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Dynamic Deformation Theory and Multiphonon Vibrational Bands in ^{154}Gd

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Abstract

Dynamic deformation theory based on a pairing plus quadrupole model (DPPQ model) is extended to 25 levels belonging to various multiphonon vibrational bands in ^{154}Gd. Although the calculated excited band energies are too high by factors of 1·4–1·7, there is generally reasonable agreement with the experimental energies, B(E2) values, magnetic moments, E2–M1 mixing ratios and E0 moments. Many new values are predicted. A brief comparison with some results derived from a new version of dynamic deformation theory, the so-called DNSB model, is also given.

1. Introduction

Recently Sousa *et al.* (1975) and Gupta *et al.* (1977*a*) have extended experimental studies of the nucleus ^{154}Gd to 25 even-parity states lying below 2·3 MeV. As these states include three 0^+, seven 2^+, two 3^+, seven 4^+, two 5^+ and four 6^+ levels, they cannot all be fitted into the three rotation–vibration bands normally considered for even–even nuclei; namely (with spins restricted to $I \leqslant 6$) the g band (0^+, 2^+, 4^+, 6^+), the β band (0^+, 2^+, 4^+, 6^+) and the γ band (2^+, 3^+, 4^+, 5^+, 6^+). For this reason, in the present paper we have attempted to extend microscopic theory of collective motion in order to seek the answers to two questions: (1) Can these 'extra' states be classified into multiphonon bands (two β-vibrational phonons, two γ-vibrational phonons etc.)? (2) Can existing microscopic theory produce results that are in reasonable agreement with the large amount of new experimental data for B(E2) values and branching ratios of transitions connecting these states?

In this study we have employed two different versions of the so-called dynamic deformation theory (Kumar 1978); these versions are designated the dynamic pairing plus quadrupole model (DPPQ) and the dynamic Nilsson, Strutinsky and Belyaev model (DNSB). The former model is the same as that used previously for samarium nuclei (Kumar 1971, 1974) and for gadolinium nuclei (Gupta 1973; Gupta *et al.* 1973, 1977*b*), except that the calculation has been extended here to a much larger number of states (25 compared with only the lowest 11 states previously). The present DNSB model is an improved version of the one used earlier in calculations for the three well-deformed nuclei ^{24}Mg, ^{102}Zr and ^{168}Er (Kumar *et al.* 1977).

A brief comparison of the DPPQ and DNSB methods used here is made in Section 2. The calculated results obtained by these two methods are compared with the experimental data for ^{154}Gd in Section 3. A few of the DPPQ results presented in Section 3 (less than 1%) have been published previously (Gupta *et al.* 1977*b*) but

the DNSB results are all new. The conclusions to be drawn from the study are given in Section 4.

2. Comparison of DPPQ and DNSB Methods

The five main steps of a calculation based on the dynamic deformation theory are as follows (see Kumar 1975 and Kumar *et al.* 1977 for details and definitions).

(1) Calculation of the single-particle basis. In addition to the single-particle energies, one needs the matrix elements of two quadrupole operators (for subsequent calculation of the mass parameters for β, γ and $\beta\gamma$ vibrations; and for the calculation of the E2 moments), three angular momentum operators (for the three moments of inertia), three spin operators (these together with the angular momentum matrix elements are needed for the calculation of the M1 moments) and one radius operator (for the E0 moments).

(2) Calculation of the U, V factors of the pairing theory.

(3) Calculation of the collective potential energy function $V(\beta, \gamma)$.

(4) Calculation of six inertial functions $B_{\beta\beta}(\beta, \gamma)$, $B_{\beta\gamma}(\beta, \gamma)$, $B_{\gamma\gamma}(\beta, \gamma)$, $\mathcal{I}_1(\beta, \gamma)$, $\mathcal{I}_2(\beta, \gamma)$ and $\mathcal{I}_3(\beta, \gamma)$, and six moment functions $Q_0(\beta, \gamma)$, $Q_2(\beta, \gamma)$, $g_1(\beta, \gamma)$, $g_2(\beta, \gamma)$, $g_3(\beta, \gamma)$ and $r^2(\beta, \gamma)$.

(5) Solution of the collective Schrödinger equation based on Bohr's (1952) collective hamiltonian for the energy levels and wavefunctions.

All these five steps in the DPPQ calculation have been modified for the present DNSB calculation. The most important differences are in steps 1 and 3, but the other differences are also listed below.

Step 1. In the DPPQ method, the single-particle field is identified with the Hartree field due to the quadrupole ($J = 2$, $T = 0$) component of the nucleon–nucleon force. Since the exchange part of the quadrupole force is neglected, it is possible to write the average potential analytically and it is not necessary to perform a Hartree type of iteration. However, such a quadrupole field leads to instability against large deformations, if a large configuration space is employed. Hence, the configuration space has to be limited to two major shells near the Fermi surface. In the DNSB method, a Nilsson (1955) type of anisotropic oscillator potential is employed to calculate the deformed single-particle basis; no attempt is made to relate this potential to a nucleon–nucleon force. In the DNSB version of the Nilsson method, the parameters of the potential are determined in such a way that the single-particle wavefunctions are independent of Z and A (Kumar *et al.* 1977). A large configuration space including $\mathcal{N} = 0$–8 major shells is employed. The various matrix elements required for step 1 had been computed previously and stored on magnetic tape.

Step 2. The modification in the DNSB method for the calculation of the U, V factors arises not from the difference in approach of the two methods, but from an improvement in the pairing theory. This improvement comes about by including the particle–hole matrix elements of the pairing force on the same footing as the particle–particle matrix elements (Kumar *et al.* 1977). It removes the divergence in the moments of inertia and the mass parameters in a situation when two single-particle levels cross the Fermi surface and the energy gap vanishes.

Step 3. In the DPPQ method, the potential energy of deformation is calculated via an expression obtained in the time-dependent Hartree–Bogolyubov treatment of

the pairing plus quadrupole hamiltonian; this is an approximation to the self-consistent Hartree–Fock approach. On the other hand, in the DNSB method the potential energy of deformation is calculated via the Strutinsky (1966) method; that is, instead of simply adding the energies of the occupied single-particle states, one calculates the potential energy as a sum of two parts: a macroscopic part arising from the deformation of the liquid drop or droplet (representing the contribution of single-particle levels far from the Fermi surface), and a microscopic part attributed to the nonuniform energy distribution of single-particle levels. While the liquid drop model (as parameterized by Seeger and Howard 1975) was employed in the first DNSB calculations (Kumar 1977; Kumar *et al.* 1977), the improved droplet model of Myers and Swiatecki (1974) and Myers (1976) has been employed in a recent calculation (Kumar 1978) and in the present one.

