

ATTEMPTS TO MEASURE THE INDUCTIVE ELEMENT ASSOCIATED WITH THE NATURAL CONVECTION OF HEAT

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Summary

A study has been made of the variation in time of the temperature of a wire immersed in a fluid and heated by a constant electric current. For a given fluid the curve obtained by plotting the ratio of the temperature of the wire to the heat input versus the time is initially the same shape for all rates of heat input. Divergences from the lowest heating rate set in only when the system of convection currents sets in. This occurs at earlier times after the commencement of heating the higher the heating rate. Expressions already developed are used to evaluate the resistive, capacitive, and inductive elements required to fit the observed transient curves. The values of the former two types of element are consistent with an assumed stagnant film of a thickness the order of 1 mm around the heated wire, but the value of the deduced inductive element is some 10^4 – 10^5 greater than that associated with the kinetic energy belonging to the system of convection currents.

I. INTRODUCTION

The work described in the previous paper (Bosworth and Groden 1960) has provided a means for evaluating certain thermal properties of a fluid from the study of convective heat transfer in the transient state. The object of the present paper is to describe some preliminary exploration of the use of the methods derived, to show the broad effects of experimental conditions, to evaluate the accuracy of the derived quantities, and to find the optimum conditions for accuracy in further work.

As a result of a complete analysis of the transient curve, four parameters associated with thermal transfer from a hot solid to the surrounding liquid could be obtained. These include two thermal resistance terms, one associated with a purely conductive flow (r in the paper cited) and one associated with a shunt flow due to convection alone (R). The other two parameters derived are C , a capacitance term and probably a measure of the thermal capacity of the stagnant film around the heated solid, and L , an inductance term tentatively associated with the kinetic energy stored in the system of circulating fluid around the heated body.

II. EXPERIMENTAL DETAILS

The system selected for experimental study consisted of a heated resistance alloy wire which was held horizontally by stout current lead and immersed in the fluid under test. The wire was heated by an A.C. current from a source of steady e.m.f. through a variable resistance and current meter. The wires used

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varied in diameter from 0.610 to 0.274 mm (23–32 S.W.G.) and the lengths varied from 1.5 to 2.5 cm. The heat flux density for unit length of a wire heated by a current I of diameter d and specific (electrical) resistance ρ is given by

$$q = 4\rho I^2 / J\pi^2 d^3, \dots\dots\dots (1)$$

with J the Joule's equivalent. For a fixed experimental set-up q is thus directly proportional to I^2 .

The temperature difference between the wire and the fluid (θ) was obtained by means of a thermocouple of wire at least 3 gauges finer than the heater wire. One junction of this couple was welded to the centre of the heater wire and the other (immersed in the fluid) was situated some 5 mm away and in the same horizontal plane. The thermocouple was connected through a variable shunt box to a recording galvanometer. Relative measures of θ can be obtained from the galvanometer reading and absolute readings can be obtained from calibration of a like couple in baths of different known temperatures.

The liquids examined were water, aqueous solutions, and various organic liquids. Some 200 ml was used in any test. No special purification was attempted. The vessel containing the liquid under test and the hot wire thermocouple device were contained in an external thermostat at a temperature ranging from 12 to 70 °C and the vessel was allowed to come to thermal equilibrium before any measure was attempted. A predetermined heating current was then switched on and the temperature difference recorder was allowed to run until the temperature became steady. When the time taken was in excess of 10 min an ordinary bench reflecting galvanometer with frequent reading could be used. Processes which could have been completed in less than 5 sec have not yet been examined, although there appears to be evidence that such fast transients could be obtained under certain conditions. Particularly quick acting recorders are necessary in the analysis of these transients and these have not yet been used in this field.

