

# MEASUREMENTS OF COUNTER-GRADIENT HEAT FLOWS IN THE ATMOSPHERE\*

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## Summary

Heat flows and temperature gradients were measured in a stable atmosphere heated by the water during a United States east coast to Bermuda aeroplane flight. The flux was computed from  $c_p \overline{\rho w' T'}$ , where  $w'$  and  $T'$  are the vertical velocities and temperature deviations of the individual gusts. It is found that the heat flow is directed up the gradient as predicted by Priestley (1954). The expression for the maximum heat flow to be expected derived by Priestley in terms of the standard deviation of the temperature fluctuations and the temperature gradient gives the correct order of magnitude for the observed flow.

## I. INTRODUCTION

The generally accepted theory of turbulent diffusion of momentum, heat, and water vapour was modelled after the molecular diffusion theory and considers the transport to be accomplished by turbulent eddies which carry the average properties of the fluid at the level of origin through some reference level. Whenever a gradient of the properties exists a net turbulent transport of the property results from the summation of all the eddies passing through the surface, provided there is no mass transport through the surface. This theory shows that the flux is proportional to both the gradient and the coefficient of turbulent mass exchange and the flux is directed down the gradient. It is concerning the point of the necessity for heat flow to be down the gradient even in a gravitational field that the theory fails to describe the actual flow of heat through the atmosphere. It has been observed that in certain cases upward flows of heat exist in the face of potential temperatures increasing with height.

The idea of warmer parcels of air rising through the atmosphere and contributing to a counter-gradient flow of heat was incorporated into the classical diffusion theory by Priestley and Swinbank (1947). They differed in their approach by considering that, at the point of origin of the eddy, the air did not necessarily have the mean temperature of that level but had the temperature  $T_0 + T''$ , where  $T_0$  is the average temperature of the level and  $T''$  is a deviation from that temperature. Treating this temperature sum in much the same manner as the authors of the classical mixing length theory, they develop the heat flow equation

$$F = \rho_0 c_p [-K(\partial T_0 / \partial z + \Gamma) + \overline{w T''}]. \quad \dots\dots\dots (1)$$

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Here  $\rho_0$  is the density corresponding to  $T_0$ ,  $K$  is the diffusion coefficient,  $\Gamma$  is the dry adiabatic lapse rate, and  $w$  is the vertical velocity of the eddy at the reference level. The authors point out that the  $w\overline{T''}$  term results in a systematic upward flux of heat irrespective of the sign of  $(\partial T_0/\partial z + \Gamma)$ . This equation cannot be checked directly because of the impossibility of observing  $T''$ . Because of this difficulty the equation is not quantitatively useful and remained without further development although it was recognized as a great advance toward a true description of heat flow in the atmosphere.

Following the presentation of the Priestley-Swinbank theory, several studies of the properties of buoyant parcels moving through an environmental air were made. One of these by Houghton and Cramer (1951) studied the effects of entrainment of environmental air upon columns of air rising as a result of its buoyancy. Equations are derived that give mass entrainment, temperature deviations, and velocities as functions of the height. These equations were set up for both dry air and saturated air from which the water vapour condenses as the air rises. Thus, assuming that the only interaction with the environment comes from the entrainment of air which carries with it its characteristic momentum and temperature anomaly, the life history of a warm parcel of air can be determined. This approach to the study of buoyant parcels was developed further by Bunker (1953), who followed the general development of Houghton and Cramer but added the effects of a turbulent atmospheric environment and a dynamic pressure drag upon the rising bubble. Equations are found which give temperature deviations and vertical velocities as functions of height when the turbulent mass exchange coefficients of the environmental air and the size of the parcels are known or assumed as functions of the height. Numerical integration of the equations is required in both of these treatments.

A theory presented by Scorer and Ludlam (1953) described a mechanism of convective rise of bubbles which erode as they penetrate the atmosphere. This mechanism is modelled after the tank experiments of Davies and Taylor (1950) and experiments of the authors with bubbles rising through tinted water. Particular emphasis is placed upon the effects of the wake of the bubble in aiding subsequent parcels to penetrate to greater heights of the atmosphere.

