu$ for track 7 in Table 1, the calculated blob densities become 45.7 and 73.2 respectively, and the corresponding gap lengths, L_G , are 27.4μ and 43.9μ in 100μ of track. Note that there is a minimum at $\alpha=0.30 \mu$.

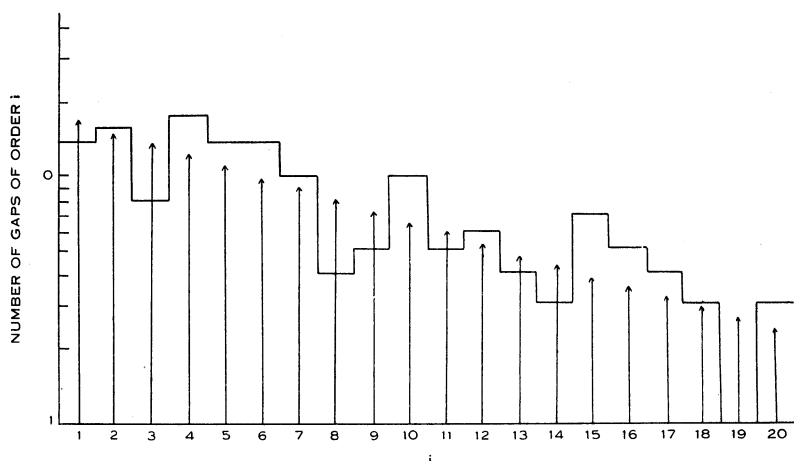


Fig. 3.—Gap size distribution. Histogram, observed; arrows, calculated.

Another way of testing the model is to compare the gap size distribution obtained from equation (2) with the observed one. A representative example is given in Figure 3 where the number of gaps of order i is plotted against i . The calculated values are derived from equation (2), using the values of T , α , and γ which fit all the tracks in the plate. The agreement is very good, especially as the histogram has not been normalized to the calculated values

V. DISCUSSION

Figure 4 shows an example of the calculated variation with p of the various track characteristics in a sample of G5 emulsion. The minimum value of the rate of energy loss gives a blob density of the order of 20–25 blobs per 100μ and this corresponds to a value of the probability p of order 0.05. The steep slope of the blob-density curve in this region confirms the well-known fact that for weakly ionizing particles the blob density is a sensitive measure of the rate of energy loss. If the ionization loss of the particle rises above about twice the minimum, however, the blob density becomes quite insensitive to changes in p , and the total and mean gap lengths become preferable. In Figure 5, the density of gaps greater than a given size is plotted against p , showing that this quantity varies rapidly with p for dense tracks, and that it does not have as extensive a plateau at medium values of p as the blob density. If the statistical accuracy is sufficient, therefore, this measure can give good discrimination, as has been found experimentally by Tennent (1953).

How closely our model approaches the real situation depends on the way in which the crystals of the undeveloped emulsion are distributed in space, and on the size distribution of the crystals. We know from the distributions of crystal sizes in Ilford G5 emulsion which have been published (Baroni and Castagnoli 1950; Pickup 1953) that the variations in crystal size are quite small, and it therefore seems reasonable to use a model in which the crystals are assumed to be all of the same size. We are left, then, with the question "How are the crystals distributed in space?", and here our model assumes extreme ordering, with all cell sizes equal, whilst O'Ceallaigh (1953) postulates complete disorder with an exponential distribution of the distances between the crystals.