Step 4. Expressions for the six inertial functions and the six moment functions are obtained in the DPPQ model via the time-dependent Hartree–Bogolyubov treatment, and in the DNSB model via the cranking plus pairing method of Belyaev (1959). Although both methods give identical expressions, the final results are quite different because of different configuration spaces and also different pairing theories.

Step 5. The DPPQ calculation employs the Kumar (1971) version of the Kumar and Baranger (1967) method of solving the collective Schrödinger equation, while the DNSB calculation employs the Kumar (1979) version of this same method, in which equation (3.159C) of Kumar (1975) is modified to obtain better wavefunction convergence.

As a final comparison between the two methods, we consider the parameters required for each calculation. In the DPPQ method there are four parameters: namely the strength of the quadrupole force, whose value is determined by fitting the ratio E_{4+}/E_{2+}; a renormalization factor, which multiplies all six inertial functions and whose value is determined by fitting the energy E_{2+}; an effective charge parameter, whose value is determined by fitting the $B(\mathrm{E2}; 0^+ \rightarrow 2^+)$ value; and a renormalization factor for the three gyromagnetic ratio functions, whose value is determined by fitting the magnetic moment μ_{2+}. These parameters can be identical for several nuclei of the same mass region, but sometimes they are quite different (Kumar 1971, 1974). Furthermore, one must search for the 'best' spherical single-particle energies, for each mass region. In contrast to the DPPQ requirements, the DNSB method is almost parameter free. It was shown previously that the same single-particle basis and parameters could be used for well-deformed nuclei ranging from ^{24}Mg to ^{168}Er (Kumar 1977; Kumar *et al.* 1977). Subsequently, in the case of the transitional germanium nuclei, only one parameter (the Strutinsky width parameter) was required to be varied from nucleus to nucleus (Kumar 1978). Now, in a more recent study of nuclei ranging from ^{12}C to ^{240}Pu (Kumar 1979), even this parameter freedom has been removed. Thus no parameter has been varied to fit the properties of ^{154}Gd in the DNSB results reported in Section 3 below.

3. Results

The level characteristics for ^{154}Gd as calculated by the present theoretical methods are given in Tables 1–12, where they are compared with the available experimental data. Brief comments on the results and their consistency with the data follow.

Table 1. K structures and level energies of positive parity states in ^{154}Gd

Classification Band	K	I	K component (%) $K=0$	$K=2$	$K=4$	Level energy (MeV) Exp.[A]	DPPQ	DNSB
g	0	0	100	—	—	0·0	0·0	0·0
		2	99·9	0·1	—	0·123	0·126	0·167
		4	99·81	0·15	0·04	0·371	0·313	0·450
		6	99·36	0·60	0·02	0·718	0·585	0·821
β	0	0	100	—	—	0·680	0·985	0·528
		2	99·0	1·0	—	0·815	1·180	0·725
		4	96·3	3·6	0·1	1·047	1·391	1·155
		6	93·0	6·7	0·3	1·366	1·680	1·705
γ	2	2	1·6	98·4	—	0·996	1·506	0·604
		3	—	100	—	1·128	1·603	0·729
		4	3·8	95·5	0·7	1·264	1·776	0·888
		5	—	99·5	0·5	1·433	1·896	1·052
		6	6·5	89·2	4·2	1·607	2·176	1·275
2β	0	0	100	—	—	1·295	1·842	1·093
		2	81·3	18·7	—	1·418	2·156	1·463
		4	69·6	21·9	8·5	1·698	2·49	2·035
$\beta\gamma$	2	2	21	79	—	1·531	2·522	1·210
		3	—	100	—	1·661	2·687	1·527
		4	8	62	30	1·790	3·082	1·624
2γ	4	4	22·5	14·0	63·5	1·646	2·843	1·398
		5	—	56	44	1·770	2·986	1·619
		6	27·3[B]	48·8[B]	20·8[B]	1·912	3·398	1·870
2γ	0	0	100	—	—		2·723	1·434
		2	95·7	4·3	—	2·081[C]	3·042	1·635
		4	94·5	4·3	1·2	2·230[C]	3·367	1·941
3β	0	0	100	—	—		2·969	2·233
		2	44	56	—	2·277[C]	3·317	2·762
		4	38·4	40	21		3·620	3·441
$\gamma+2\beta$	2	2	36	64	—		3·903	2·096
		3	—	100	—		3·749	2·481
		4	36·7	58·9	4·4		4·585	2·852
$2\gamma+\beta$	4	4	16	4	79		4·263	

[A] Experimental values from Sousa *et al.* (1975) and Gupta *et al.* (1977*a*).
[B] Remaining contribution belongs to the $K=6$ component.
[C] See text for discussion of these assignments.

Table 2. DPPQ calculated average shapes, intrinsic quadrupole moments and g values for levels in ^{154}Gd

Classification Band	K	I	Shape factors β_{rms}	γ_{rms}	Q (e b)	g value (nm)
g	0	0	0·262	13·7°	—	—
		2	0·270	13·2	6·29	0·42
		4	0·275	12·9	6·35	0·41
		6	0·284	12·5	6·45	0·41
β	0	0	0·279	10·4	—	—
		2	0·289	9·9	6·18	0·38

Table 2 (*Continued*)

Classification Band	K	I	Shape factors β_{rms}	γ_{rms}	Q (e b)	g value (nm)
β	0	4	0·310	9·8	6·45	0·37
		6	0·327	10·0	6·64	0·35
γ	2	2	0·255	21·9°	5·68	0·43
		3	0·265	21·0	—	0·42
		4	0·274	20·0	6·13	0·42
		5	0·278	19·7	6·07	0·43
		6	0·293	18·5	6·26	0·42
2β	0	0	0·286	2·1	—	—
		2	0·317	6·4	5·97	0·37
		4	0·317	9·3	5·55	0·37
$\beta\gamma$	2	2	0·307	14·5	4·79	0·42
		3	0·297	16·8	—	0·42
		4	0·317	17·8	3·9	0·40
2γ	4	4	0·300	17·5	4·32	0·39
		5	0·293	19·6	4·01	0·41
		6	0·344	14·4	7·31	—
2γ	0	0	0·305	7·8	—	—
		2	0·265	24·4	5·48	0·42
		4	0·281	23·4	5·59	0·42
3β	0	2	0·318	11·7	15·2	0·43
		4	0·325	12·7	3·38	0·39
$\gamma+2\beta$	2	2	0·330	14·1	7·13	0·45
		3	0·336	15·1	—	0·42
		4	0·345	15·3	8·2	—
		5	0·330	16·8	5·4	—
$2\gamma+\beta$	4	4	0·315	20·6	6·68	0·43