An extremely valuable treatment of the problem of buoyant parcels was developed by Priestley (1953). Time derivative equations of the vertical velocity and temperature excess  $T'$  of the parcel following the mean motion are developed taking into account mixing of momentum and heat of the parcel air with the environment, which has the temperature  $T_e$ . The simultaneous equations are:

$$\dot{w} = (g/T_e)T' - k_1 w, \dots\dots\dots (2)$$

$$\dot{T}' = -w(\partial T_e/\partial z + \Gamma) - k_2 T'. \dots\dots\dots (3)$$

Here  $k_1$  and  $k_2$  are mixing rates of momentum and heat. They are defined in terms of the radius  $R$  of the parcel and the diffusivity coefficient  $K$  by the following relations, where  $c_1$  and  $c_2$  depend upon the form of the parcel:

$$k_1 = c_1 K_1/R^2, \quad k_2 = c_2 K_2/R^2. \dots\dots\dots (4)$$

The second order differential equation was formed

$$\ddot{w} + (k_1 + k_2)\dot{w} + \kappa^2 w = 0 \quad \dots\dots\dots (5)$$

and solved in conjunction with the previous equations for  $w$  and  $T'/T_e$  as functions of time and the mixing rates. When these solutions are examined for various combinations of environmental lapse rates and mixing rates it is clear that the parcels may have three modes of motion: oscillatory, asymptotic, and absolute buoyancy.

In a subsequent paper, Priestley (1954) has developed the equations describing the motion and temperature relations of a buoyant parcel to the point where heat flows can be calculated under given conditions. Using either the oscillatory motion or the asymptotic motion case, the products of  $wT'$  and  $T'^2$  are averaged over the path of the parcel to get the heat flow and the standard deviation ( $\sigma_T$ ) of the temperature. The relation is found

$$F_H/\rho c_p = \sigma_T^2(g/T_e) \frac{2k_1}{(g/T_e)(\partial T_e/\partial z + \Gamma) + k_1 k_2 + 2k_1^2}, \quad \dots\dots\dots (6)$$

which gives the heat flow in terms of the temperature fluctuations, the lapse rate, and the mixing rates.

To investigate the dependence of the convective heat flow as a function of lapse rate and other quantities, several parameters are defined and substituted into the above equation. From a plot of these parameters it is apparent that a maximum value of the heat flow exists for a given mixing rate ratio. This is interpreted to mean that an optimum size of parcel exists which can transport a maximum amount of heat for a given degree of turbulence and thermal stability. The upper limit for the heat flux can now be expressed as

$$F_H = \frac{\rho c_p \sigma_T^2}{\sqrt{\{(3T_e/g)(\partial T_e/\partial z + \Gamma)\}}}, \quad \dots\dots\dots (7)$$

when the reasonable assumption is made that  $k_1 = k_2$ .

In view of these theories set forth by the various workers, which have had little confirmation or checking with observations, it is desired to obtain data that can be used for this purpose. The flow of cool air over warm water offers circumstances ideal for the study of the gravitational effect upon heat flow and the application of the concepts and equations of Priestley. Usually in these cases, the turbulence in the lowest 50 m is too small to transport all of the heat crossing the air-water interface, with the result that a large super-adiabatic lapse rate is established. The resulting instability allows warm buoyant parcels of varying sizes to rise through the air, thereby transporting heat upward at a faster rate than that of turbulent diffusion. This system carries the heat to higher levels where it is finally mixed into the environmental air. As a result of the small heat diffusion in the lowest levels and the greater transport to higher levels, a potential temperature minimum can be established in the 50–300 m level in the cool air. As a temperature inversion usually exists at the top of the mixed layer the stability of the air is increased further by downward diffusion of heat from this source. Under the conditions just described and at

altitudes up to the inversion many of the various types of heat flow will be large and easily measured.

