Aust. J. Phys., 1982, **35**, 409–14

Parametric Instabilities in a Magnetized Plasma

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Abstract

This paper makes a study of stimulated Raman scattering and stimulated Brillouin scattering of an incident electromagnetic pump wave in a magnetized plasma. The background magnetic field is taken to be parallel to the pump electric field. The growth rates of the two stimulated-scattering instabilities are found to be reduced in the presence of the background magnetic field.

1. Introduction

The parametric interaction of an intense coherent electromagnetic wave with collective modes in a plasma was investigated by Bornatici *et al.* (1969), Liu and Rosenbluth (1972), Drake *et al.* (1974), among others, in an attempt to provide an interpretation of observed phenomena in laser-produced plasmas and plasma-heating experiments. One of the consequences of the interaction of an incident (pump) electromagnetic wave with a plasma is the parametric excitation of two plasma waves. If the latter are both purely electrostatic, they are eventually absorbed in the plasma and this decay process then leads to enhanced (or anomalous) absorption of the incident electromagnetic wave. If one of the excited plasma waves is electromagnetic, it can escape from the plasma and show up as enhanced (or stimulated) scattering of the incident electromagnetic wave. This process can be of two types according to whether the other excited plasma wave is a Langmuir wave (stimulated Raman scattering) or an ion-acoustic wave (stimulated Brillouin scattering).

Experiments of Stamper *et al.* (1971) showed that intense spontaneously generated magnetic fields are present in laser-produced plasmas. These magnetic fields are usually strong enough to modify the spectrum of electrostatic modes in the plasma but not strong enough to influence the characteristics of propagation of the incident and scattered electromagnetic modes. The purpose of this paper is to study the stimulated Raman scattering and stimulated Brillouin scattering of an incident electromagnetic wave in a magnetized plasma.

2. A Prototype for Parametric Instabilities

As a prototype for the parametric processes discussed in this paper, consider a system acted on by an oscillatory pump (of large magnitude) of the form

$$Z(t) = 2Z_0 \cos\omega_0 t, \tag{1}$$

where Z_0 is taken to be a constant if one restricts consideration to initial stages of the ensuing instabilities in the system so that any depletion of the pump is then negligible. The pump induces a coupling between two natural modes of oscillation $X(t)$ and $Y(t)$ (with characteristic frequencies ω_1 and ω_2 respectively) say, in the form (Nishikawa 1968)

$$(\mathrm{d}^2/\mathrm{d}t^2+\omega_1^2)X(t) = \lambda Y(t)Z(t), \tag{2}$$

$$(\mathrm{d}^2/\mathrm{d}t^2+\omega_2^2)Y(t) = \mu X(t)Z(t), \tag{3}$$

where λ and μ are coupling constants which are assumed to be such that $(\lambda\mu)$ is real and positive.

Upon Fourier transforming according to

$$Q(t) = \int_{-\infty}^{\infty} Q(\omega)\exp(-\mathrm{i}\,\omega t)\,\mathrm{d}\omega, \tag{4}$$

equations (2) and (3) give

$$(\omega^2-\omega_1^2)\,X(\omega) = -\lambda Z_0\{Y(\omega-\omega_0)+Y(\omega+\omega_0)\}, \tag{5}$$

$$(\omega^2-\omega_2^2)\,Y(\omega) = -\mu Z_0\{X(\omega-\omega_0)+X(\omega+\omega_0)\}. \tag{6}$$

Consider a resonant situation with

$$\omega_0 \approx \omega_1+\omega_2 \tag{7}$$

and retain in equations (5) and (6) only the terms $X(\omega)$ and $Y(\omega-\omega_0)$, so that the dispersion relation follows

$$(\omega-\omega_1)(\omega-\omega_0+\omega_2)+(\lambda\mu Z_0^2/4\omega_1\,\omega_2) = 0. \tag{8}$$