Table 3. Absolute B(E2) values, B(E2) ratios and g values for levels in ^{154}Gd

Note that here and in the following tables, the errors in the experimental values are given by number in parentheses which are the uncertainties in the corresponding last digits, e.g. 3·85(15) is to be read as 3·85±0·15 while 0·015(4) means 0·015±0·004

Transition I_i	I_f	B(E2) values (e^2 b^2) Exp.[A]	DPPQ	DNSB	Transition I_i	I_f	B(E2) values (e^2 b^2) Exp.[B]	DPPQ	DNSB
0_g	2_g	3·85(15)	3·86	3·32	0_g	$2_{\beta\gamma}$		0·0002	0·007
0_g	2_γ	0·143(11)	0·139	0·132	4_g	2_g	1·178(39)	1·162	1·086
0_g	2_β	0·015(4)	0·019	0·020	6_g	4_g	1·376(60)	1·344	1·379
0_g	$2_{2\beta}$		0·007	0·027					

Transition ratio	B(E2) ratios Exp.[B]	DPPQ[C]	DNSB	g-band level	g values (nm) Exp.[D]	DPPQ	DNSB
$(4_g\rightarrow2_g)/(2_g\rightarrow0_g)$	1·52(10)	1·51	1·64	2_g	0·427(114)	0·42	0·49
$(6_g\rightarrow4_g)/(4_g\rightarrow2_g)$	1·17(8)	1·16	1·27	4_g		0·41	0·50

[A] Experimental results from Ronningen *et al.* (1977).
[B] Experimental results from Ward *et al.* (1975)
[C] The rotational model values for these B(E2) ratios are 1·41 and 1·10 respectively.
[D] Experimental g value from Ben-Zvi *et al.* (1970).

Table 4. Absolute B(E2) values for $\beta\rightarrow$g band transitions in ^{154}Gd

Transition		B(E2) values ($10^{-2}e^2b^2$)[A]				Transition		B(E2) values ($10^{-2}e^2b^2$)[A]	
I_β	I_g	Exp. (*a*)	Exp. (*b*)	Exp. (*c*)	DPPQ	I_β	I_g	Exp. (*a*)	DPPQ
0	2	21(3)	31		22	4	2	0·35(8)	0·60
2	0	0·48(4)	0·48(4)	0·30(8)	0·37	4	4	3·8(6)	3·37
2	2	4·0(4)	4·0		3·3	4	6	11·9(25)	8·57
2	4	11·9(8)	12·0		8·66	6	4	0·27(10)	0·09
						6	6	3·3(10)	3·52

[A] Experimental values: (*a*) deduced from the branching ratios given by Rud *et al.* (1971) and the B(E2; $0_g\rightarrow 2_\beta$) value from Riedinger *et al.* (1969); (*b*) from Riedinger *et al.* (1969); (*c*) from Hamilton (1976).

Table 5. B(E2) ratios for transitions from β and γ bands in ^{154}Gd

Transition ratio		Exp. E_γ (keV)	B(E2) ratios		
I_i	I_f/I_f'	ratio	Exp.[A]	RV[B]	DPPQ
2_β	$4_g/2_g$	444·4/692·4	2·75(8)[a]	1·80	2·61
	$0_g/2_g$	815·6/692·4	0·121(4)[a]	0·70	0·11
	$0_g/4_g$	815·6/444·4	0·045(4)[a]	0·39	0·043
4_β	$6_g/4_g$	329·5/676·6	2·38(8)[a,b]	1·75	2·54
	$2_g/4_g$	924·6/676·6	0·086(3)[a]	1·10	0·18
	$2_g/6_g$	924·6/329·5	0·032(2)[a]	0·63	0·07
6_β	$4_g/6_g$	995/648	0·08(3)[c]		0·024
2_β	$0_\beta/0_g$	134·8/815·5	125(6)[d]		205
4_β	$2_\beta/2_g$	232·2/924·6	410(25)[a]		200
2_γ	$4_g/2_g$	625·2/873·2	0·144(5)[a,e]	0·05	0·09
	$0_g/2_g$	996·3/873·2	0·46(1)[a]	0·70	0·56
	$0_g/4_g$	996·3/625·2	3·2(2)[a,e]	13·9	6·3
	$0_\beta/0_g$	315·6/996·3	0·140(7)[d]		0·03
	$2_\beta/2_g$	180·7/873·2	1·03(23)[d]		1·47
	$0_\beta/2_\beta$	315·6/180·7	0·063(14)[d]		0·01
3_γ	$2_g/4_g$	1004·8/756·9	1·06(4)[a]	2·5	1·41
	$2_\beta/2_g$	312·3/1004·8	0·289(13)[d]		0·035
	$4_\beta/4_g$	80·4/756·9	50(25)[d]		1·6
	$4_\beta/2_\beta$	80·4/312·3	182(91)[d]		34
	$2_\gamma/2_g$	131·6/1004·8	17(1)[d]		26
4_γ	$6_g/4_g$	545·6/892·7	0·27(4)[a,f]	0·09	0·37
	$2_g/4_g$	1140·9/982·7	0·14(1)[a]	0·34	0·32
5_γ	$4_g/6_g$	1061·2/714·6	0·74(15)[e]	1·75	0·78
6_γ	$4_g/6_g$	1235·6/888·8	0·08(2)[e]	0·27	0·14

[A] Experimental ratios: (*a*) weighted average of the values given by Meyer (1968) and Riedinger *et al.* (1970); (*b*) Meyer's (1968) value of 5·91 was in error (Gupta *et al.* 1977*a*); (*c*) Rud *et al.* (1971); (*d*) Zolnowski *et al.* (1971); except that they gave the reciprocal ratio by mistake for B(E2; $3_\gamma\rightarrow 4_\beta/4_g$); (*e*) Sousa *et al.* (1975); cf. their values of 0·114(22) for B(E2; $2_\gamma\rightarrow 4_g/2_g$) and 4·13(83) for B(E2; $2_\gamma\rightarrow 0_g/4_g$); (*f*) intensity of the 545·6 keV γ ray given by Gupta *et al.* (1977*a*).

[B] Calculated values from the rotation–vibration model.

Level energies and wavefunctions

The percentage of the K components in the wavefunctions and the calculated level energies from both theoretical models are given in Table 1. The levels have been grouped into different 'rotational' bands according to the largest K components and E2 decay characteristics (discussed below). The comparison with the experimental values shows that the DPPQ calculated excitation energies of the vibrational bands are too high by factors of 1·4–1·7. On the other hand, in the DNSB calculation the vibrational bandhead energies are too low by 0·1–0·4 MeV, but the agreement improves for the higher spin band members.