The experimental factors which might be expected to affect the nature of the convective transient are the diameter of the wire, its temperature excess θ , and the thermal nature of the fluid, including the thermal conductivity and the convective modulus. Changes in θ provide the easiest experimental variation, limitation being set only by the onset of boiling at the upper limit and the sensitivity of the galvanometer at the lower limit. However, as will be seen later, there is a great tendency for the time constants to become excessively long at low values of θ and the ultimate limiting factors may be the stability of the heating current and of the thermostat bath. Changes in d with the present form of device are limited to a comparatively narrow range. Below 32 S.W.G. it is difficult to get suitable thermocouples which do not produce an appreciable thermal disturbance at the centre of the heated wire. Wires greater than 22 S.W.G. tend to draw an excessive heating current and seriously disturb external thermostating. The range of thermal conductivity in non-metallic liquids is comparatively restricted (Bosworth 1952). On the other hand there are readily available fluids covering a wide range of convective moduli; this

modulus is an experimental variable which may be readily changed by using viscous liquids at different compositions or temperatures. Mixtures used include the systems glycerol-water and sucrose-water.

III. EVALUATION OF EXPERIMENTAL DATA

As already intimated, relative values of the heat flux q and the temperature difference θ may be obtained more readily than absolute figures. However, the first step in the deduction of the circuit parameters is concerned solely with dimensionless quantities, A and B , which are ratios of times and impedances respectively as derived from the transient curves. From A and B , using data from the tables and curves given in the previous paper (Bosworth and Groden 1960), it is possible to derive the quantities X, Y , and R/r , all of which are

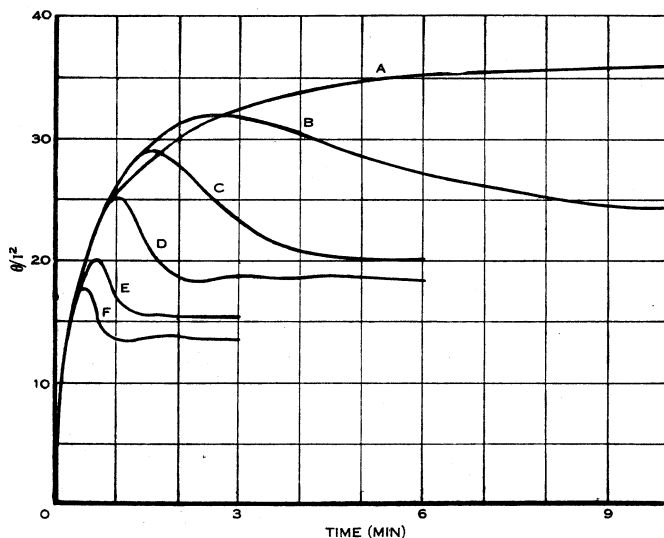


Fig. 1.—Transient curves, plotted as θ/I^2 against time.

dimensionless. To proceed further it is necessary to take into account the absolute value of t_1 and then get the quantity \sqrt{LC} , or alternatively to take into account the final thermal impedance Z_0 which is equal to $Rr/(R+r)$ or θ/I^2 for any fixed experimental set up. From the known absolute (or relative) values of θ/q it is then possible to derive R and r . From

$$\sqrt{L/C} = YR, \quad \dots\dots\dots (2)$$

and from \sqrt{LC} it is then possible to evaluate both L and $1/C$ on the same (or relative) scale.

As an illustration we will consider data derived from a wire of diameter 0.05105 cm immersed in 60.4 per cent. w/w sucrose solution at 12 °C. The heating current ranged from 0.230 A through 0.362, 0.540, 0.790, 1.220, to 1.590 A. The transient curves, plotted in the form of θ/I^2 versus time, are shown in Figure 1. The lowest heating rate gave a curve with a very indistinct maximum, at least within the time of the experiment recorded, while the curves

associated with the higher heating rates showed transients passing successively faster and faster through a maximum and then approaching the final value from above. Some of the curves show signs of a severely damped oscillation after passing through the first maximum.

