RADIO SOURCE COUNTS AT 408 MHz

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Abstract

Radio source counts at 408 MHz are investigated using the data from the first Molonglo catalogue (MC1) and an all-sky catalogue of strong sources. Earlier results are qualitatively confirmed. Possible explanations of the high exponent of the number–flux density relation between sources of strong and intermediate flux density are discussed briefly, but no firm conclusions seem possible.

I. Introduction

The apparently anomalous distribution of radio source flux densities has been familiar for many years and has led to the construction of numerous elaborate cosmological models. More recent investigations of the $\log N$–$\log S$ relation at high frequencies have indicated, however, that the anomaly is less marked than had been accepted on the basis of the low frequency results. Since most cosmological model-making has been based on the Cambridge source counts at 408 MHz (Pooley and Ryle 1968) it seems worth while making use of the first Molonglo survey at the same frequency (Davies *et al.* 1973; henceforth referred to as Paper I) as a check. In addition the all-sky catalogue of strong sources prepared by Robertson (1973, present issue pp. 403–16; henceforth referred to as Paper II) allows us to define the high flux density part of the curve with greater certainty than was possible at the time of the earlier Cambridge work.

Eventually the Molonglo catalogue survey, together with deep surveys which are being made to much lower flux density levels, should allow the $\log N$–$\log S$ relation to be defined even more accurately at 408 MHz. However, the reduction of the accumulated data has proved to be very time consuming and so we have taken advantage of the preparation of the first catalogue (MC1) in Paper I to make a preliminary investigation. Because of the small area covered by the catalogue, results obtained from it are limited in statistical accuracy above 3 f.u. However, when this catalogue is combined with the all-sky catalogue in Paper II, the critical range above 1 f.u. is reasonably well covered, and this is the region where the anomaly was most apparent. The MC1 catalogue also extends downwards reliably to about 0·25 f.u., a region where the Cambridge counts based on a synthesized fan beam were possibly subject to some confusion. Because of the high resolution of the Molonglo pencil beam radio telescope ($<3'$ arc), confusion effects are negligible over the whole range of the present counts. The only significant instrumental corrections necessary were for the effects of random noise on the weakest sources.

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Two areas of the Molonglo catalogue have been selected for analysis: those extending in right ascension from $01^h 30^m$ to $06^h 45^m$ and $08^h 15^m$ to $16^h 30^m$ with corresponding declination ranges of $-19°·5$ to $-22°·25$ and $-19°·4$ to $-22°·15$ (epoch 1950). The total area amounts to 0·160 sr and contains 1053 sources or approximately 6600 sources sr^{-1} with flux densities above 0·22 f.u. In calculating source number densities the total area is reduced somewhat by the exclusion of regions where calibration signals had been inserted or where records were faulty. These excluded areas depend on the flux density but do not exceed 4% of the total area selected. The flux densities are measured on the Wyllie (1969) scale.

For sources stronger than 10 f.u. the catalogue of Paper II, which covers 10·1 sr, has been used. It has been shown that the systematic errors in this catalogue are appreciably smaller than the statistical uncertainties and may therefore be ignored. The flux densities are also given on the Wyllie scale.

TABLE 1

DIFFERENTIAL SOURCE COUNTS AT 408 MHz FROM COMBINED MC1 AND ALL-SKY CATALOGUES

ΔF_i (f.u.)	$\Delta N_{obs,i}$	Mean x_i/y_i	$\Delta N_{corr,i}$	$\Delta N_{corr,i}/\Omega$ (sr^{-1})
0·22–0·31	276	0·78	348	2262
0·31–0·44	244	1·01	238	1550
0·44–0·62	186	1·09	171	1093
0·62–0·88	155	1·06	146	912
0·88–1·25	72	1·03	70	437
1·25–1·77	53	1·02	52	325
1·77–2·50	26	1·01	26	162
2·50–3·54	17	1·00	17	106
3·54–5·00	9	1·00	9	56
5·00–7·07	6	1·00	6	37
7·07–10·0	5	1·00	5	31
10·0–14·1	84		84	8·3
14·1–20·0	28		28	2·8
20·0–28·3	20		20	2·0
28·3–40·0	14		14	1·4
⩾40·0	14		14	1·4