Average shapes, intrinsic quadrupole moments and gyromagnetic ratios

The characteristic β_{rms}, γ_{rms}, intrinsic quadrupole moments Q and gyromagnetic ratios μ_I/I (g values) as calculated by the DPPQ method for the various bands are given in Table 2. The quantities β_{rms} and γ_{rms} provide rough measures of the average nuclear shape when the nucleus is in the state (α, I) and are defined via the equations

$$\langle \alpha, I \,|\, \beta^2 \,|\, \alpha, I \rangle = \beta_{\text{rms}}^2, \qquad \langle \alpha, I \,|\, \beta^3 \cos 3\gamma \,|\, \alpha, I \rangle = \beta_{\text{rms}}^3 \cos 3\gamma_{\text{rms}}. \tag{1}$$

B(E2) *values for g-band transitions*

The calculated absolute B(E2) values, B(E2) ratios and g values involving the ground (g) band are compared with the experimental results in Table 3. In each case the calculated DPPQ values agree with experiment within the experimental errors. There is not such good agreement with the DNSB values, although the results are still reasonable, particularly considering that no effective charge has been included in the calculations and that no theoretical parameter has been varied to fit the properties of ^{154}Gd.

B(E2) *values, B*(E2) *ratios and g values*

The calculated absolute B(E2) values for $\beta \rightarrow$ g band transitions are given in Table 4, which shows that the DPPQ method gives reasonable agreement with experiment up to the highest spins ($I = 6$) considered in this study. A similar comparison for the B(E2) ratios for transitions from the β and γ bands is given in Table 5. For β-band decay, the agreement with experiment is good for the 2_β and 4_β states, while it is fair for the 6_β state; the large ratios for intraband ($\beta \rightarrow \beta$) to interband ($\beta \rightarrow$ g) transitions are also reproduced. For γ-band decay, the experimental values for $\gamma \rightarrow$ g transitions are reproduced within a factor of 2, but the comparison is not as good as this for $\gamma \rightarrow \beta$ transitions. The largest discrepancy occurs for the ratio $(3_\gamma \rightarrow 4_\beta)/(3_\gamma \rightarrow 4_g)$. However, in this case the $3_\gamma \rightarrow 4_\beta$ transition has an energy of only 80 keV and a relative intensity of $0{\cdot}01 \pm 0{\cdot}01$ (Meyer 1968) and it would be hard to resolve in experiment from the strong Compton background. (Note that the B(E2) ratio for $(3_\gamma \rightarrow 4_\beta)/(3_\gamma \rightarrow 4_g)$ reported by Zolnowski *et al.* (1971) is mistakenly given as the reciprocal ratio.)

For the higher lying bands considered below, both the experimental γ-ray energies and intensities used to derive the B(E2) ratios are given for the sake of clarity (Tables 6–9).

Table 6. B(E2) ratios for transitions from third $K^\pi = 0^+(2\beta)$ band in ^{154}Gd

Transition ratio I_i	I_f/I_f'	Exp. I_γ ratio[A]	Exp. E_γ (keV) ratio	B(E2) ratios Exp.[A]	DPPQ
$2_{2\beta}$	$0_g/2_g$	0·02/0·01	1418·4/1295·5	>1·4[B]	29·5
	$4_g/2_g$	0·13/0·01	1047·4/1295·5	>38[B]	54
	$0_\beta/2_\beta$	0·021/0·096	737·7/602·8	0·06(1)	0·01
	$4_\beta/2_\beta$	0·015/0·096	371/602·8	2·0(7)	2·7
	$0_\beta/0_g$	0·021/0·02	737·7/1418·4	26(5)	0·6
	$2_\beta/2_g$	0·096/0·01	602·8/1295·5	450[B]	2210
	$2_\beta/2_\gamma$	0·096/0·003	602·8/422·1	>4·8	4·4
	$2_\gamma/2_g$	0·003/0·01	422·1/1295·5	>96[B]	505
	$4_\beta/4_g$	0·015/0·13	371/1047·4	20(5)	110
	$0_{2\beta}/0_\beta$	0·02/0·021	125·4/737·7	$6(3)\times10^3$	8×10^3
	$0_{2\beta}/2_g$	0·02/0·01	125·4/1295·5	$2·4\times10^3$ [B]	$1·4\times10^4$
$4_{2\beta}$	$4_\beta/4_g$	0·03/0·005	650·6/1327	>176[C]	$1·3\times10^5$

[A] Experimental ratios from the I_γ results of Meyer (1968), except for I_γ(1295·5 keV) = 0·01 (cf. 0·026 from Meyer) and I_γ(737·7 keV) = 0·021(3) from Gupta (1973).
[B] A 75% M1 content in the $2_{2\beta}\rightarrow2_g$ transition γ ray of 1295·5 keV (see Table 11) would increase these ratios four times.
[C] The 1327 keV transition is predicted to be pure M1 (E2 ~ 0·25%), which would increase the B(E2) ratio to 7×10^4.

Table 7. B(E2) ratios for transitions from second $K^\pi = 2^+(\beta\gamma)$ band in ^{154}Gd

Transition ratio I_i	I_f/I_f'	Exp. I_γ ratio[A]	Exp. E_γ (keV) ratio	B(E2) ratios Exp.[A]	DPPQ
$2_{\beta\gamma}$	$0_g/2_g$	0·017/0·06	1531·4/1408·1	0·17(4)	0·013
	$4_g/2_g$	0·124/0·06	1160·0/1408·1	4·86(73)	0·09
	$0_\beta/2_\beta$	0·65/0·5	850·6/715·8	0·55(2)	1·0
	$4_\beta/2_\beta$	0·014/0·5	483·7/715·8	0·20(5)	1·6
	$3_\gamma/4_\gamma$	0·08/0·04	403·5/267·4	0·25[B]	0·95
	$0_\beta/0_g$	0·65/0·017	850·6/1531·4	713(51)	550
	$2_\beta/2_g$	0·5/0·06	715·8/1408·1	246(36)	7·1
$3_{\beta\gamma}$	$4_g/2_g$	0·032/0·14	1290·0/1537·8	0·55(13)	1·9
	$4_\beta/2_\beta$	0·26/1·55	613·3/845·4	0·84(4)	0·7
	$2_\beta/2_g$	1·55/0·14	845·4/1537·8	219(11)	375
	$4_\beta/4_g$	0·26/0·032	613·3/1290·0	333(19)	134
	$4_\gamma/2_\gamma$	0·087/0·082	397·1/664·7	13·6(8)[C]	13
	$4_\gamma/4_g$	0·087/0·032	397·1/1290·0	943(53)[C]	350
	$4_\beta/4_\gamma$	0·26/0·087	613·3/397·1	0·35(2)	0·38
	$2_\beta/2_\gamma$	1·55/0·082	845·4/664·7	5·7(3)	7·25
	$2_\gamma/2_g$	0·082/0·14	664·7/1537·8	38·4	51
$4_{\beta\gamma}$	$2_g/4_g$	0·006/0·01	1667·3/1419·2	0·26(5)	0·30
	$6_g/4_g$	⩽0·11/0·01	1072·2/1419·2	⩽5·7	0·8