The major technique employed in this study is the measurement of heat flow from an aeroplane by averaging the cross products of the temperature and vertical velocity deviations. Such values have been obtained at numerous heights and localities where counter-gradient fluxes are suspected to exist. Observations were obtained from an aeroplane flying at levels from 30 to 1700 m and to distances up to 1100 km off shore. Radiosonde and radiowinds observed at land, ship, and island stations were used as supplementary sources of information. The instrumentation of the aeroplane and the technique of obtaining vertical and horizontal turbulent fluctuations of the air velocity have been described by Bunker (1955), hence only the briefest mention of these topics will be made. A modification of the reduction system is used now which gives reliable values over flight paths of many kilometres in length. This change involves the use of deviations of the airspeed and attitude from a mean over the entire run rather than over 10 sec. The fluctuations are determined from the measured responses of a vertically mounted accelerometer, a gyroscope, and a pressure gauge connected to a pitot-static tube. Temperature fluctuations are found from the response of a thermopile. Once a time series of  $w_i$ , the vertical turbulent component of the wind, and  $T_i$ , the temperature fluctuations, read to  $0.1^\circ\text{C}$  are obtained, then cross products and root mean square deviations can be formed, giving  $(\overline{w'^2})^{\frac{1}{2}}$ ,  $(\overline{T'^2})^{\frac{1}{2}}$ , and  $c_p \overline{\rho w' T'}$ , the heat flux. Particular attention will be paid here to the heat flow calculation, and an interpretation of this observed quantity in terms of diffusion and buoyant transport will be discussed. Heat flows into the air mass averaged over all or part of the distance covered by the air in its travel over the water surface have been calculated from observed individual temperature changes, local changes, and mean vertical velocities in the air mass. From the mean temperatures observed at various heights by the aeroplane, the gradients of the potential temperature can be found and the way is clear for a discussion of Priestley's maximum buoyant heat flow and associated problems of turbulent mass exchange.

## II. OBSERVATIONAL DATA OBTAINED DURING THE POLAR OUTBREAK OVER THE ATLANTIC OCEAN, JANUARY 18, 1955, RHODE ISLAND TO BERMUDA

This flight to Bermuda was made at about the middle of a 3-week period of predominant north-westerly flow off the east coast of North America. A low pressure system passed to the north of New England on the morning of January 17 and north-west winds prevailed until a coastal storm interrupted them temporarily on the afternoon of the 19th. The skies were clear over the land at take-off time and remained so as the air passed over the shallow coastal waters. As warmer waters were reached, a few cumulus clouds appeared and rapidly built up to a complete overcast of stratocumulus as the air flowed over the very warm waters of the Gulf Stream. In this region the aeroplane was flying in and out of scud at 500 m. No showers were noted at any time. Some 300 km beyond the Gulf Stream breaks in the overcast appeared and grew until only six-tenths of the sky was covered by cumulus clouds at Bermuda.

The resistances of a thermistor mounted in a housing attached ahead of the nose of the aeroplane were recorded continuously throughout the flight. From these resistances the air temperatures were found, corrected for the dynamic heating of the aeroplane's motion, and potential temperatures computed. Water temperatures were obtained from mean maps and the intake water temperatures of the R.V. *Atlantis* which travelled the course to Bermuda January 19–22, 1955. These water temperatures are treated as air temperatures and reduced to potential temperatures. This procedure is justified on the basis that the air immediately in contact with the water takes on the water's temperature.