By putting

$$\omega = \Omega+\mathrm{i}\,\gamma, \tag{9}$$

equation (8) gives us

$$\gamma^2\{1+(\Delta^2/4\gamma^2)\} = \lambda\mu Z_0^2/4\omega_1\,\omega_2, \tag{10}$$

where

$$\Delta \equiv \omega_0-\omega_1-\omega_2.$$

The maximum growth rate occurs at perfect match ($\Delta = 0$), and is given by

$$\gamma_{\max} = (\lambda\mu Z_0^2/4\omega_1\,\omega_2)^{1/2}. \tag{11}$$

3. Stimulated Raman Scattering

Consider a homogeneous plasma with a uniform background magnetic field $\boldsymbol{B}_0$. A large amplitude plane-polarized electromagnetic pump wave

$$\boldsymbol{E}_t = 2\boldsymbol{E}_t\cos(\boldsymbol{k}_t\,.\,\boldsymbol{x}-\omega_t\,t), \tag{12}$$

with $\boldsymbol{E}_t$ parallel to $\boldsymbol{B}_0$, is incident on the plasma. The equilibrium state is comprised of electrons oscillating with velocity

$$\boldsymbol{v}_{\mathrm{t}} = (2e\boldsymbol{E}_{\mathrm{t}}/m\omega_0)\sin(\boldsymbol{k}_{\mathrm{t}}.\boldsymbol{x} - \omega_{\mathrm{t}} t), \tag{13}$$

m being the mass of the electron, in the incident electric field $\boldsymbol{E}_{\mathrm{t}}$, with the ions remaining stationary and making up a neutralizing background. Let us perturb this equilibrium and study the time development of these perturbations using the linearized fluid equations and Maxwell's equations. In the stimulated Raman scattering process in a magnetized plasma, where only the electrons participate, the pump wave $(\omega_{\mathrm{t}}, \boldsymbol{k}_{\mathrm{t}})$ decays into an electromagnetic wave $(\omega_{\mathrm{t}'}, \boldsymbol{k}_{\mathrm{t}'})$ and a modified Langmuir wave $(\omega_{\mathrm{l}}, \boldsymbol{k}_{\mathrm{l}})$ with the constraints

$$\omega_{\mathrm{t}} = \omega_{\mathrm{t}'} + \omega_{\mathrm{l}}, \qquad \boldsymbol{k}_{\mathrm{t}} = \boldsymbol{k}_{\mathrm{t}'} + \boldsymbol{k}_{\mathrm{l}}. \tag{14}$$

One obtains for the Langmuir wave

$$(\partial n_{\mathrm{l}}/\partial t) + N_0 \nabla . \boldsymbol{v}_{\mathrm{l}} = 0, \tag{15}$$

$$\begin{aligned}\frac{\partial \boldsymbol{v}_{\mathrm{l}}}{\partial t} + \frac{3KT_{\mathrm{e}}}{mN_0}\nabla n_{\mathrm{l}} + \frac{e}{m}\boldsymbol{E}_{\mathrm{l}} + \frac{e}{mc}\boldsymbol{v}_{\mathrm{l}} \times \boldsymbol{B}_0 \\ = -(\boldsymbol{v}_{\mathrm{t}}.\nabla \boldsymbol{v}_{\mathrm{t}'} + \boldsymbol{v}_{\mathrm{t}'}.\nabla \boldsymbol{v}_{\mathrm{t}}) - (e/mc)(\boldsymbol{v}_{\mathrm{t}} \times \boldsymbol{B}_{\mathrm{t}'} + \boldsymbol{v}_{\mathrm{t}'} \times \boldsymbol{B}_{\mathrm{t}}),\end{aligned} \tag{16}$$

where n_{l} is the perturbation in the number density, $\boldsymbol{v}_{\mathrm{l}}$ the velocity, T_{e} the temperature of the electrons, N_0 the number density in the unperturbed state, and note from

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \tag{17}$$

that

$$(e/mc)\boldsymbol{B}_{\mathrm{t}} = \nabla \times \boldsymbol{v}_{\mathrm{t}} \qquad \text{and} \qquad (e/mc)\boldsymbol{B}_{\mathrm{t}'} = \nabla \times \boldsymbol{v}_{\mathrm{t}'}. \tag{18}$$