From readings taken from the third curve we obtain the following four characteristics :

$$\begin{array}{ll} t_0 & 27 \text{ sec} \\ t_1 & 92 \text{ sec} \\ Z_0 & 20.4 \text{ arbitrary units} \\ Z_1 & 29.9 \text{ arbitrary units} \end{array} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} A=3.40, \\ \\ B=1.415. \end{array}$$

From Figure 4 of the previous paper it is now possible to read off both

$$X=2.3 \text{ and } R/r=1.80,$$

from which

$$Y=2.75.$$

From the value of Z_0 (measured in arbitrary units as θ/I^2) we then arrive at the values

$$R=57.2 \text{ and } r=31.7 \text{ arbitrary units.}$$

From the values of R and Y we obtain

$$\sqrt{(L/C)}=157 \text{ in the same arbitrary units,}$$

and finally using the value of t_1 and the ratio $\sqrt{(LC)}/t_1$, depending on X and reading off from Table 2 of the previous paper, we get

$$\sqrt{(LC)}=131 \text{ sec.}$$

Accordingly

$$\begin{array}{l} L=20600 \text{ arbitrary units, and} \\ C=0.83 \text{ reciprocal units.} \end{array}$$

IV. EXPERIMENTAL RESULTS

Analysis of the curves shown in Figure 1 may be carried out by the method outlined above. However, it is more convenient to convert the purely relative figures to absolute figures from the known diameter and electrical resistivity of the wire concerned, using equation (1). For a wire of diameter 0.05105 cm with specific resistivity $\rho=10^{-4} \Omega \text{ cm}$, the heat flux per unit length becomes

$$q=0.071I^2 \text{ cal cm}^{-1} \text{ sec}^{-1}, \quad \dots\dots\dots (3)$$

so that in turn $rR/(R+r)$ becomes

$$(\theta/q)_{\text{final}}=14(\theta/I^2)_{\text{final}}.$$

The appropriate values of the circuit parameters derived from the six curves shown in Figure 1 are given in Table 1, while Table 2 contains corresponding values from the same wire in different solutions and Table 3 for different wires in various liquids.

TABLE 1

THERMAL CIRCUIT PARAMETERS ASSOCIATED WITH A WIRE OF DIAMETER 0.0511 CM IN 60.4 PER CENT. SUCROSE SOLUTION AT 12 °C

θ (degC)	r (degC cm sec cal ⁻¹)	R (degC cm sec cal ⁻¹)	L (degC cm sec ⁻² cal ⁻¹)	C (cal (degC) ⁻¹ cm ⁻¹)
1.7	540	2700	25,000,000	0.160
3.15	465	1200	1,300,000	0.065
5.9	445	800	280,000	0.060
11.5	400	750	160,000	0.049
23.2	376	535	35,000	0.041
34.4	364	410	12,000	0.054

TABLE 2

THERMAL CIRCUIT PARAMETERS ASSOCIATED WITH A WIRE OF 0.0511 CM DIAMETER IN SUCROSE SOLUTION

Strength of Solution	Bath Temper- ature	θ (degC)	r (degC cm sec cal ⁻¹)	R (degC cm sec cal ⁻¹)	L (degC cm sec ² cal ⁻¹)	C (cal (degC) ⁻¹ cm ⁻¹)
60.4%	16 °C	2.6	440	2300	1,700,000	0.072
		3.7	380	1700	860,000	0.049
		5.5	400	1260	410,000	0.045
		9.2	460	1200	170,000	0.044
		12.8	440	780	78,000	0.049
		18.5	410	700	53,000	0.036
		25.5	390	700	39,000	0.030
		34	390	490	11,000	0.045
		59	380	350	3,100	0.046
		63	440	280	2,500	0.035
		79	520	170	710	0.037
		86	620	63	510	0.048
		112	320	280	1,400	0.029
		138	245	190	1,100	0.046
	40.5 °C	3.4	650	2600	2,740,000	0.048
		7.6	460	590	49,000	0.043
		13.0	320	680	19,000	0.043
		42	290	360	2,900	0.041
	69 °C	5.6	420	1400	240,000	0.022
		17.1	500	400	24,000	0.022
		24.6	430	300	2,000	0.037
		45.6	490	200	410	0.021
81.2%	18 °C	132	520	520	12,800	0.050
55.4%	19 °C	6.5	430	640	4,200	0.045
		40.0	340	250	220	0.054