[A] Experimental ratios from the I_γ results of Meyer (1968) and the B(E2) values from Zolnowski *et al.* (1971), except that Meyer's incorrect result for I_γ(1419·2 keV) has been replaced by Gupta's (1973) value. Note also that Meyer obtained his I_γ results for the 483·7 and 1531·4 keV transitions by peak shape fitting, but these transitions were not observed by Sousa *et al.* (1975).
[B] The 267·4 keV γ ray was assigned by Meyer (1968) to two alternative transitions, namely $2\rightarrow4$ and $4\rightarrow2$.
[C] If the 397·1 keV γ ray is from a doublet (Meyer 1968), the B(E2) ratio will be reduced.

Third $K^\pi = 0^+(2\beta)$ band

As can be seen from Table 6, the preferential decay from the 2β band to the one-phonon β band rather than to the γ band or the g band is reproduced qualitatively by the present DPPQ calculations, but the B(E2) ratios to different members of the β and g bands agree with experiment in some cases and not in others. The $2_{2\beta} \rightarrow 2_g$ (1295 keV) transition is only 25% E2 and the $4_{2\beta} \rightarrow 4_g$ (1327 keV) transition is almost pure M1, according to our calculated values for δ(E2/M1) (see Table 11; note that these transitions are too weak for an angular correlation experiment). Now, in ^{156}Gd the $2_{2\beta} \rightarrow 2_g$ transition is known to be only 13 ± 3 % E2 (Hamilton 1976) and the calculated $4_{2\beta} \rightarrow 4_g$ transition is almost pure M1 (Gupta *et al.* 1977*b*). Thus the large M1 components predicted for the similar transitions in ^{154}Gd appear to be quite reliable. If such large M1 components had been taken into account in Table 6, the agreement between theory and experiment would have been much better.

From the above results we can note that the $I_\beta \rightarrow I_g$ transitions are predominantly E2 in both ^{154}Gd and ^{156}Gd while the $I_{2\beta} \rightarrow I_g$ transitions have large M1 components, and this difference between the two $K^\pi = 0^+$ vibrational bands is reproduced correctly by the dynamic deformation theory. However, although the bandhead energy of the 2β band for ^{154}Gd is almost twice that of the β band and a preferential decay to the β band is observed, a two-phonon description of this band does not provide a complete picture, for the energy ratio differs considerably as we move through the gadolinium nuclei: it is 1·7, 1·9 at $N = 88, 90$ but only 1·1, 1·2 at $N = 92, 94$ (Sakai and Rester 1977).

Second $K^\pi = 2^+(\beta\gamma)$ band

The 2^+ state at 1531·4 keV decays primarily to the 0^+ and 2^+ members of the β band. The calculated B(E2) branching ratio agrees qualitatively with experiment, as can be seen from Table 7. Meyer (1968) obtained the experimental intensity of the 483·7 keV $2_{\beta\gamma} \rightarrow 4_\beta$ transition by a peak shape fit, but this transition has not been observed in later experiments. The relative intensity of the $2_{\beta\gamma} \rightarrow 2_g$ (1408 keV) transition obtained by Sousa *et al.* (1975) was 0·17 compared with the value of 0·06 from Meyer (1968).

None of the experimental B(E2) ratios given in Table 7 have been corrected for possible M1 admixtures and hence they cannot be properly compared with the theoretical values at present. However, the $\beta\gamma(2^+)$ character of the 2^+ and 3^+ members of the band is well supported by both the theoretical and experimental values in Table 7. These states decay predominantly to the β band or the γ band and rarely to the g band. The assignment of the 4^+ member is somewhat tentative because, although the $4_{\beta\gamma} \rightarrow 4_g$ transition has been confirmed in coincidence experiments (Gupta *et al.* 1977*a*), only an upper limit has been obtained for the intensity of the $4_{\beta\gamma} \rightarrow 6_g$ transition.

First $K^\pi = 4^+(2\gamma)$ band

Sousa *et al.* (1975) determined the B(E2) ratios for transitions to the g band in ^{154}Tb decay and found poor agreement with the values predicted by a rotation–vibration model. First-order mixing with the γ band improved the model values but the need for a more complicated mixing was evident. Considerable band mixing with sizable $K = 0$ and 2 contributions is predicted by the present theory (Table 1). The

Table 8. B(E2) ratios for transitions from first $K^\pi = 4^+(2\gamma)$ band in ^{154}Gd

Transition ratio I_i	I_f/I_f'	Exp. I_γ ratio	Exp. E_γ (keV) ratio	B(E2) ratios Exp.[A]	RV[B]	DPPQ
$4_{2\gamma}$	$3_\gamma/2_\gamma$	0·56/1·0	518·0/649·5	1·72(17)	0·56	1·3
	$4_\gamma/2_\gamma$	0·09/1·0	382·1/649·5	1·33(22)	0·20	1·0
	$2_\beta/4_\beta$	0·06/0·13	830·4/598·2	0·10(3)		1·6
	$3_\gamma/4_\gamma$	0·56/0·09	518·0/382·1	1·30(22)	2·8	1·3
	$2_g/4_g$	0·03/0·03	1522·8/1274·7	0·43(15)		0·8
	$6_g/4_g$	0·03/0·03	928·2/1274·7	4·9(16)		0·6
	$2_\gamma/2_\beta$	1·0/0·06	649·5/830·4	56·5(80)		13
	$2_\gamma/2_g$	1·0/0·03	649·5/1522·8	2220		44
	$4_\gamma/4_\beta$	0·09/0·13	382·1/598·2	6·6(10)		20
	$4_\gamma/4_g$	0·09/0·03	382·1/1274·7	750		30
$5_{2\gamma}$	$4_\gamma/3_\gamma$	3·1/3·5	506·4/642·3	2·82(56)	1·0	2·1
	$5_\gamma/3_\gamma$	1·6/3·5	337·9/642·3	7·07(216)	0·49	2·8
	$4_\gamma/4_\beta$	3·1/1·9	506·4/722·5	9·1(40)		1·6
	$4_4/4_\gamma$	2·5/3·1	124·4/506·4	850		17
$6_{2\gamma}$	$6_\gamma/5_\gamma$	0·37/1·0	304·8/479·2	3·5(8)	0·59	1·6

[A] Experimental ratios from Sousa *et al.* (1975).
[B] Calculated values from rotation–vibration model.

