Properties of the
Lowest \( T = 2 \) State
in \(^{24}\text{Mg}\)

J. C. P. Heggie and H. H. Bolotin
School of Physics, University of Melbourne, Parkville, Vic. 3052.

Abstract
The lowest \( T = 2 \) state in \(^{24}\text{Mg}\) was investigated using the \(^{23}\text{Na}(p, \gamma)^{24}\text{Mg}\) reaction. The excitation energies of this level and of the two lower \( T = 1 \) levels subsequently fed in its \( \gamma \)-ray decay were determined to be \( 15436.4 \pm 0.6, 10711.7 \pm 0.8 \) and \( 9967.5 \pm 0.5 \) keV respectively. Accurate \( \gamma \)-ray decay branching ratios of these three levels were obtained, and a comparison was made with the analogous \( \Delta T = 1 \) transitions in \(^{24}\text{Na}\). From a simultaneous measurement of the radiative strength and the \(^{23}\text{Na}(p, \alpha, \gamma)^{20}\text{Ne}\) strength, partial widths were deduced for the \( \gamma \)-ray, proton and \( \alpha \)-particle decays of the \( T = 2 \) level. The total width of the level was found to be \( 1.02 \pm 0.34 \) keV.

Introduction
In recent years several \( 0^+ \), \( T = 2 \) states in \( T_z = 0 \), \( A > 20 \) nuclei have been located, identified and studied (McGrath et al. 1967, 1970; Riess et al. 1967; Snover et al. 1969; Jelley et al. 1972; Kuan et al. 1972; Szücs et al. 1973; Vernotte et al. 1973). Whilst their particle decays are of interest, as this information provides a sensitive measure of the isospin impurity admixed into these states by charge-dependent forces, it is by virtue of the forbidden nature of these decays that these levels can be observed as narrow resonances in capture reactions and their subsequent \( \gamma \)-ray decay studied.

The lowest \( T = 2 \) state in \(^{24}\text{Mg}\) was first observed in the isospin-allowed reaction \(^{24}\text{Mg}(t, p)^{26}\text{Mg}\) by McGrath and coworkers (McGrath et al. 1967, 1970) at an excitation energy of 15.43 MeV. They established that the principal decay modes were by proton emission to the ground state of \(^{23}\text{Na}\) and by \( \alpha \)-particle decay to the first excited state of \(^{20}\text{Ne}\). Accordingly, it was not surprising that Riess et al. (1967) were able to find the \( T = 2 \) state as a narrow resonance in the reaction \(^{23}\text{Na}(p, \gamma)\) at the same energy as they observed a dip in the \(^{23}\text{Na}(p, p_0)\) yield. Using a large NaI detector they were able to identify two \( \gamma \)-ray decays from resonance leading directly to two \( J^e = 1^+ \), \( T = 1 \) levels. Subsequently, Szücs et al. (1973), employing a high resolution Ge(Li) detector, reported more accurately determined \( \gamma \)-ray branching ratios and excitation energies of the intermediate \( T = 1 \) levels, together with a more precisely established excitation energy of \( 15434 \pm 3 \) keV for the \( T = 2 \) state itself.

The present investigation was undertaken to independently confirm the results of Szücs et al. (1973) and to measure the strengths in the \((p, \alpha_1 \gamma)\) and \((p, p' \gamma)\) channels. Taken in conjunction with the results of McGrath et al. (1970), the simultaneous measurement of the \((p, \gamma)\) and \((p, \alpha_1 \gamma)\) yields allows the total width of the resonance to be ascertained.
Experimental Procedure

The proton beam from the University of Melbourne 5U Pelletron was used to populate the $T = 2$ state in $^{24}\text{Mg}$ via the reaction $^{23}\text{Na}(p, \gamma)$. Targets (2–3 keV thick for 4 MeV protons) were prepared by the evaporation of Na metal or NaOH from tantalum boats onto 0.254 mm (0.01 inch) thick gold backings. Direct air cooling of the gold backings allowed beams of up to 4 $\mu$A to be sustained on target for long periods without significant target deterioration.