TABLE 3
THERMAL CIRCUIT PARAMETERS FROM VARIOUS SOURCES

Wire Diameter (mm)	Liquid	Bath Temperature	θ (degC)	r (degC cm sec cal ⁻¹)	R (degC cm sec cal ⁻¹)	L (degC cm sec ² cal ⁻¹)	C (cal (degC) ⁻¹ cm ⁻¹)
0.457	40% sucrose solution*	19 °C	2.8	530	680	53,000	0.083
			4.3	840	400	8,700	0.077
0.457	70% sucrose*	—	47	440	1160	285,000	0.044
0.1016	water	22 °C	26	12.1	42.1	2,700	0.168
0.1016	carbon tetrachloride (?)	—	4.0	90	330	280,000	0.084
0.1016	toluene	—	2.4	64	620	18,000	0.170
			3.1	61	510	3,800	0.089
			4.4	65	350	3,500	0.092
			37	55	130	2,600	0.090

* Data from published curves (Bosworth 1946a).

V. ACCURACY

Before considering possible relationships between the circuit parameters and the properties of the heated wire and the convective medium, some thought must be given to possible sources of error. Two obvious sources of inaccuracy immediately come to mind. The device used cannot strictly be described by any simple mathematical expression. The small thermocouple has been used to measure the temperature difference between the central portion of the hot wire and the surrounding fluid. In practice, of course, the presence of the couple alters the electrical conductivity of the central portion of the wire and so disturbs the rate of heat generation. Again, the arms of the couple carry heat away from the heated zone and so tend again to depress the temperature of the zone for which a temperature measurement is required. Disturbing influences may be reduced by using finer wires for the thermocouple and may become insignificant with sufficiently fine wire. However, in an initial broad study of the allied phenomena this particular aspect has not yet received quantitative attention.

For any particular experimental set-up, assuming that the actual temperature is a representative central temperature of the wire, then the accuracy for all four parameters is largely dominated by the relationship between the A 's, or ratio of the times for attaining standard points on the transient, and the B 's, or ratio of the temperature differences. Figure 4 in the previous paper shows this relationship. At a value of B equal to 1.2, which is the average value of this experimental ratio recorded in the data used in Table 1, a 1 per cent. error in the ratio A will

give rise to 3 per cent. error in the ratio of R/r and thus subsequently in the derived parameters. Again, at a value of A equal to 2.5 (the average of the experimental figures used above) an error of 1 per cent. in the value of B will give rise to a 10 per cent. or larger error in the ratio of R/r and thus in subsequently derived parameters. Accordingly, it is vitally important, if reliable circuit parameters are to be obtained, that the measurements of B should be made with the utmost care. It is indeed fortunate that the amplitudes of transient records can be recorded and measured with considerable accuracy—with a higher accuracy for example, than that which may be recorded for the time required to reach a turning point. In the latter case the flat nature of the maximum and the asymmetrical shape of the peak considerably limits the accuracy of assessment of the true turning point. However, assuming that we can measure Z_1 (or the amplitude at the peak) with a reasonably high accuracy the value of all subsequent data will primarily be limited by the accuracy with which we may measure Z_0 (or the final impedance). For this reason it is most desirable that current stabilizers should be used in the heating circuit. Any fluctuation in the rate of heating will vitiate attempts to attain a highly accurate measurement of B as Z_1/Z_0 . Again, if the transient involves a particularly long time the experimentalist may read an apparent value of Z which has in fact not settled down to the final Z_0 . Again, if the period is long and as a considerable source of internal heat must be applied to the fluid during the course of the transient, the apparent Z_0 may in fact not be the true Z_0 because the properties of the fluid have been changed. Long-period transients may therefore be expected to give false readings of the thermal circuit parameters.