Table 9. B(E2) ratios for transitions from possible $K^\pi = 0^+(2\gamma)$ band in ^{154}Gd

Transition ratio I_i	I_f/I_f'	Exp. I_γ ratio	Exp. E_γ (keV) ratio	B(E2) ratios Exp.[A]	DPPQ
$2_{2\gamma}$	$3_\gamma/2_\gamma$	1·0/0·66	953·1/1084·3	2·9	5·0
	$4_\beta/2_\beta$	0·83/0·14	1033·3/1265·3	16	159
	$2_\gamma/2_\beta$	0·66/0·14	1048·3/1265·3	10·2	690
$4_{2\gamma}$	$2_g/4_g$	0·6/0·79	2106·9/1858·8	0·4	1·9
	$3_\gamma/2_\gamma$	1·0/0·43	1101·9/1234·0	4	56
	$3_\gamma/2_g$	1·0/0·6	1102/2107	42	15
	$3_\gamma/4_g$	1·0/0·79	1102/1859	17	30

[A] Experimental ratios derived for the 2081·0 and 2230 keV levels from Sousa *et al.* (1975), with the present assignments of $2^+_{2\gamma}$ and $4^+_{2\gamma}$ respectively to these levels.

calculated B(E2) ratios given in Table 8 show reasonable agreement for most transitions. A preferential decay to the γ band rather than the β band is indicated, but there is also significant decay to the g band. This type of decay reminds one of the Davydov–Filippov (1958) model, where the lowest $K^\pi = 0^+, 2^+, 4^+, \ldots$ bands arise from the rotation of an asymmetric top. Indeed, it may be noted here that such a decay pattern is also allowed in the present theory since, although we have adopted the Bohr–Mottelson (1953, 1975) classification of bands in terms of β and γ phonons, we have not taken the nucleus to be an axially symmetric rotor with β–γ vibrations but rather, via the collective Schrödinger equation, have placed no restraints on the shape that the nucleus may assume.

Fourth $K^\pi = 0^+(2\gamma)$ band

Meyer (1968) proposed that two states at 1838·0 and 2093·8 keV were the 2^+ and 4^+ members of a $0^+(2\gamma)$ band. However, this classification was not confirmed in the

later experiments by Sousa *et al.* (1975), who instead proposed several levels with $I^\pi = 1^+, 2^+, 3^+$ and 4^+ in the 2·0–2·4 MeV region. The lowest of these, the 2081·0 keV level, was established through energy fits of decays to 2_γ and 3_γ states and possible decays to 2_β and 4_β states. We have assigned this level to be $2^+_{2\gamma}$ on the basis of a comparison between the calculated and experimental B(E2) ratios (see Table 9); in particular the fact that it decays preferentially to the 2_γ level rather than to 2_β. For similar reasons, we have assigned the 2230 keV level, which was established by Sousa *et al.* through coincidences for transitions to 2_g, 4_g and 3_γ states with a weak transition to the 2_γ state, to be a $4^+_{2\gamma}$ level.

Fifth $K^\pi = 0^+(3\beta)$ band

Sousa *et al.* (1975) proposed the 2119·7 and 2187·2 keV levels as $I^\pi = 1^+$ and 2^+ and the 2266, 2277 and 2305 keV levels as $I^\pi = 2^+, 3^+$ and 4^+, on the basis of the observed appropriate coincidences, except for the 2266 keV level which was proposed on the basis of its energy. Excluding the first two levels which we regard as belonging to a $K^\pi = 1^+$ band, we propose the latter three states as possible members of a $0^+(3\beta)$ band. The experimental B(E2) ratios for decay from these levels are compared in Table 10 with those corresponding to decay from the calculated 2^+ and 4^+ states (classified as 3β-band members in Table 1). From the relatively better agreement for the experimental 2277 keV level with the calculated 2^+ state we assign this level as $2^+_{3\beta}$. However, the absence of a strong transition from the 2277 keV level to a 2β-band member makes the present assignment somewhat tentative.

E2/M1 *mixing ratios*

The mixing ratios δ(E2/M1) derived from the calculated E2 and M1 matrix elements and the experimental transition energies are compared with experiment in Table 11. The general feature that $\beta\rightarrow$g and $\gamma\rightarrow$g transitions are largely E2 even in the case of transitional nuclei is shown by both the theoretical and experimental results. The DPPQ calculated δ values for $\gamma\rightarrow$g transitions are too large (or the calculated M1 components are too small), but the signs and the general trends are given correctly. This is encouraging, and we have therefore also included in Table 11 theoretical predictions for a number of unknown cases, in the hope that these might provide further guidance in elucidating the structure of the higher bands which generally decay via low intensity transitions, for which angular correlation experiments are almost impossible. Although the DNSB calculation provides better agreement with the δ values for $\gamma\rightarrow$g transitions, it is much worse for $\beta\rightarrow$g transitions, where even the sign is wrong in one case. This discrepancy probably could be removed by improving the theory.

Electric monopole transitions and X(E0/E2) values

The calculated E0 matrix elements and the ratios X(E0/E2) for both ^{154}Gd and ^{156}Gd are compared with experiment in Table 12. The results of a similar calculation for ^{156}Gd have been reported previously (Gupta *et al.* 1977*b*) but the E0 matrix elements were not given at that time. The matrix elements in Table 12*a* are of the form $\rho(\text{E0}; i\rightarrow f) = \langle f | r^2/R^2 | i \rangle$, where the nuclear radius $R = 1{\cdot}2A^{\frac{1}{3}}$ fm. The theoretical values have been calculated using Reiner's (1961) relation for r^2/R^2, which

Table 10. B(E2) ratios for transitions from possible $K^\pi = 0^+(3\beta)$ band in ^{154}Gd

Transition ratio		Exp. B(E2) ratios for E_x (keV)[A]			DPPQ calc. B(E2) ratios[B]	
I_i	I_f/I_f'	E_x = 2266	2277	2305·8	$2_{3\beta}$	$4_{3\beta}$
$2_{3\beta}, 4_{3\beta}$	$4_g/2_g$	1·4	0·3	7·5	0·06	0·5
	$4_\beta/2_\beta$		N[C]	3·7	2·5	0·2
	$3_\gamma/2_\gamma$		9·3	4·3	11	0·9
	$4_\gamma/2_\gamma$		1·8	1·4	1·6	24
	$2_\beta/2_g$	3·0	N[C]	40	0·8	0·2
	$4_\beta/4_g$		33	20	32	0·03
	$4_\gamma/2_g$		12·5	84	0·8	0·1

[A] Experimental ratios from the data of energy levels and γ-ray transitions given by Sousa *et al.* (1975).
[B] For the calculated members of the 3β band as specified in Tables 1 and 2.
[C] Not available; the likely transition to the 2_β state lies close to a 1458·4 keV intense γ ray.