Gamma rays produced in the reaction were detected by two Ge(Li) counters having active volumes of 70 and 40 cm$^3$ and resolutions at 1.33 MeV $\gamma$-ray energy of 2.3 and 2.1 keV respectively. The $\gamma$-ray spectra were calibrated using the well-known lines in $^{16}\text{O}$ and $^{15}\text{N}$ of energy 6129.4 $\pm$ 0.2 and 5269.9 $\pm$ 0.3 keV (Endt and van der Leun 1973) resulting from the contaminant reactions $^{19}\text{F}(p, \gamma\gamma)$ and $^{18}\text{O}(p, \gamma\gamma)$ respectively. These calibration lineshapes show the effect of a Doppler shift arising from that portion of the decays which take place while the product nuclei are in flight. However, each line also displays a salient peak corresponding to the unshifted $\gamma$-ray energy; this results from decays which occur only after the nuclei have been brought to rest. Our energy calibration was based wholly on the unshifted peak energies. Digital gain and zero stabilization were employed throughout, and small nonlinearities in the amplifier–ADC systems were accounted for using precision pulse generators. The effect of Doppler shift on the $^{24}\text{Mg}$ $\gamma$-ray lines was accounted for by an appropriate average taken over the detection angle presented by the active volume of the $\gamma$-ray detector in the geometry employed. In addition, this calculational procedure was checked by recording $\gamma$-ray spectra simultaneously at both 0° and 90° to the incident beam direction. A further energy correction was made to account for nuclear recoil during $\gamma$-ray emission. The errors assigned to the $^{24}\text{Mg}$ transition energies incorporate the uncertainties associated with these corrections. The largest source of error was the uncertainty in the determination of the peak centroid.

Excitation functions measured over a limited energy region served to locate the resonance. Subsequently, both on- and off-resonance runs were performed, and level energies and branching ratios were determined from a comparison of these spectra. Finally, using the same target and experimental conditions, the strength of the resonance was extracted by comparing the resulting yield with that obtained from the narrow $^{23}\text{Na}(p, \gamma)$ resonance at 1417 keV.

Results

Fig. 1 illustrates measured excitation functions in the neighbourhood of the $^{24}\text{Mg} T = 2$ resonance for several different channels. With the exception of Fig. 1e, the experimental points in Fig. 1 were obtained from the measured intensities of the appropriate full-energy peaks in the $\gamma$-ray spectra recorded with the 70 cm$^3$ Ge(Li) detector positioned 1.5 cm from the target. Each point corresponds to a total integrated charge of 2000 $\mu$C. The resonance is most clearly evident in Fig. 1e, where the $\gamma$-ray energy window was selected to optimize the $\gamma$-ray yield from the decay of the intermediate $J^e = 1^+$, $T = 1$ levels (Riess et al. 1967; Szücs et al. 1973). It is clear that there is some strength in the $(p, x, \gamma)$ channel (Fig. 1d), confirming the findings of McGrath et al. (1967, 1970), whilst the situation in the $(p, p'\gamma)$ channels (Figs 1a–1c) remains unclear.
Fig. 1. Excitation functions in the neighbourhood of the $T = 2$ resonance in $^{24}$Mg obtained by bombarding $^{23}$Na with protons. The data were obtained using a 70 cm$^3$ Ge(Li) γ-ray detector placed at 55° to the beam direction and are for the indicated reaction channels. The lines drawn between data points only serve to guide the eye.
Fig. 2. Spectra recorded at 0° to the beam in the region of the primary γ-ray transitions from the resonance to the $J^* = 1^+$, $T = 1$ intermediate levels: (a) on resonance, (b) off resonance and (c) the difference between on and off resonance. The brackets designate (from left to right) the positions of the double-escape, single-escape and full-energy peaks of the labelled transitions. The decay from the resonance R to the 10712 keV level is not immediately apparent in (a) but, following suitable background subtraction, it appears as a weak line in (c).
Fig. 3. Spectra recorded at 0° to the beam in the region of the secondary γ-ray transitions arising from the population of the $T = 2$ resonance, showing: (a) those attributed to the resonance, (b) those attributed to a broad underlying resonance $R'$. 

$E_p = 3911$ keV

$E_p = 3906$ keV
Based on the calibration of the beam-analysing magnet and assuming a thick-target yield curve, the resonance is found at a proton energy of $3910 \pm 1$ keV with a total width of $<1.8$ keV. The corresponding excitation energy in $^{24}\text{Mg}$ is $15438.3 \pm 1.8$ keV which is a little higher than, although still in agreement with, our final value of $15436.4 \pm 0.6$ keV obtained from a consideration of all Ge(Li) spectra. The former value is expected to be less reliable because of uncertain amounts of carbon and oxygen on the front face of the target, which have not been corrected for. The latter final value is in excellent agreement with the earlier work of Reiss et al. (1967) and Szücs et al. (1973).