On the other hand, if the thermal transient is associated with too short a period the galvanometer or recorder may not follow truly the whole course of the transient, particularly as the low resistance associated with the thermocouple circuit may cause serious overdamping of the galvanometer. Errors due to this cause usually first show up in abnormally high values of t_0 (the time taken first to cross Z_0) relative to the value of t_1 . Overdamping of the galvanometer gives a spuriously low value of A , which has shown up in many instances in which a bench galvanometer has been replaced by a quick-acting potentiometric recorder which gives an apparently much higher value of A for the same conditions as those studied with a reflecting galvanometer. Generally speaking therefore the method described in these two papers can be applied to very quick-acting transients only if suitable quick-acting recorders are used.

A study of the effect of errors due to mis-estimation of the final value of Z_0 may be carried out on curve A of Figure 1. The final value of Z obviously is attained outside the range of time recorded in the figure given (that is, up to 10 min) and has to be taken as 32 arbitrary units. Had we postulated that at the time of 30 min Z had in fact not yet reached its final value and that instead a longer time would have given $Z_0=31$, then the values of X and Y which had been computed as 11.6 and 4.65 would instead be 14.2 and 10.3, with the associated circuit parameters in the order given in Table 1: 580, 1620, 42×10^6 , and 0.150. Conversely, an assumption that due to heating of the fluid the value of 32 had

already overshoot the final true value and $Z_0=33$ is a better representation, then the four parameters would have been estimated as at 510, 5050, 14×10^6 , and 0.185.

The apparent abnormally high values of C , R , and L at the lowest heating rate cannot therefore be accepted at this stage and a more carefully designed experiment appears to be required.

VI. CONCLUSIONS FROM EXPERIMENTAL DATA

The variation in the four circuit parameters, as shown in Tables 1-3, while subject to fairly large errors, particularly at the lowest heating rate, are sufficiently pronounced to make a series of broad generalizations.

All the four properties appear to change somewhat with change in the nature of the convective liquid and with the size of the heating surface. However, the most pronounced change is that associated with the change in the overall temperature difference θ . The two properties r and C , those which have tentatively been associated with the conductive contribution to the total heat flux, are practically unchanged with change of θ . On the other hand, the properties R and L , tentatively attributed to the convective contribution to the total heat flux, both decrease rapidly with increase of θ , the change in L being particularly pronounced. This behaviour, of course, is, at least in its general characteristics, in accordance with the well-known features of convective transfer. However, a quantitative examination of the magnitude of the quantities concerned could add considerable knowledge to this subject.

The capacity term, for example, appears to vary with the diameter of the wire concerned. An average figure of $0.050 \text{ cal cm}^{-1} (\text{degC})^{-1}$ was obtained for a wire of 0.511 mm diameter. This figure may be compared with the thermal capacity per centimetre of the wire itself. The corresponding capacity C^1 of the wire is

$$C^1 = \frac{1}{4} \pi \rho c d^2 \text{ cal cm}^{-1} (\text{degC})^{-1},$$

where ρ is the density of the wire, c its specific heat, and d its diameter.

Inserting numerical values we get

$$C^1 = 1.7 \times 10^{-3} \text{ cal cm}^{-1} (\text{degC})^{-1},$$

which is only about 1/30 of the measured value (C) of the capacity per unit length of the wire. The thermal capacity of whatever is contributing to the effective C as deduced from the nature of the thermal transient is thus much larger than the thermal capacity of the heated solid alone. The thickness of the fluid layer surrounding the wire required to give a capacity of $0.05 \text{ cal cm}^{-1} (\text{degC})^{-1}$, using the Janovsky and Archangelsky (1928) data for the volumetric specific heat of sucrose solution, amounts to 1.1 mm. Taking the capacity of the toluene films surrounding a wire of 0.1016 mm diameter as $0.09 \text{ cal cm}^{-1} (\text{degC})^{-1}$ (Table 3) gives a corresponding sheet thickness of 1.0 mm. These figures may be compared with the estimated film thickness of 1 mm in mild thermal convection given by Eckert and Soehngen (1952).