II. ANALYSIS

We have used the differential form of the source counts in the analysis since it has been shown by Jauncey (1967) that the integral counts, as used by Pooley and Ryle (1968), tend to smooth any changing slopes and give statistically invalid results in the determination of slopes and particularly of their uncertainties. We have adopted a logarithmic interval of flux density, which corresponds to a flux density ratio of $\sqrt{2}$. The number of sources $\Delta N_{obs,i}$ observed in each flux density interval ΔF_i is listed in Table 1 together with the corrected numbers $\Delta N_{corr,i}$, to be described in the following paragraphs, and the corrected number density $\Delta N_{corr,i}/\Omega$ (sr^{-1}). Above 10 f.u. the data were taken from Paper II and below 10 f.u. from Paper I. To remove the effects of resolution on the counts, integrated flux densities were used in all ranges, and thus the strong sources were treated similarly to the weak.

It is well known that instrumental effects are usually important in compiling a catalogue and the corrected numbers in Table 1 include our estimates of such effects. The corrections are not large except for the lowest flux density range which is regarded as uncertain. The errors in the observed numbers arise almost entirely from three basic causes:

(1) noise fluctuations and calibration errors, which produce random errors in the flux densities;

(2) exclusion from the counts of sources not detected in more than one observing session, which reduces the numbers with low flux density;

(3) removal of sidelobe responses from the records, which results in the removal of some genuine sources.

The effects of (1) and (2) dominate and in comparison (3) may be safely ignored. The basic problem in correcting these errors is that of determining the best estimate of the true flux density distribution $P(S)\,\mathrm{d}S$ from the observed distribution $P(F)\,\mathrm{d}F$. These distributions are linked by the error distribution $P(F|S)\,\mathrm{d}F$, which is the probability that a source of flux density S will be observed with a flux density between F and $F+\mathrm{d}F$. From the nature of probability it follows that

$$P(F) = \int_0^\infty P(F|S)\,P(S)\,\mathrm{d}S. \tag{1}$$

The ratio of the expected number of sources allocated to the ith flux density interval in the presence of errors in the measured flux density to that in the absence of errors is given by x_i/y_i, where

$$x_i = \int_{F_i}^{F_{i+1}} P(F)\,\mathrm{d}F = \int_{F_i}^{F_{i+1}} \mathrm{d}F \int_0^\infty P(F|S)\,P(S)\,\mathrm{d}S \tag{2}$$

and

$$y_i = \int_{F_i}^{F_{i+1}} P(S)\,\mathrm{d}S. \tag{3}$$

To obtain corrections to the numbers of sources, a knowledge of $P(F|S)$ is necessary. This may be obtained directly from the errors derived in Paper I. These errors do not include the effects of confusion but, because of the large number of beam areas per source, this correction is negligible. We may allow for both of the errors (1) and (2) by considering the function $P(F|S)$ to be made up of the product of two independent parts p_1 and p_2, in which p_1 is determined by the effects of random noise and p_2 is determined by the probability of confirmation.

It is shown in Paper I that the random errors in flux density closely follow a Gaussian distribution, and thus

$$p_1(F|S) = (2\pi\sigma^2)^{-\frac{1}{2}}\exp\{-(F-S)^2/2\sigma^2\}. \tag{4}$$

Here we need only consider the effects of random noise when determining σ. Calibration errors affect all flux density ranges equally and therefore do not affect the slope of the counts and, in any case, have a negligible effect on actual numbers (see Paper II). The appropriate value of σ, taken directly from Paper I, is $0 \cdot 087$ f.u. for a single measurement.