Table 11. E2/M1 mixing ratios for transitions in ^{154}Gd

Transition		Exp. E_γ	δ(E2/M1)			Transition		Exp. E_γ	δ(E2/M1)
I_i	I_f	(keV)	Exp.[A]	DPPQ	DNSB	I_i	I_f	(keV)	DPPQ
2_β	2_g	692·4	$8\cdot3^{+1\cdot6}_{-1\cdot1}$	4·9	−10·1	$2_{\beta\gamma}$	2_g	1408·5	−12·6
4_β	4_g	676·6	2·2(9)	2·1	69·6		2_β	715·8	−4·4
6_β	6_g	648		1·2	9·0		2_γ	535·0	−11·9
							3_γ	403·5	−62
2_γ	2_g	873·2	−9·2(5)	−41	−16·3				
3_γ	2_g	1004·8	−7·9(3)	−128	−15·3	$3_{\beta\gamma}$	2_g	1537·8	−5·4
3_γ	4_g	756·9	−6·1(2)	−80	−8·7		4_g	1290·0	−7·1
4_γ	4_g	892·7	−4·0(4)	−12·4	−6·2		2_β	845·4	178
5_γ	4_g	1061·2		−47	−6·2		4_β	613·3	27·8
5_γ	6_g	714·6		−33	−4·8		2_γ	664·7	−45
6_γ	6_g	888·8		−5·9	−3·8		4_γ	397·1	18
$2_{2\beta}$	2_g	1295·5		0·6		$4_{\beta\gamma}$	4_g	1419·2	−35
	2_β	602·8		3·1					
	2_γ	422·1		138		$4_{2\gamma}$[B]	4_g	1274·7	−6·8
							4_β	589·2	−0·6
$4_{2\beta}$	4_g	1327		0·05			4_γ	382·1	5·5
	4_β	650·6		1·8			3_γ	518·0	400
3_γ	2_γ	131·6		1300		$5_{2\gamma}$[B]	4_g	1399·5	−2·0
	2_β	312·3		7·2			4_β	722·5	193
	4_β	80·4		11·3			4_γ	506·4	26·2
							5_γ	337·9	11·4
							4_4	124·4	7·1

[A] Experimental ratios from Gupta *et al.* (1977*a*).
[B] For the 2γ ($K = 4$) band.

is obtained by assuming a uniform charge distribution over an ellipsoid and keeping terms up to second order in β. The ratios in Table 12*b* are given by

$$X(\mathrm{E0/E2};\, i\to f) = e^2R^4\rho^2(\mathrm{E0};\, i\to f)/B(\mathrm{E2};\, i\to f)\,.$$

Where comparisons can be made with experiment, there is generally reasonable agreement for both the calculated ρ and X values.

Table 12. Electric monopole matrix elements and E0/E2 ratios for ^{154}Gd and ^{156}Gd

Values are tabulated for (*a*) the E0 matrix elements $\rho(E0; i \to f)$ and (*b*) the ratios $X(E0/E2; i \to f)$ as defined in Section 3

(*a*) $\rho(E0; i \to f)$

Transition $i \to f$		ρ(E0) values ^{154}Gd		^{156}Gd		Transition $i \to f$		ρ(E0) values ^{154}Gd	^{156}Gd
I_i	I_f	Exp.[A]	DPPQ	Exp.[A]	DPPQ	I_i	I_f	DPPQ	DPPQ
0_β	0_g	0·41(9)	0·37	0·41(5)	0·36	$0_{2\beta}$	0_g	0·03	0·02
2_β	2_g	0·10–0·59	0·38		0·34	$2_{2\beta}$	2_g	−0·02	−0·02
4_β	4_g	0·06–0·22	0·37		0·34	$4_{2\beta}$	4_g	−0·03	−0·03
6_β	6_g	0·09–0·35	0·36						
						$0_{2\beta}$	0_β	0·58	0·62
2_γ	2_g		0·04		0·05	$2_{2\beta}$	2_β	0·57	0·66
4_γ	4_g		0·06		0·11	$4_{2\beta}$	4_β	0·52	0·60
6_γ	6_g		0·06						
						$2_{2\beta}$	2_γ	−0·09	0·07
2_γ	2_β		0·06		0·04	$4_{2\beta}$	4_γ	−0·04	0·05
4_γ	4_β		0·13		0·08				
6_γ	6_β		0·18			$4_{2\gamma}$[B]	4_g	0·05	0·01
						$4_{2\gamma}$[B]	4_γ		−0·12

(*b*) $X(E0/E2; i \to f)$

Transition $i \to f$		X(E0/E2) values ^{154}Gd		^{156}Gd		Transition $i \to f$		X(E0/E2) values ^{154}Gd	^{156}Gd
I_i	I_f	Exp.[D]	DPPQ	Exp.[D]	DPPQ	I_i	I_f	DPPQ	DPPQ
0_β[C]	0_g[C]	0·11(3)	0·11	0·18(4)	0·26	$4_{2\gamma}$[B]	4_g	0·31	0·55
2_β	2_g	0·45(4)	0·75	0·50(8)	0·64				
4_β	4_g	0·60(18)	0·70	0·57(10)	0·49	$4_{2\gamma}$[B]	4_γ	0·16	
2_γ	2_g		0·005		0·010	$0_{2\gamma}$[E]	0_g	0·028	
4_γ	4_g		0·013		0·006	$2_{2\gamma}$[E]	2_g	34	
6_γ	6_g		0·017						
						$0_{3\beta}$	0_g	0·48	
$0_{2\beta}$	0_g		0·40	0·06(2)	0·028	$2_{3\beta}$	2_g	1·17	
$2_{2\beta}$	2_g		1·06		0·048				
$4_{2\beta}$	4_g		2·32		0·22				

[A] Experimental values from Aldushchenkov and Voinova (1973).
[B] For the 2γ ($K = 4$) band.
[C] For all $X(0_i \to 0_f)$ values, the E2 transition refers to $0_i \to 2_f$ where 0_f and 2_f belong to the same band.
[D] Experimental values from Rud and Nielsen (1970) and Rud *et al.* (1971).
[E] For the 2γ ($K = 0$) band.