Five times *en route* to Bermuda the aeroplane was flown from the cloud base down to 90 m above the water to obtain vertical temperature gradients, turbulence, and heat flow data. The temperatures obtained during these descents had to be corrected for the horizontal gradients since 30–40 km was

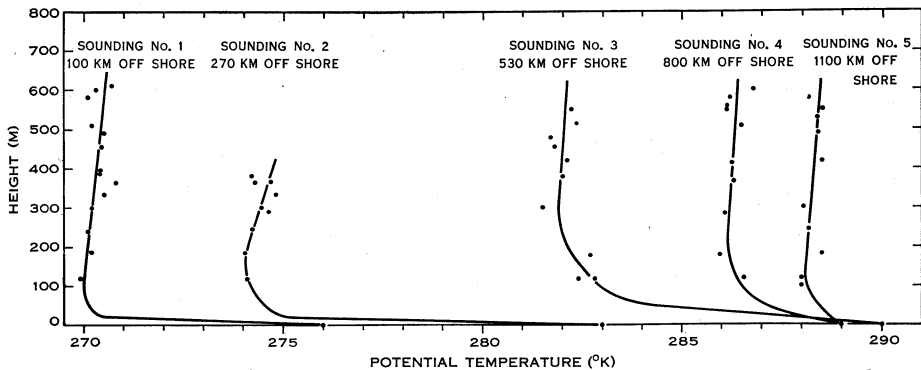


Fig. 1.—Potential temperature gradients, Rhode Island to Bermuda.

traversed during a descent and ascent. The temperatures, corrected to mean off-shore distances, have been plotted on a height-temperature diagram, and presented in Figure 1. As conclusions of considerable importance will be drawn from the nature of these gradients great care was taken that no systematic errors were introduced by the reduction system and the flight procedure of flying faster at the lowest elevations. From numerous flight tests performed it is concluded that if any systematic errors exist they are less than  $0.1^{\circ}\text{C}$ . An uncertainty of  $0.1^{\circ}\text{C}$  over a 500 m height range is equivalent to an uncertainty of  $2 \times 10^{-6} \text{ cm}^{-1}$  in the gradient determination. The observed gradients are several times this amount. It is noted that a slight but real stability exists in the upper 90 per cent. of the sub-cloud layer. Although no temperatures other than the water temperatures were obtained below the 30 m level it is obvious that the air in this region is very unstable.

Fluctuation data obtained from 17 horizontal runs made during the five descents already mentioned were reduced to time series of  $w_i$  and  $T_i$  and subsequently to series of  $w'$  and  $T'$ . From these series values of  $(\overline{w'})^{\frac{1}{2}}$ ,  $(\overline{T'})^{\frac{1}{2}}$ , and  $c_p \rho \overline{w' T'}$  have been computed and tabulated in Table 1. With these values are

listed information concerning an identifying run number, distance off shore, height above the sea surface, and the vertical gradients of the potential temperature measured from the temperature-height diagram, Figure 1. The last column gives the maximum heat flow as computed from Priestley's theory. This will be discussed later.

A potential temperature cross section of the atmosphere between Rhode Island and Bermuda, presented here as Figure 2, has been drawn to show the temperature changes taking place in the air mass as it moves over the water and to show other observed quantities. The ordinate of the diagram is atmospheric pressure in millibars, while the abscissa is the double entry of the Greenwich Mean Time of a particular observation made from the aeroplane, and of the

TABLE 1  
HEAT FLOWS COMPUTED FROM FLUCTUATION DATA, JANUARY 18, 1955  
Atlantic Ocean, Rhode Island to Bermuda. 1820-2210 hr G.M.T.

Run Number	Distance (km)	Height (m)	$(\overline{w'^2})^{\frac{1}{2}}$ (cm sec <sup>-1</sup> )	$(\overline{T'^2})^{\frac{1}{2}}$ (°C)	$\overline{c_p \rho w' T'}$ (mcal cm <sup>-2</sup> sec <sup>-1</sup> )	dθ/dz (cm <sup>-1</sup> × 10 <sup>5</sup> )	$F_{Hmax.}$ (mcal cm <sup>-2</sup> sec <sup>-1</sup> )
974	100	520	86	0.05	0.1	1.1	0.2
975	100	335	121	0.07	1.7	1.1	0.4
976	100	150	94	0.08	1.1	1.1	0.6
977	270	305	107	0.08	1.1	3.0	0.4
978	270	150	135	0.19	3.9	-1.0	—
979	530	460	174	0.10	2.1	0.5	1.3
980	530	120	177	0.18	1.3	-8.0	—
981	800	550	122	0.09	1.1	0.6	1.0
982	800	305	134	0.10	2.0	0.6	1.2
983	800	120	138	0.11	2.1	-7.0	—
984	1100	550	129	0.07	0.8	0.8	0.5
987	1100	305	151	0.08	1.1	0.8	0.7
990	1100	90	138	0.13	2.4	?	—