The probability that a source of observed flux density F and true flux density S is confirmed by a second observation is the probability that a source of flux density S is observed above the detection threshold D_0 and is given by

$$p_2(S) = \int_{D_0}^{\infty} (2\pi\sigma^2)^{-\frac{1}{2}} \exp\{-(S-D)^2/2\sigma^2\}\, \mathrm{d}D \tag{5}$$

and

$$P(F|S) = p_1(F|S)p_2(S). \tag{6}$$

This result is directly applicable when the catalogued flux density is that of one observation only which has been confirmed, a common situation since the integrated flux densities in MC1 can only be well evaluated when the source is near the central beams. When two observations were averaged to give a flux density, equation (6) applies as an approximation by using in p_1 the error appropriate to the mean of two observations. Although this is no longer strictly Gaussian because of the selection procedure used, it is evident from Paper I that the discrepancy is small and may be safely ignored.

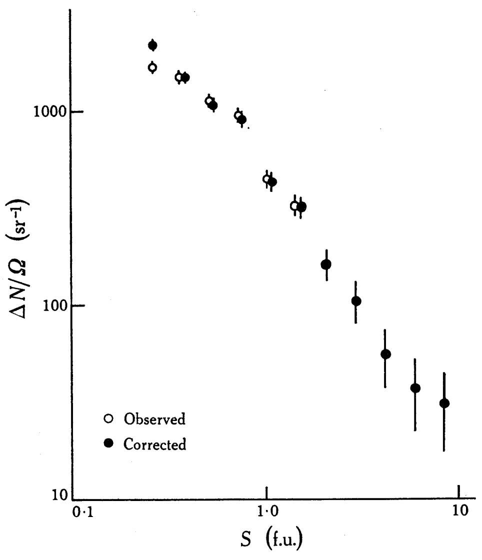

Fig. 1.—Differential counts for sources in the MC1 catalogue as a function of integrated flux density S below 10 f.u. A logarithmic interval in S, corresponding to a flux density ratio of $\sqrt{2}$, has been used.

Evaluation of the correction requires a knowledge also of $P(S)\,\mathrm{d}S$, but this is the quantity to be derived. Strictly speaking an iterative process is required for a solution, but the correction is only weakly dependent on the form of $P(S)\,\mathrm{d}S$ because of the strong effect of p_2 on the correction factors so that, for small corrections, convergence is rapid. From a rough estimate of the corrections and a plot of the data, we took $P(S)\,\mathrm{d}S = KS^{-1 \cdot 9}\,\mathrm{d}S$ for the region below 1 f.u. where the corrections begin to be appreciable. The correction factors x_i/y_i were then calculated from equations (2) and (3) for flux densities obtained from one or two observations and the value of D_0 used in preparing the MC1 catalogue ($0 \cdot 18$ f.u.). Corrections were then calculated separately in each interval for the cases of one or two observations. The appropriate mean value of x_i/y_i is listed in Table 1.

In Figure 1, a plot of the raw differential counts and the corrected counts for the data taken from the MC1 catalogue is given for the tabulated logarithmic intervals.