By using (18), equation (16) becomes

$$\frac{\partial \boldsymbol{v}_{\mathrm{l}}}{\partial t} + \frac{3KT_{\mathrm{e}}}{mN_0}\nabla n_{\mathrm{l}} + \frac{e}{m}(\boldsymbol{E}_{\mathrm{l}} + c^{-1}\boldsymbol{v}_{\mathrm{l}} \times \boldsymbol{B}_0) = -\nabla(\boldsymbol{v}_{\mathrm{t}}.\boldsymbol{v}_{\mathrm{t}'}). \tag{19}$$

Taking the divergence of equation (19), using

$$\nabla . \boldsymbol{E}_{\mathrm{l}} = -4\pi e n_{\mathrm{l}} \tag{20}$$

and equation (15) and considering propagation perpendicular to $\boldsymbol{B}_0$ (i.e. $\boldsymbol{k}_{\mathrm{l}}.\boldsymbol{B}_0 = 0$), one obtains

$$(\partial^2/\partial t^2 + \omega_{\mathrm{u}}^2)n_{\mathrm{l}} = N_0 \nabla^2(\boldsymbol{v}_{\mathrm{t}}.\boldsymbol{v}_{\mathrm{t}'}), \tag{21}$$

where

$$\omega_{\mathrm{u}}^2 = \omega_{\mathrm{l}}^2 + \omega_{\mathrm{ce}}^2, \qquad \omega_{\mathrm{l}}^2 = \omega_{\mathrm{pe}}^2 + 3k_{\mathrm{l}}^2(KT_{\mathrm{e}}/m),$$

$$\omega_{\mathrm{ce}} = eB_0/mc, \qquad \omega_{\mathrm{pe}}^2 = 4\pi N_0 e^2/m.$$

Next, from

$$\nabla^2 \boldsymbol{E}_{\mathrm{t}'} - \frac{1}{c^2}\frac{\partial^2 \boldsymbol{E}_{\mathrm{t}'}}{\partial t^2} = -\frac{4\pi e}{c^2}\frac{\partial}{\partial t}(n_{\mathrm{l}}\boldsymbol{v}_{\mathrm{t}}), \tag{22}$$

one obtains for the scattered electromagnetic wave

$$\left(\frac{\partial^2}{\partial t^2}+\omega_{t'}^2\right)\boldsymbol{E}_{t'} = 4\pi e\frac{\partial}{\partial t}(n_1 \boldsymbol{v}_t), \tag{23}$$

where

$$\omega_{t'}^2 = \omega_{pe}^2 + k_{t'}^2 c^2,$$

and we have used the fact that the magnetic field $\boldsymbol{B}_0$ is not strong enough to influence the characteristics of propagation of the incident and the scattered electromagnetic waves.

Noting that

$$\nabla^2(\boldsymbol{v}_t \cdot \boldsymbol{v}_{t'}) = -\frac{e^2 k_1^2}{m^2\omega_t\omega_{t'}}\boldsymbol{E}_t \cdot \boldsymbol{E}_{t'}, \qquad \frac{\partial}{\partial t}(n_1 \boldsymbol{v}_t) = -\frac{e\omega_{t'}}{m\omega_t} n_1 \boldsymbol{E}_t, \tag{24a, b}$$

equations (21) and (23) become

$$(\partial^2/\partial t^2 + \omega_u^2)n_1 = \lambda_R E_t E_{t'}, \tag{25}$$

$$(\partial^2/\partial t^2 + \omega_{t'}^2)E_{t'} = \mu_R E_t n_1, \tag{26}$$

where

$$\lambda_R = -\frac{\omega_{pe}^2}{\omega_t\omega_{t'}}\frac{k_1^2}{4\pi m}(\hat{\boldsymbol{e}}_t \cdot \hat{\boldsymbol{e}}_{t'}), \qquad \mu_R = -\frac{\omega_{t'}}{\omega_t}\frac{\omega_{pe}^2}{N_0}(\hat{\boldsymbol{e}}_t \cdot \hat{\boldsymbol{e}}_{t'});$$

$\hat{\boldsymbol{e}}_t$ and $\hat{\boldsymbol{e}}_{t'}$ are the directions of polarization of $\boldsymbol{E}_t$ and $\boldsymbol{E}_{t'}$.