off shore distance from the Rhode Island coast. The heavy lines are isopleths of potential temperature computed from both aeroplane measurements, represented here by dots, and radiosonde measurements, represented by crosses within circles. Cloud reports are entered along the top and at the flight level. The potential temperature of the air in contact with the water is entered in the ellipses at the sea-level pressure line. The turbulence encountered during the flight is indicated by the root-mean-square vertical velocities in cm sec<sup>-1</sup> entered in the boxes located at the proper pressure and time. Lastly the heat flows in mcal cm<sup>-2</sup> sec<sup>-1</sup> computed from the cross products of the vertical velocities and the temperature deviations are entered with an arrow representing the direction of flow.

The sensible heat flowing into the moving air column has been calculated by measuring the temperature change per 100 km as observed from the aeroplane and by measuring local changes and subsidence rates by use of radiosonde and

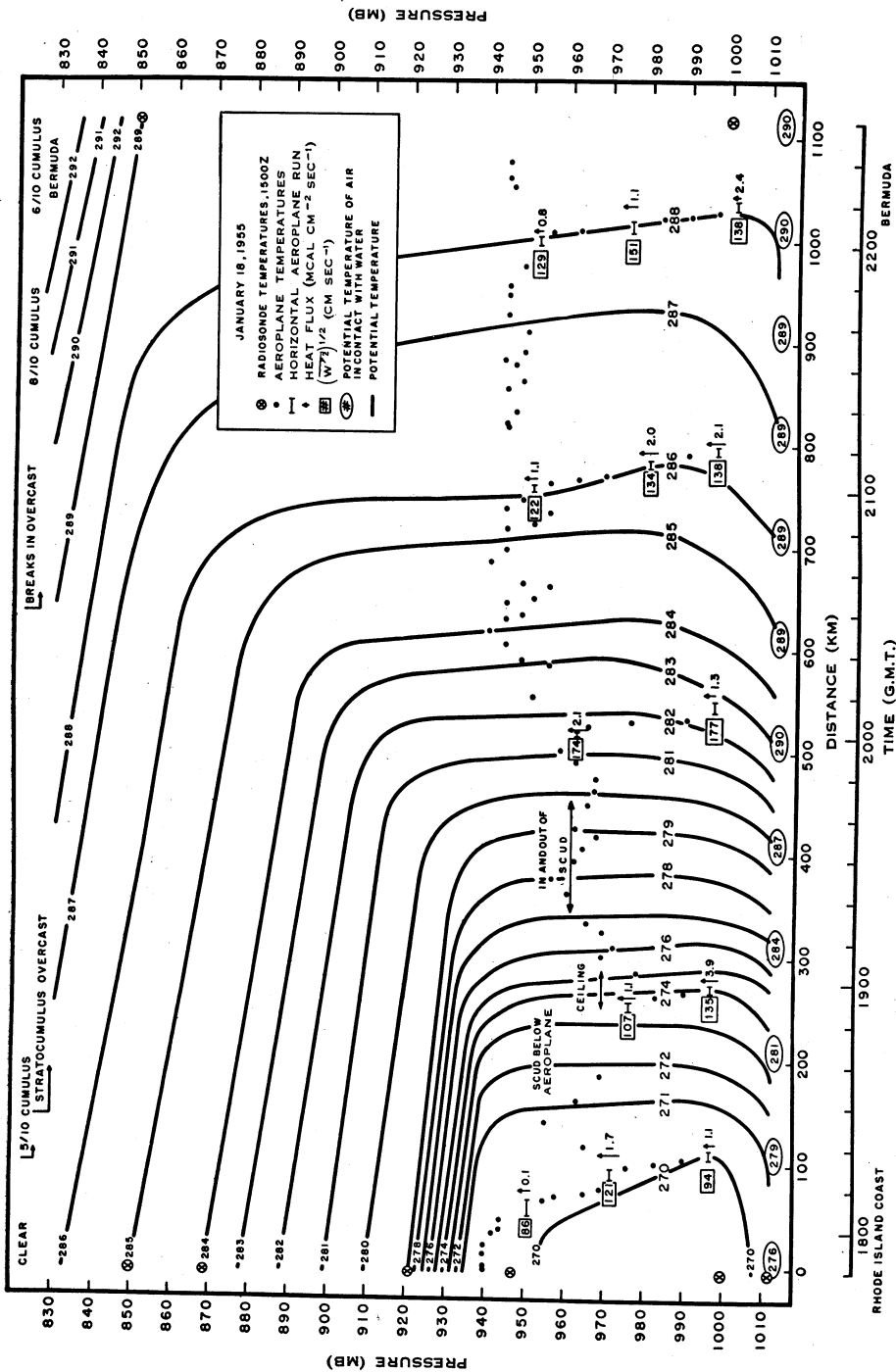


Fig. 2.—Atmospheric cross section from Rhode Island to Bermuda.

radiowind reports. The temperature changes were measured in two height ranges, one below the inversion and one in the inversion. From these figures the amount of heat added was found and the rate of addition calculated by dividing by the time of transit,  $8 \times 10^3$  sec. Adding the rates of heating for the two regions gives the total heating assuming no subsidence and no local changes of temperature. The average over the entire area was found to be  $5.3 \text{ mcal cm}^{-2} \text{ sec}^{-1}$ . To evaluate the heating arising from the neglected effects the terms in the following equation were determined

$$\left(\frac{dQ}{dt}\right)_{l \& s} = \left(\frac{dp}{g}\right) c_p \left[ \frac{dT}{dt} + \left(\frac{p}{10^6}\right)^{R/c_p} w \frac{d\theta}{dz} \right], \dots\dots\dots (8)$$