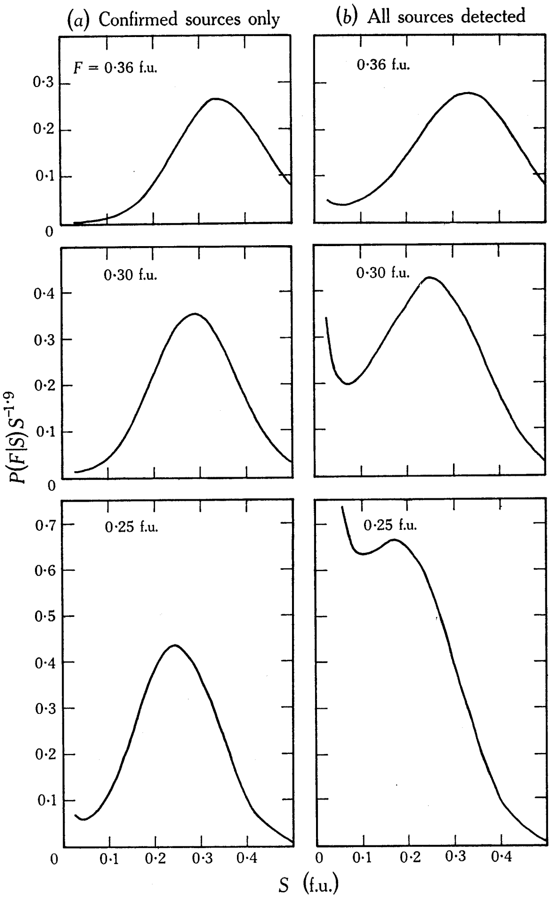

Fig. 2.—The integrands $P(F|S)S^{-1 \cdot 9}$ in x_i for the indicated observed flux densities F: (a) for confirmed sources $P(F|S) = p_1 p_2$, and (b) for all detected sources $P(F|S) = p_1$. (See text.)

It is apparent that there is an abrupt change of slope just below 1 f.u. Maximum likelihood fits to the corrected grouped data (see e.g. Crawford *et al.* 1970) give a slope of $-0 \cdot 9 \pm 0 \cdot 1$ below $0 \cdot 88$ f.u.,* as used in the corrections, and a slope of $-1 \cdot 6 \pm 0 \cdot 2$ above $0 \cdot 88$ f.u., where corrections are negligible.

Evidence for the validity of the method of correction is presented in Figure 2. Here we have plotted as functions of flux density S both $p_1 p_2 S^{-1 \cdot 9}$ and $p_1 S^{-1 \cdot 9}$,

* It should be noted that, for any power-law model, the slope of the differential counts using logarithmic intervals is the same as that of the integral counts.

which represent the integrand in x_i (equation (2)) for the cases of confirmed sources and all sources detected respectively. The correction factors are then proportional to the areas under these curves. It is clear that, for the confirmed sources, the areas and therefore the corrections are very well defined for sources with flux densities above $F = 0\cdot30$ f.u. and reasonably well defined above $F = 0\cdot25$ f.u. but that for all sources, the corrections become completely indeterminate at $0\cdot25$ f.u. due to the large noise fluctuations and are only reasonably well defined above about $F = 0\cdot36$ f.u. For this reason, only confirmed sources have been used in the counts. We conclude that the correction in the lowest flux density interval is uncertain but corrections above $0\cdot30$ f.u. are quite reliable.

III. Results

To present the results as clearly as possible, we have plotted in Figure 3 the ratio of the corrected number of sources in a given flux density interval to the number expected in a static Euclidean universe (in which $P(S)\,\mathrm{d}S = KS^{-2\cdot5}\,\mathrm{d}S$). The latter

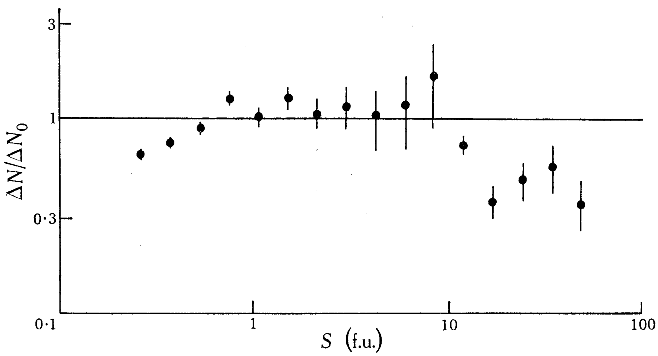

Fig. 3.—Ratio of the present differential counts ΔN, for sources obtained from paper I for $S < 10$ f.u. and paper II for $S > 10$ f.u., relative to the differential counts ΔN_0 expected in a static Euclidean universe.