Equations (25) and (26) are of the same form as (2) and (3), so that the maximum growth rate for stimulated Raman scattering is given from equation (11) as

$$\gamma_R = \frac{\omega_{pe}^2 k_1 |\hat{\boldsymbol{e}}_t \cdot \hat{\boldsymbol{e}}_{t'}| E_t}{4\omega_t\{\omega_{t'}\pi N_0 m(\omega_l^2+\omega_{ce}^2)^{1/2}\}^{1/2}}. \tag{27}$$

Observe that the growth rate is reduced in the presence of a background magnetic field. In the absence of the latter, (27) reduces to the one deduced by Bornatici *et al.* (1969).

4. Stimulated Brillouin Scattering

In the stimulated Brillouin scattering process the pump wave (ω_t, k_t) decays into an electromagnetic wave $(\omega_{t'}, k_{t'})$ and an ion-acoustic wave (ω_s, k_s), with the constraints

$$\omega_t = \omega_{t'} + \omega_s, \qquad \boldsymbol{k}_t = \boldsymbol{k}_{t'} + \boldsymbol{k}_s. \tag{28a, b}$$

Here both electrons and ions participate in the motion of the ion-acoustic wave. One obtains for the electrons moving in this wave

$$(\partial n_{es}/\partial t) + N_0 \nabla \cdot \boldsymbol{v}_{es} = 0, \tag{29}$$

$$\begin{aligned} \frac{KT_e}{mN_0}\nabla n_{es} + \frac{e}{m}\left(\boldsymbol{E}_s + \frac{1}{c}\boldsymbol{v}_{es} \times \boldsymbol{B}_0\right) &= -(\boldsymbol{v}_t \cdot \nabla \boldsymbol{v}_{t'} + \boldsymbol{v}_{t'} \cdot \nabla \boldsymbol{v}_t) - (e/mc)(\boldsymbol{v}_t \times \boldsymbol{B}_{t'} + \boldsymbol{v}_{t'} \times \boldsymbol{B}_t) \\ &= -\nabla(\boldsymbol{v}_t \cdot \boldsymbol{v}_{t'}), \end{aligned} \tag{30}$$

where we have ignored the electron inertia, and have assumed that the electrons respond isothermally to the ion-acoustic wave. One has for the ions moving in the wave

$$(\partial n_{is}/\partial t)+N_0\nabla\cdot\boldsymbol{v}_{is} = 0, \tag{31}$$

$$\frac{\partial \boldsymbol{v}_{is}}{\partial t}+\frac{3KT_i}{MN_0}\nabla n_{is}-\frac{e}{M}\left(\boldsymbol{E}_s+\frac{1}{c}\boldsymbol{v}_{is}\times\boldsymbol{B}_0\right) = 0, \tag{32}$$

with

$$\nabla\cdot\boldsymbol{E}_s = 4\pi e(n_{is}-n_{es}), \tag{33}$$

where M is the mass of an ion, and T_i the temperature of the ions.

Taking the divergence of equation (30), using equations (29) and (33), and considering propagation perpendicular to $\boldsymbol{B}_0$ (i.e. $\boldsymbol{k}_s\cdot\boldsymbol{B}_0 = 0$), one obtains

$$n_{es} = \frac{1}{1+k_s^2\lambda_D^2+\omega_{ce}^2/\omega_{pe}^2}\left(n_{is}+\frac{N_0}{\omega_{pe}^2}\nabla^2(\boldsymbol{v}_t\cdot\boldsymbol{v}_{t'})\right), \tag{34}$$

where

$$\lambda_D^2 = KT_e/m\omega_{pe}^2.$$

Further, taking the divergence of equation (32) and using equations (31), (33) and (34), one obtains

$$\left(\frac{\partial^2}{\partial t^2}+\omega_s^2+\omega_{ci}^2\right)n_{is} = \frac{(m/M)N_0\nabla^2(\boldsymbol{v}_t\cdot\boldsymbol{v}_{t'})}{1+k_s^2\lambda_D^2+\omega_{ce}^2/\omega_{pe}^2}, \tag{35}$$

where

$$\omega_s^2 = \frac{k_s^2C_s^2}{1+k_s^2\lambda_D^2+\omega_{ce}^2/\omega_{pe}^2}+3k_s^2\frac{KT_i}{M}, \qquad C_s^2 = KT_e/M.$$