where  $dQ/dt)_{l \& s}$  is the heating due to subsidence and local changes in temperature where  $dp$  is the change in pressure in baryes through the layer concerned. The  $dT/dt$  term was measured by using the Bermuda radiosonde for two times 12 hr apart. The vertical velocity,  $w$ , was computed from the radiowinds from Nantucket, Mass., Washington, D.C., and Bermuda, by computing mass divergences. These divergences were calculated by the Bellamy (1949) method. Assuming the vertical velocity to be zero at the 1000 mb level, it is found that the air is subsiding at the rate of  $0.6 \text{ cm sec}^{-1}$  at 850 mb over the entire region. While it is admitted that the computation of divergences and velocities from so few wind observations is generally inaccurate, in the present case the wind field was steady for long periods and over large areas, so the computed velocity is considered reasonably reliable. The average rate of heating due to these two terms is  $1.3 \text{ mcal cm}^{-2} \text{ sec}^{-1}$  in the 1000–800 mb layer. When this is subtracted from the average total heat flow measured for the same layer, we get  $4.0 \text{ mcal cm}^{-2} \text{ sec}^{-1}$ . The maximum heat flow found for the 100 km regions was found to be  $8.0 \text{ mcal cm}^{-2} \text{ sec}^{-1}$  for the 200–300 km region. It is of interest to compare these values with those entered in Table 1. The average for the total area,  $4.0 \text{ mcal cm}^{-2} \text{ sec}^{-1}$ , represents satisfactory agreement, all difficulties considered, with  $2.4 \text{ mcal cm}^{-2} \text{ sec}^{-1}$ , the average of the five cross product heat flows taken at the lowest level of a given series. Likewise the heat flow for the 200–300 km region computed from the change in heat content,  $8.0 \text{ mcal cm}^{-2} \text{ sec}^{-1}$ , shows satisfactory agreement with the cross products method value of  $3.9 \text{ mcal cm}^{-2} \text{ sec}^{-1}$ .