number has been normalized to give a ratio of unity for the flux interval with base level $F_i = 0\cdot88$ f.u. Because no significant information about the slope of the source counts is obtained above about 40 f.u., where there is a total of only 14 sources, an "equivalent number" has been plotted in the 40–56·6 f.u. interval. This equivalent number represents all the sources stronger than 40 f.u. for an assumed integral slope of $-1\cdot5$, which is the initial slope of all cosmologies, and is given by

$$\Delta N = N(1-\beta^{1\cdot5}), \qquad (7)$$

where ΔN is the number of sources expected with flux densities between S_1 and S_2 ($S_1 < S_2$), N is the total number of sources with flux densities above S_1, and β is the ratio S_1/S_2.

Finally, to compare our corrected counts directly with the original Cambridge counts we have adopted the integral form as used by Pooley and Ryle (1968) and plotted both Cambridge and Molonglo results on the one diagram in Figure 4.

As the Cambridge flux densities were based on the CKL flux density scale, which is approximately 10% lower than the Wyllie scale used for the Molonglo catalogue, they have been increased by the appropriate amount. The error bars shown are equal to $\sqrt{N}$, where N is the total number of sources defining each point, and so are relevant for determining the statistical agreement between the two sets of results but not for determining the reliability of the derived slopes (see e.g. Jauncey 1967).

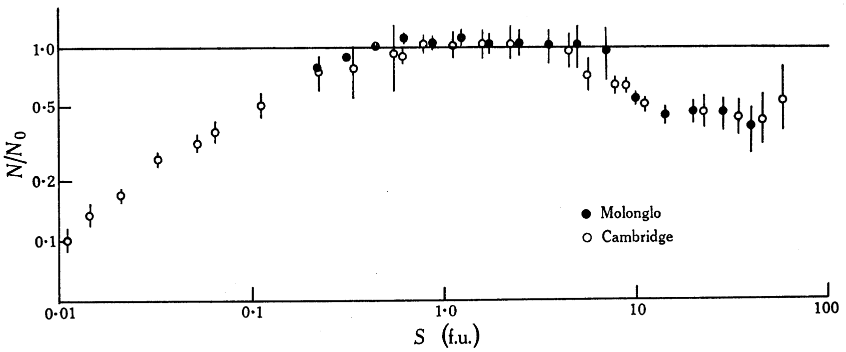

Fig. 4.—Direct comparison of the integral source counts obtained from Molonglo (present results) and Cambridge (Pooley and Ryle 1968). N represents the observed number of sources and N_0 the number expected in a static Euclidean universe.

IV. Discussion

Examination of Figure 4 shows that, when the counts are plotted in integral form, there is no significant difference between the Cambridge and Molonglo results. In fact, the agreement is striking in view of the different sources of the data. Nevertheless, when considering our results in the differential form given in Figure 3 we do not find compelling evidence for source evolution, whereas Pooley and Ryle (1968) invoked evolutionary processes to explain the form of their curve at both high and low flux densities. We do not wish to undertake a detailed analysis of the source counts until further data become available and so our present discussion is confined to some general remarks.

Examination of Figure 3 shows that there is no unique slope that can be fitted to the source counts but that three regions can be recognized: a high flux density region extending down to about 15 f.u., in which the slope cannot be distinguished from the $-1\cdot5$ value expected; an intermediate region between $0\cdot7$ f.u. and about 7 f.u., in which the source density has increased by a factor of about two but again where the slope may be close to $-1\cdot5$; and, finally, a region below $0\cdot7$ f.u., in which the slope is substantially flatter than $-1\cdot5$.

The discrepancy between the high and intermediate flux density regions is a major one and is clearly responsible for the steep slopes found in early surveys. It seems unlikely that this effect arises purely as a result of chance fluctuations in a uniform distribution of sources. For example, the probability that the observed distribution above $2\cdot5$ f.u. arises as a chance fluctuation from a random population of slope $-1\cdot5$ has been estimated from the χ^2 test to be $\sim 0\cdot001$.