Using

$$\nabla^2\boldsymbol{E}_{t'}-\frac{1}{c^2}\frac{\partial^2\boldsymbol{E}_{t'}}{\partial t^2} = -\frac{4\pi e}{c^2}\frac{\partial}{\partial t}(n_{es}\boldsymbol{v}_t) \tag{36}$$

and equation (34), one obtains for the scattered electromagnetic wave

$$\left(\frac{\partial^2}{\partial t^2}+\omega_{t'}^2\right)\boldsymbol{E}_{t'} = \frac{4\pi e}{1+k_s^2\lambda_D^2+\omega_{ce}^2/\omega_{pe}^2}\frac{\partial}{\partial t}(\boldsymbol{v}_t n_{is}). \tag{37}$$

From (24), equations (35) and (37) can be written as

$$(\partial^2/\partial t^2+\omega_s^2+\omega_{ci}^2)n_{is} = \lambda_B E_t E_{t'}, \tag{38}$$

$$(\partial^2/\partial t^2+\omega_{t'}^2)E_{t'} = \mu_B E_t n_{is}, \tag{39}$$

where

$$\lambda_B = -\frac{(\omega_{pe}^2/\omega_t\omega_{t'})(k_s^2/4\pi M)}{1+k_s^2\lambda_D^2+\omega_{ce}^2/\omega_{pe}^2}(\hat{e}_t\cdot\hat{e}_{t'}),$$

$$\mu_B = -\frac{(\omega_{t'}/\omega_t)(\omega_{pe}^2/N_0)}{1+k_s^2\lambda_D^2+\omega_{ce}^2/\omega_{pe}^2}(\hat{e}_t\cdot\hat{e}_{t'}).$$

Equations (38) and (39) are again of the same form as (2) and (3), so that the maximum growth rate for stimulated Brillouin scattering is given from equation (11) as

$$\gamma_{\mathrm{B}} = \frac{\{\omega_{\mathrm{pe}}^2 k_{\mathrm{s}}/\omega_{\mathrm{t}}(4\pi M N_0)^{1/2}\}\,|\,\hat{e}_{\mathrm{t}}\cdot\hat{e}_{\mathrm{t'}}\,|\,E_{\mathrm{t}}}{2(1+k_{\mathrm{s}}^2\lambda_{\mathrm{D}}^2+\omega_{\mathrm{ce}}^2/\omega_{\mathrm{pe}}^2)\{\omega_{\mathrm{t'}}(\omega_{\mathrm{s}}^2+\omega_{\mathrm{ci}}^2)^{1/2}\}^{1/2}}. \tag{40}$$

Observe that the growth rate is again reduced in the presence of a background magnetic field. In the absence of the latter, equation (40) reduces to the one deduced by Liu and Rosenbluth (1972).

References

Bornatici, M., Cavaliere, A., and Engelmann, F. (1969). *Phys. Fluids* **12**, 2362.

Drake, J., Kaw, P. K., Lee, Y. C., Schmidt, G., Liu, C. S., and Rosenbluth, M. N. (1974). *Phys. Fluids* **17**, 778.

Liu, C. S., and Rosenbluth, M. N. (1972). Institute for Advanced Study, Princeton, Rep. No. COO 3237–11.

Nishikawa, K. (1968). *J. Phys. Soc. Jpn* **24**, 916.

Stamper, J. A., Papadopoulous, K., Sudan, R. N., Dean, S. O., McLean, E. A., and Dawson, J. M. (1971). *Phys. Rev. Lett.* **26**, 1012.

Manuscript received 16 December 1981, accepted 27 January 1982