### III. DESCRIPTION OF CONVECTIVE PARCELS AND ENVIRONMENT

Before checking the heat flows computed according to any theory against the observed heat flows, a few values and averages have been assembled that will give a clearer conception of the convective regime existing in the study area. First it should be pointed out that at the levels at which the aeroplane was flying it is easy to pick out the individual buoyant parcels from the environmental air. Visual inspection of the horizontal  $w'$ ,  $T'$  time series shows clearly that the environmental air is relatively quiet and homogeneous and that the convective parcels break through this quiet air as isolated well-defined turbulent disturbances of warmer updrafts and colder downdrafts. This regime contrasts strongly with a wind field characterized by mechanical turbulence in which no warm updrafts or cool downdrafts can be recognized although  $w'$ ,  $T'$  cross products



show that an appreciable heat flow exists. In the present study, the existence of these parcels is much more obvious at the lower levels (150–300 m) where the temperature fluctuations are greater, than at the higher levels (600 m). At the higher levels the cool downdrafts disappear so that only the environmental air and the warm updrafts are observable by inspection.

TABLE 2  
OBSERVED SIZES OF COOL AND WARM AIR PARCELS

Size Range (m)	Number of Cool Air Parcels	Number of Warm Air Parcels
50	6	7
50–100	7	7
100–200	3	6
200–300	1	6
300–400	0	3
400–500	0	1

Three temperature deviation groups have been defined by  $T' \leq -0.2^\circ\text{C}$ ,  $-0.1^\circ\text{C} \leq T' \leq 0.1^\circ\text{C}$ , and  $0.2^\circ\text{C} \leq T'$ , to aid in describing the observed parcels. These groups are intended to represent cool descending parcels, environmental air, and warm rising parcels respectively. The diameters of these cool and

TABLE 3  
ANALYSIS OF TEMPERATURE CLASS PROPERTIES

Run Number	Temperature Class ( $^\circ\text{C}$ )	Number of 20 m Intervals	Average Temperature ( $^\circ\text{C}$ )	Average Vertical Velocity (cm sec $^{-1}$ )	Heat Flow (mcal cm $^{-2}$ sec $^{-1}$ )
977	$-0.2 \leq T \leq 0.3$	165	0.006	— 4.9	1.13
977	$0.2 \leq T \leq 0.3$	45	0.11	15	0.41
977	$-0.2 = T$	15	—0.18	—60	0.25
977	$-0.1 \leq T \leq 0.1$	105	—0.013	— 5.5	0.50
978	$-0.3 \leq T \leq 0.6$	165	0.004	0.6	3.92
978	$0.2 \leq T \leq 0.6$	28	0.346	162	2.88
978	$-0.3 \leq T \leq 0.2$	9	—0.244	—96	0.36
978	$-0.1 \leq T \leq 0.1$	128	—0.053	—28	0.71
982	$-0.2 \leq T \leq 0.3$	165	0.002	2.0	1.99
982	$0.2 \leq T \leq 0.3$	15	0.25	186	1.15
982	$-0.2 = T$	2	—0.20	—64	0.08
982	$-0.1 \leq T \leq 0.1$	148	—0.02	—16	0.80

warm parcels have been determined for all 13 horizontal runs made during the flight to Bermuda. During these runs which totalled about 40 km only 47 parcels were observed. Their sizes are tabulated in Table 2.

It should be noted in interpreting these size distributions that parcels larger than 1000 m would be difficult to recognize by the method used. This

is true since the defined temperature groups are with respect to a line of regression of temperature upon time along the 3000 m flight path. Also it is likely that parcels of greater size have smaller amplitudes of the temperature deviations and hence may be unobservable with the present thermopile. In any case these parcels contribute the major portion of the heat flows tabulated in Table 1.