Attributing cosmological significance to the anomaly does not seem completely natural because the change in number density appears to occur over a limited flux density range, whereas one would expect evolutionary effects to be spread over a wide range because of the large dispersion in the luminosity distribution. The significance of the apparently sharp change in number density has been investigated by fitting to the data a uniform slope over the range $2 \cdot 5$–40 f.u. A slope of $-1 \cdot 9$ is found to give the best least squares fit and the probability that the observed numbers could arise from a random distribution of this slope is found to be $\sim 0 \cdot 25$. Thus this interpretation is not implausible, and it can be checked further when the statistical reliability is improved by extending the area of the Molonglo catalogue.

An explanation of the anomaly which deserves consideration is that the sources are organized in some form of spatially clustered or hierarchical distribution. The likelihood of chance fluctuations from the $-1 \cdot 5$ slope is then much increased. If present, the clustering cannot be very marked as it has never been positively established by any statistical checks. However, we have noticed a curious distribution among the strong sources which is suggestive of clustering although without real statistical significance: there is a gap of $1 \cdot 8:1$ between the flux densities of the eighth and ninth strongest sources whereas, in a *uniform* distribution, about 11 sources would be expected in this flux density range following the eighth source. Moreover, the eight strongest sources show some grouping on the celestial sphere which appears nonrandom, although not significantly so at a probability level of about $0 \cdot 2$. Thus it does seem possible that most of these eight sources, all radio galaxies, form a local physical grouping.

Suggestions have been made that the anomalous numbers are limited to the sources of highest surface brightness, quasars or the unidentified sources. There does appear to be some recent evidence of a significant difference between the number counts of sources of high and low surface brightness at 1400 MHz (Bridle *et al.* 1972) but we do not have the necessary data to check this at 408 MHz. Also Munro (1971) has found that unidentified sources at 408 MHz show a significantly steeper slope than identified sources but he points out that this effect is expected as a result of the preferential identification of strong sources. Furthermore, there seems little indication of a substantial change in the form of the $\log N$–$\log S$ relation at the highest frequencies, where the proportion of bright sources and quasars is much higher (see e.g. Kellermann *et al.* 1971).

Thus we conclude that the reason for the anomaly at high and intermediate flux densities is unclear. It seems that further statistical tests are unlikely to result in any definitive conclusions, although improved statistical reliability in the region just below 10 f.u. would be helpful and a further investigation of the degree of randomness in the distribution would also be worth while. More promising, however, would be systematic studies of the physical properties and identifications of large numbers of the sources involved.

Pooley and Ryle (1968) also invoked evolution to explain the sharp cutoff they found in the numbers of the faintest sources. The survey in Paper I does not go deep enough to check this conclusion although the qualitative features are confirmed. A deep survey of part of the region of the MC1 catalogue is being currently analysed and it is hoped that this will provide the necessary data.

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VI. References

Bridle, A. H., Davis, M. M., Fomalont, E. B., and Lequeux, J. (1972).—*Nature Phys. Sci.* **235**, 123.
Crawford, D. F., Jauncey, D. L., and Murdoch, H. S. (1970).—*Astrophys. J.* **162**, 405.
Davies, I. M., Little, A. G., and Mills, B. Y. (1973).—*Aust. J. Phys. astrophys. Suppl.* No. 28.
Jauncey, D. L. (1967).—*Nature* **216**, 877.
Kellermann, K. I., Davis, M. M., and Pauliny-Toth, I. I. K. (1971).—*Astrophys. J.* **170**, L1.
Munro, R. E. B. (1971).—*Aust. J. Phys.* **24**, 743.
Pooley, G. G., and Ryle, M. (1968).—*Mon. Not. R. astr. Soc.* **139**, 515.
Robertson, J. G. (1973).—*Aust. J. Phys.* **26**, 000.
Wyllie, D. V. (1969).—*Mon. Not. R. astr. Soc.* **142**, 229.