**Table 1. Comparison of level energies and branching ratios**

<table>
<thead>
<tr>
<th>Szücs et al. (1973)</th>
<th>Present work$^a$</th>
<th>Transition $E_l \rightarrow E_f$ (keV)</th>
<th>Branching ratio ($%$) for transition $^{1369}$SzUcs (1973)</th>
<th>Present work</th>
<th>Suggested value$^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15434 ± 2</td>
<td>15436·4±0·6</td>
<td>15436 → 1369</td>
<td>&lt;2</td>
<td>&lt;4·5</td>
<td>&lt;2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9967</td>
<td>83±2</td>
<td>78·9±2·4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10712</td>
<td>17±2</td>
<td>21·1±2·4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>12816</td>
<td>&lt;10±2</td>
<td>12±2</td>
</tr>
<tr>
<td>10711 ± 2</td>
<td>10711·7±0·8</td>
<td>10712 → 0</td>
<td>91±7</td>
<td>73±5</td>
<td>73±5</td>
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<td></td>
<td></td>
<td></td>
<td>1369</td>
<td>&lt;23</td>
<td>27±5</td>
</tr>
<tr>
<td>9967 ± 2</td>
<td>9967·5±0·5</td>
<td>9967 → 0</td>
<td>63±3</td>
<td>65·0±2·0</td>
<td>65±2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1369</td>
<td>32±6</td>
<td>35·0±2·0</td>
</tr>
</tbody>
</table>

$^a$ Present results for level energies and errors are recommended as best available values.  
$^b$ The suggested branching ratios for the 15436 keV level have been renormalized (see text) to account for a 10% branch to the 12816 keV level which could not be observed in the present work or that of Szücs et al. (1973).

Typical examples of on- and off-resonance $\gamma$-ray spectra are shown in Figs 2 and 3. The detector angle with respect to the beam direction was 0°. Decays resulting from the $T = 2$ resonance and from a broad underlying resonance are distinguished by labelling these resonances R and R' respectively in Figs 2 and 3. The primary decay from the resonance level to a state at 10712 keV is not immediately obvious in Fig. 2a because it is masked by lines from the contaminant $^{18}$O$(p, \alpha)$ reaction. However, after suitable background subtraction, this weak transition shows up clearly in Fig. 2c. The present results suggest that the $T = 2$ resonance decays to just two levels at 10712 and 9967 keV; these levels being the analogues of $J^n = 1^+$ levels in $^{24}$Na at 472 and 1347 keV respectively. However, as pointed out by Start et al. (1973), a third weak transition to a level at 12816 keV, which is the analogue of the 3589 keV state in $^{24}$Na, is also expected. This transition was sought but, unfortunately, it was masked by the very strong background from the $(p, p'\gamma)$ reaction and could not be observed in the present work. Furthermore, since the 12816 keV level is itself proton unstable, any subsequent $\gamma$-ray decay from this level would also be too weak to observe. Accordingly, in Table 1, where the results of the present investigations are compared with those of Szücs et al. (1973), we have allowed for a 10% branch from the $T = 2$ resonance to the 12816 keV level based on the strength (Start et al. 1973) of the analogous transition in $^{24}$Na (see Fig. 4).

As seen in Table 1, with the exception of the results for the branching ratios of the 10712 keV level, the agreement between the present work and that of Szücs et al.
(1973) is excellent. Szücs et al. were unable to observe a transition from this level to the 1369 keV state and thus they placed an upper limit of <23% on its intensity. In contrast, this transition was apparent in all our on-resonance spectra, accumulated using either NaOH or Na metal targets and viewed at detector angles of 0° and 90°. The branching ratio of 27%±5% obtained in the present work is not in serious disagreement with the limit of <23% proposed by Szücs et al., but it does imply a considerably different branching ratio for the decay of this level to the ground state.

![Diagram of γ-ray decay modes](image)

**Fig. 4.** Comparison of the γ-ray decay modes of the lowest $T = 2$ states in $^{24}$Na and $^{24}$Mg. Excitation energies are in keV. The branching ratios for $^{24}$Na were taken from Start et al. (1973), while those for $^{24}$Mg have been taken from the present Table 1. The 15436 → 12816 keV transition in $^{24}$Mg (shown dashed) was not observed but is inferred from the analogous transition from the $T = 2$ state in $^{24}$Na.

Fig. 4 presents a comparison of the γ-ray decay of the lowest $T = 2$ states in $^{24}$Mg and $^{24}$Na. The obvious strong similarity in the decay of these states lends considerable weight to the selection rule (first proposed by Warburton and Weneser 1969) that the branching ratios for $\Delta T = \pm 1$ decays of analogue states should be identical.