The value of the parameter r in Tables 1–3 must be associated with the flow of heat from the wire before a condition of convection sets in. This involves an unsteady state problem in heat flow in a cylindrical system and is best not exhaustively treated until more complete and more accurate data have been compiled. Suffice it to say, however, that the solution will depend only on the diameter of the wire (possibly with correction due to the presence of the thermocouple) and on the thermal conductivity of the liquids. Analysis thus offers a possibility of eliminating the disturbing effects of convection on the measurements of conduction in mobile liquids, and this technique may well lead to a quick laboratory method for comparing the thermal conductivities of mobile liquids.

The factors R , which are highly sensitive to the temperature excess θ , are in turn indices to the convective moduli. As such these factors describe the ways in which the additional heat flow rate varies with the overall temperature difference in natural thermal convection.

The factors L were, in the first instance, thought to be interpreted as the measure of the kinetic energy stored in the system of circulating convection currents per unit of squared thermal flux. One is tempted to write, on analogy with the corresponding expression in electrodynamics, for the kinetic energy K ,

$$K = \frac{1}{2} L q^2. \quad \dots\dots\dots (4)$$

This, however, on the basis of the definitions and units so far used, is dimensionally incorrect in just the same way as would the expression

$$V = \frac{1}{2} (\int q dt)^2 / C$$

be dimensionally incorrect for the thermal (potential) energy stored in the capacity.

These formal difficulties could, according to the author (Bosworth 1946*b*), be neatly avoided by a redefinition of the thermal flux as a definition of a flow of entropy. Equation (3) then becomes

$$q = \frac{0.071}{T} I^2 \text{ E.U. cm}^{-1} \text{ sec}^{-1}, \quad \dots\dots\dots (5)$$

with T the absolute temperature of the sink. On the basis of the units previously used, namely, joules (degC)⁻¹ cm⁻¹, q becomes

$$q = \frac{0.30}{T} I^2 \text{ thermal amperes/cm.} \quad \dots\dots\dots (6)$$

On this basis the various properties listed in Tables 1–3 could then be changed to new units as thermal ohms.cm (or (degC)² sec cm joules⁻¹) for the second and third columns, thermal henry cm (or (degC)² sec² cm joules⁻¹) for the fourth column, and thermal farads per cm (or joules cm⁻¹ (degC)⁻²) for the last. The second, third, and fourth columns should be multiplied by $T/4.2$ and the fifth by $4.2/T$.

Equation (4) now becomes dimensionally correct and K is measured in joules per cm of the heated wire. As we are concerned with the convective contribution to the heat current we put

$$q = \theta/R$$

or

$$K = \frac{1}{2}L\theta^2/R^2. \quad \dots\dots\dots (7)$$

To find the way in which the kinetic energy changes with the temperature difference θ , it is therefore necessary to find the relationship between L/R^2 and θ . Data from the Tables 1 and 2 have been plotted in Figure 2 in the form $\log L/R^2$