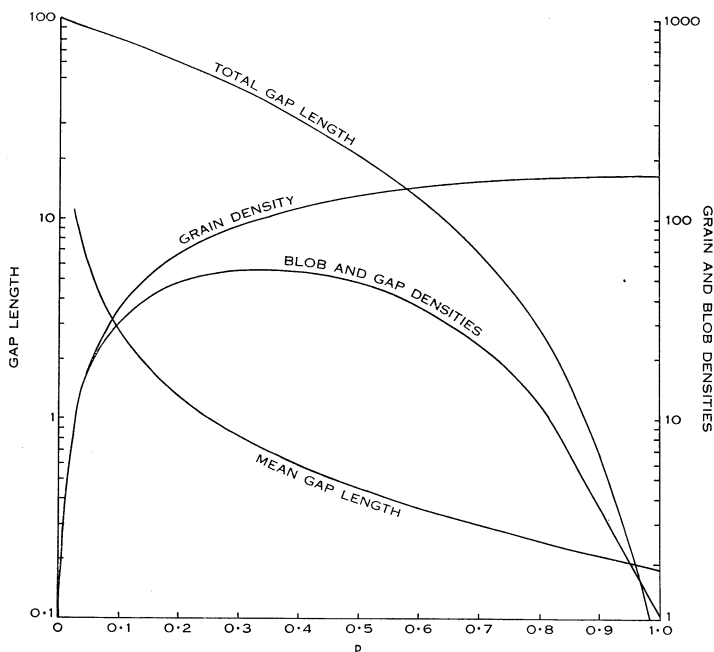


Fig. 4.—Variation of track characteristics with p in a sample of G5 emulsion ($\alpha=0.29 \mu$, $\gamma=2.24$).

Which of these two basic assumptions is nearer to the truth seems to be a question which can only be decided by experiment after the full consequences of the assumption of disorder, made by O'Ceallaigh (1953), have been worked out, allowing one to compare the numerical predictions of both theories. Meanwhile we feel that the somewhat amazing agreement between the experimental data and the detailed numerical results derived from our model is sufficient to show that it must be adequate in most cases of practical importance.

There is, however, one point where there is considerable disagreement between our model and O'Ceallaigh's. O'Ceallaigh's gap width distribution is a continuous exponential function, whilst ours (equation (2)) is a discrete set of values decreasing according to a power law. Now it is easily shown (O'Ceallaigh 1953) that a continuous exponential distribution would make the mean gap width independent of the degree of grain growth (γ in our notation), so that w

could be used as a measure of the probability p unaffected by physical development. On the other hand, if the distribution is as given in equation (2) then there is a dependence on the fractional part of γ , $\Gamma - \gamma$, and this is particularly important when p is large, that is, for dense tracks. In view of these uncertainties we feel that the claim (O'Ceallaigh 1953 ; Johnston and O'Ceallaigh 1954) that the mean gap width is independent of the amount of grain growth by physical development requires experimental proof before it can be accepted.

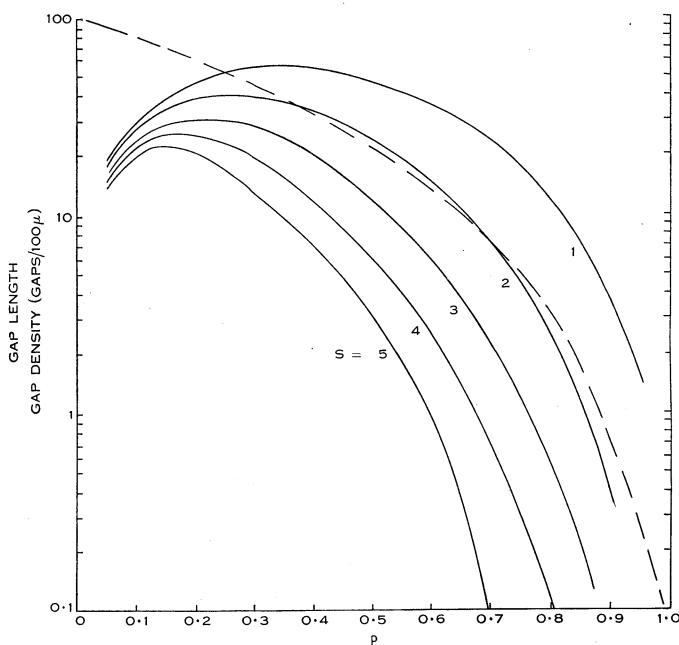


Fig. 5.—Density of gaps in order $\geq s$ in G5 emulsion ($\alpha=0.29 \mu$, $\gamma=2.24$). Broken line: total gap length per 100 μ .

VI. ACKNOWLEDGMENTS

The authors would like to express their thanks to Dr. E. P. George for a number of helpful discussions ; they are greatly indebted to Mr. C. Waller for much useful advice on the properties of nuclear research emulsions.

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