Using a strength of $30.5 \pm 6.6$ eV (Switkowski et al. 1975) for the $^{23}$Na(p, γ) resonance at 1417 keV and assuming semithick-target yields, the $T = 2$ resonance
strength of
\[ \Gamma_p \Gamma_{\gamma}/\Gamma = 1.95 \pm 0.50 \text{ eV} \]
was obtained from a comparison of the yields of the two resonances measured with
the same target and identical geometry. Coupled with the value of \( \Gamma_p/\Gamma = 0.74 \pm 0.07 \)
obtained by McGrath et al. (1970), a total radiative width of \( \Gamma_{\gamma} = 2.6 \pm 0.6 \text{ eV} \) is
extracted. This value, combined with the suggested branching ratios in Table 1,
yields partial widths of \( 1.8 \pm 0.5, 0.50 \pm 0.13 \) and \( 0.26 \pm 0.07 \text{ eV} \) for the decays
from the resonant state to the levels at 9967, 10712 and 12816 keV respectively
(the latter strength is inferred rather than measured). Under the assumption of pure
M1 character for these transitions, the corresponding transition probabilities in
Weisskopf units are \( 0.53 \pm 0.13, 0.23 \pm 0.06 \) and \( 0.36 \pm 0.09 \) respectively; values
which are typical of \( \Delta T = \pm 1 \) allowed M1 decays in self-conjugate nuclei (Endt
and van der Leun 1974). Furthermore, the branching ratio limit of \(<2\% \) for a
possible \( \Delta T = 2 \) transition from the \( T = 2 \) state to the first excited state at 1369 keV
implies an upper limit of \( 0.05 \text{ eV} \) or 0.03 W.u. for the E2 strength. This low limit is
hardly surprising in view of the expected one-body nature of the electromagnetic
operator which prohibits the change in the total isospin of a nucleus from exceeding
unity.

Finally, in a manner similar to that used to obtain the radiative strength above,
the strength in the \( (p, x_1 \gamma) \) channel was obtained as
\[ \Gamma_p \Gamma_{x_1}/\Gamma = 0.17 \pm 0.04 \text{ keV}. \]
The results of McGrath et al. (1970) can then be used to obtain \( \Gamma_p = 0.75 \pm 0.25, \Gamma_{x_1}
= 0.22 \pm 0.06 \) and \( \Gamma = 1.02 \pm 0.34 \text{ keV} \), the latter value being quite consistent
with the earlier result of \( \Gamma < 1.8 \text{ keV} \) obtained from considerations of the leading
edge of the thick-target yield curve in Fig. 1.

Conclusions

The results of the present investigation have led to a more precise determination
of the energy of the lowest \( T = 2 \) state in \( ^{24}\text{Mg} \). If we use the known masses for the
\( T = 2 \) state in \( ^{24}\text{Na} \) (Wapstra 1971; Start et al. 1973) and the ground state of \( ^{24}\text{Ne} \)
(Silbert and Jarmie 1961), the isobaric mass equation predicts the excitation energy
of the lowest \( T = 2 \) state in \( ^{24}\text{Al} \) and the mass excess of \( ^{24}\text{Si} \) as \( 5955 \pm 12 \) and
10759 \( \pm 34 \text{ keV} \) respectively. The relatively large errors associated with these predictions
arise mainly from the uncertainty in the mass of \( ^{24}\text{Ne} \), which has been measured
to an accuracy of only \( \sim 10 \text{ keV} \) (Silbert and Jarmie 1961) using the reaction
\( ^{22}\text{Ne}(t,p)^{24}\text{Ne} \). It would clearly be advantageous to repeat that experiment using present
day techniques. As yet, no measurements of the masses of the \( T = 2 \) state
of \( ^{24}\text{Al} \) or the ground state of \( ^{24}\text{Si} \) have been made. However, based on the above
predictions, \( ^{24}\text{Si} \) is expected to be proton stable by 3.30 MeV and \( ^{24}\text{Al} \) \( (T = 2) \) to
be proton unstable to levels in \( ^{23}\text{Mg} \) below 4.09 MeV.

The branching ratios for the \( T = 2 \) and the two \( T = 1 \) levels in \( ^{24}\text{Mg} \) have been
measured. With the exception of the decay of the 10712 keV level, for which a previously
unreported branch has been observed in the present work, our results are in
excellent agreement with previously reported measurements. The present results
confirm the selection rule which stipulates that \( \Delta T \geq 2 \) electromagnetic transitions
are not allowed in nuclei. Furthermore, comparisons of these findings with those on the analogous transitions in $^{24}\text{Na}$ support the suggested rule (Warburton and Weneser 1969) that the branching ratios for $\Delta T = \pm 1$ decays of analogue states should be identical.

Finally, the results confirm that proton decay to the $^{23}\text{Na}$ ground state and $\alpha$-particle decay to the first excited state of $^{20}\text{Ne}$ are the principal decay modes of the lowest $T = 2$ level in $^{24}\text{Mg}$. The present work, taken in conjunction with the work of McGrath et al. (1970), has allowed the partial widths $\Gamma_p$ and $\Gamma_{\alpha 1}$ and the total width $\Gamma$ to be extracted.

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References


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