Calculated potential and other functions

For the sake of brevity we have not presented contour plots of the calculated potential functions, inertial functions and wavefunctions of ^{154}Gd since these are qualitatively similar to those presented earlier for ^{152}Sm (Kumar 1974, 1975) and ^{156}Gd (Gupta *et al.* 1977*b*). However, a few characteristics may be noted for comparison: The calculated potential function of ^{154}Gd has a prolate minimum at $\beta = 0{\cdot}24$ and $\gamma = 0°$ with a deformation energy (from a spherical shape) of 3·7 MeV (this energy was 4·5 MeV for ^{156}Gd and 3·1 MeV for ^{152}Sm), and the prolate–oblate difference is 2·8 MeV. The ground state lies 1·82 MeV above the potential minimum

(which is then the energy of zero-point motion) so that the first three bands (i.e. most of the members of those bands considered here) lie in the prolate minimum below the spherical barrier (which is 1·88 MeV above the ground state). The higher bands are mostly above the spherical and oblate barriers and hence exhibit considerable mixing of prolate, asymmetric, oblate and spherical shapes.

4. Conclusions

The nucleus ^{154}Gd with 90 neutrons and 64 protons is a transitional nucleus on the deformed side of the spherical–deformed shape transition boundary at $N = 88$–90. It is rather similar to ^{152}Sm, another $N = 90$ nucleus, but, in the case of ^{154}Gd, experimental results are available for many more higher bands than the lowest three (g, β and γ). These data for the higher bands provide more stringent tests of a microscopic theory of collective motion. To the best of our knowledge, the present study is the first attempt to make a detailed comparison between theory and experiment for 25 states of the same nucleus.

The present dynamic deformation theory combined with the pairing plus quadrupole model has passed all tests surprisingly well. True, the calculated energies of the excited bands are too high by a factor of 1·4–1·7, but the B(E2) ratios, and even the absolute B(E2) values, are given remarkably well for states as high in excitation energy as 2·3 MeV. Thus our answer to the second question raised in the Introduction is: a qualified yes. This result seems to suggest that the geometry of the nuclear wavefunctions is given correctly by the 'generalized rotational basis' (Kumar 1975) used in the dynamic deformation theory. This part of the theory is simply a generalization of the rotational model, which predicts that B(E2) ratios involving only one or two bands in terms of Clebsch–Gordan coefficients depend only on the (K, I) characteristics of the initial and final states, and *not* on the exact nature of the different bands. The B(E2) values involving three bands do depend on the nature of the bands involved, but the present study suggests that the rotational model classification of bands in terms of β–γ phonons is meaningful even for the higher bands considered (four 2-phonon bands, and three 3-phonon bands). Thus our answer to the first question raised in the Introduction also is: yes.

Deviations from the rotational model can be attributed in the present type of calculation to K-mixing and to changes in the average nuclear shape as the nucleus is excited (or decays) from one state to another. The dynamic deformation theory allows us to take advantage of the rotational model on one hand (where various states are grouped into bands, each band with a definite and common intrinsic structure), and the shell model on the other hand (where each state represents a different configuration mixing or nucleon distribution). The latter behaviour is allowed in the dynamic deformation theory via the fact that the average nuclear shape (to be more precise, the probability distribution as a function of the nuclear shape) is allowed to be different in each nuclear state, and that each nuclear shape represents a different nucleon distribution or configuration mixing.

It would be very easy to attribute the discrepancies between theory and experiment noted in Section 3 above to the noncollective or two quasi-particle states. However, this would not be correct. It is true that two quasi-particle states do not appear explicitly in the final wavefunctions (they are, however, employed for calculating the inertial functions and the gyromagnetic ratio functions), but these (and higher) quasi-particle states are included in the dynamic deformation theory via the dynamics

or the vibrations, since each vibrational phonon represents a linear combination of two, four, ... quasi-particle states.

The main restriction of the present dynamic deformation theory comes from the assumption of a particular symmetry of the states, namely a D_2 symmetry with $(r_1 r_2 r_3) = (+ + +)$ (see Bohr and Mottelson 1975, p. 178), where r_k is the eigenvalue of the rotation operator $R_k(\pi)$. This assumption dictates the symmetry of the rotational part of our wavefunctions, the allowed K values and the symmetry of the allowed intrinsic states (which are the time-reversal-invariant BCS zero quasi-particle states in the present theory). States not considered are:

(1) Negative-parity states. These, of course, do not mix with the even-parity states considered here, and their calculation would require, in the present type of theory, basis wavefunctions of a different symmetry.

(2) States of positive parity but with $(r_1 r_2 r_3) = (+ - -), (- + -)$ and $(- - +)$. These states are absolutely essential if one wants to calculate 1^+ states (which are not allowed for $(+ + +)$ symmetry). However, their mixing with the levels considered here for the 25 states of ^{154}Gd cannot be very important, otherwise the calculated B(E2) values would have been quite wrong (recall that the symmetry of the wavefunctions is crucial to the rotational model type patterns of B(E2) values).

It seems to us that the main cause of the discrepancies between the theory and experiment (which occur mainly in the energy region of the vibrational bands) is the truncated configuration space used in the DPPQ method, which is restricted to two major oscillator shells. The inertial renormalization parameter, which multiplies all the calculated inertial functions, provides only a crude way of taking into account the effects of the 'inert' core. It could be argued that the core contributions are different for the moments of inertia and the vibrational mass parameters and hence different renormalization coefficients should be used for the two types of functions. However, this would not help since such a procedure would upset the symmetry properties of the six inertial functions which must reduce in the limit of small deformations to Bohr's (1952) form, where all six functions depend on a single mass parameter except for some β- and γ-dependent functions dictated by the symmetry conditions (Bohr 1952; Kumar and Baranger 1967).

As has been remarked many times before, the problem of truncation of the configuration space is not simply a computer time (cost) problem. If one merely expands the space without improving the nucleon–nucleon interaction, the calculated results can get worse instead of better. This has been our experience with several versions of the quadrupole force, where the radial matrix elements have been calculated according to different prescriptions; for instance, replacement of the oscillator by a Woods–Saxon potential, restriction to $\Delta N = 0$ matrix elements only, etc. As noted in Section 2 above, calculations can now be made over a fairly complete configuration space ($\mathcal{N} = 0$–8 major oscillator shells) with the recently developed DNSB version of the dynamic deformation theory, although such calculations still require much larger and faster computers than those used for most DPPQ calculations. However, the brief comparison made here between the DPPQ and DNSB results indicates once more that agreement with experiment is not improved simply by expanding the configuration space. Nevertheless, probably a more extensive study of the DNSB parameters is required before it can be definitely concluded that this method is generally

inferior to the DPPQ method, rather than just that the latter method happens to work better for those mass regions where theoretical parameters have been established quite well after many years of study of various nuclear properties.

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