The sensible heat transported by air parcels belonging to each of the defined temperature classes has been computed for three horizontal runs picked at random from the 13 available. For each run and class, the number of 20 m intervals of the run occupied by each class, the average temperature deviation, the average vertical velocity, and the heat flow computed from the  $w'$ ,  $T'$  cross product have been tabulated in Table 3.

This collection of data gives a clear picture of the properties of the convective parcels and their environment, but is too incomplete to serve as a check upon the merits of the several parcel theories already discussed.

#### IV. COMPARISON OF THEORIES OF HEAT FLOW WITH THE DATA

A study of Figure 1 shows that the gradient of the potential temperature, except for the lowest 100 m or so, was positive throughout all of its journey from the mainland to Bermuda in spite of heating from the bottom. The discussion of systematic errors in the previous section shows that the gradients are real and that the classical heat flow theory can break down as has been pointed out previously by Priestley and Swinbank (1947).

The heat flow of the environmental air ( $-0.1 \leq T \leq 0.1$ ) was computed with the idea in mind that the correlation between the vertical velocity deviations and the temperature deviations might be negative in accordance with the classical diffusion theory, which says that heat should flow down the gradient. The heat flow was found to be positive in all three cases, two of these cases having positive gradients, thus giving additional confirmation of the inadequacy of classical theory in the presence of a gravitational field. As already stated, it has been anticipated that if a temperature class with sufficiently small limits had been analysed a negative correlation would have been found. This was based on the concept that this temperature class represented the undisturbed environmental air and that for small deviations neither organized overturning nor the weak buoyancy forces of Priestley and Swinbank would cause the  $w'T''$  term of (1) to predominate over the diffusion term. As the thermopile could not resolve temperature variations of less than  $0.1^\circ\text{C}$ , no answer could be obtained to this question.

The present set of observations is not well adapted to shed light upon the validity or weaknesses of the three parcel theories of Houghton and Cramer, Bunker, and Scorer and Ludlum in their present form. No definitive statistical study of the sizes, velocities, and temperature amplitudes at various heights and distances off shore has been attempted. This was not done because, after a brief inspection of the records, it was evident that a very large range existed in these variables and that a very small sample had been taken in each location. It was felt that no conclusions could be drawn concerning the validity of the

equations from this type of data without further development of the equations on a statistical basis.

As Priestley's equation (7) for the heat flow is expressed in observable statistical parameters, a comparison between his predicted maximum heat flows and  $c_p \bar{\rho w' T'}$  can be made. It is assumed that in the present case the heat flow mechanism of the atmosphere became adjusted in such a manner that a close approximation to maximum flow is attained. The maximum heat flow has been computed from (7) by substituting the observed value of  $\overline{T'^2}$  for  $\sigma_T^2$  as defined. In substituting  $\overline{T'^2}$  for  $\sigma_T^2$ , one is replacing  $\sigma_T^2$  by a value that is a time-weighted average of the temperature deviations of both parcels moving through the flight path and the environmental air. Equation (7) then gives the time-weighted maximum heat flow rather than the heat flow due to a given parcel of the optimum size. The value obtained in this manner is equivalent to the heat flow determined from  $c_p \bar{\rho w' T'}$ . These values have been tabulated in the last column of Table 1. A glance will show that these are values of the same order of magnitude but smaller than the values of  $c_p \bar{\rho w' T'}$  listed in column 6 of the table.

The agreement as to sign and order of magnitude proves that the theory is sound and that the underlying assumptions must be basically correct. The fact that the predicted values are smaller than the observed values may be caused by discrepancies between the assumptions made concerning impressed temperatures in a stable atmosphere and the condition of the temperatures and vertical velocities of the buoyant parcels obtaining at the base of the stable region of the atmosphere. If this is the case then a different expression for the original conditions of the parcel might be developed that would fit the observed conditions even better. Similarly, a more carefully controlled experiment might fit the conditions treated by Priestley more exactly and lead to a closer agreement.

#### V. ACKNOWLEDGMENTS

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