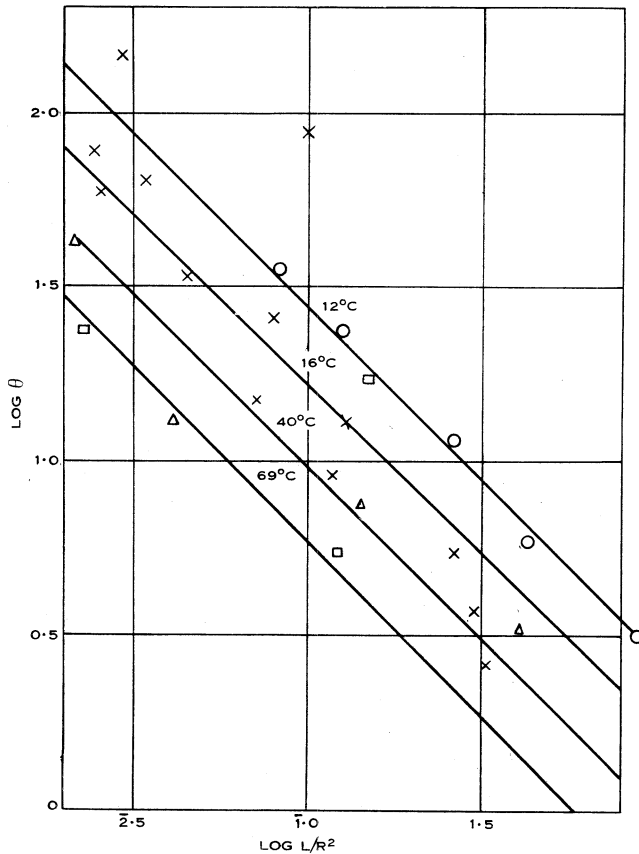


Fig. 2.—Relation between L/R^2 and θ .

versus $\log \theta$. The differently marked points corresponding to different bath temperatures fall on different parallel straight lines, all having the slope -1.0 . We accordingly put

$$L/R^2 = a\theta^{-1},$$

with the factor a as a constant for any particular wire in any particular fluid. Thus,

$$K = \frac{1}{2}a\theta, \quad \dots\dots\dots (8)$$

so that it appears that the kinetic energy of the convective current increases linearly with the temperature excess. For the 0.511 mm wire in 60.4 per cent. sucrose solution at 12 °C the quantity a takes the value $0.041 \text{ J (degC)}^{-1} \text{ cm}^{-1}$, and at 16 °C, $0.025 \text{ J (degC)}^{-1} \text{ cm}^{-1}$. Accordingly, $K=0.020 \text{ J/cm}$ at 12 °C, and 0.012 J/cm at 16 °C, for a temperature rise of 1 degC and with somewhat lower figures in the less viscous fluids. Now the total volume of fluid in which the hot wire was immersed was some 200 ml, and at the most some 30 ml/cm of wire could be involved in the system of convection currents. The expected kinetic energy in such a system could only be of the order of ergs/cm. The phenomenon giving rise to the inductive features of thermal convection is of one or two orders of magnitude larger than that expected from the kinetic energy of the system of convection chimneys.

Presumably any process in which the heat flow rate is partially conditioned by states of the system arising from absorption of energy in a non-thermal form from the initial heat flow will add an inductive-like element to the thermal circuit. The storage of heat in a process of surface evaporation would act in this way. In some instances cited in Table 2 the liquid was actually caused to boil on the wire (although the liquid medium as a whole did not boil). While in these instances the "inductance" was larger than the extrapolated value from the non-boiling region, however, the change was not one of a different order of magnitude, as might have been expected. Accordingly, there arises the suggestion that the unexpectedly large values associated with thermal induction without surface boiling may be due to surface chemical phase change, for example, to deaeration of the liquid near the wire and subsequent re-resolution of microscopic bubbles in the slightly cooler region. This suggests that thermal transients could profitably be studied after complete deaeration of fluids under test.

As an alternative interpretation of the high value of thermal inductance there may be cited our general lack of complete knowledge of the mechanism involved in the passage of layers of different fluid density over a solid surface—the so-called tea-pot effect (Reiner 1956). The tendency of flowing streams of different density to adhere to solid surfaces appears to be several orders of magnitude greater than that expected from normal surface (tension) phenomena. Reiner has suggested that the creation of micro-eddies is responsible. If upheld, a similar mechanism may also be the cause of the apparently high kinetic energy term connected with the act of setting up the convection chimney.

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