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Continuous Localisation–Delocalisation Transition at Intermediate Electron Densities*

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Abstract

We find in 2D electron layers in quantum transistors that the interplay between the electron correlations and their interactions with defects in the semiconductor substrate generates a continuous localisation–delocalisation transition for intermediate electron densities ($5 \lesssim r_s \lesssim 9$). We distinguish this transition from the discontinuous metal–insulator transition which is observed at lower electron densities ($r_s \gtrsim 10$). The approach we use is based on the behaviour of electrons at low densities. We take into account the interactions between electrons and also their interactions with disorder. We determine a zero temperature phase diagram of localised and delocalised states as a function of electron and impurity densities. The phase boundary of the continuous transition is determined by the localisation length of the electrons.

1. Introduction

The metal–insulator transition in 2D electron or hole layers in n–Si MOSFET, p–SiGe, and p–GaAs–AlGaAs devices is observed only at very low densities (Kravchenko *et al.* 1994, 1996; Popovic *et al.* 1997; Simmons *et al.* 1998*a*, 1998*b*; Lam *et al.* 1997; Coleridge *et al.* 1997). Since the average energy associated with the interaction between the electrons or holes is much larger than the Fermi energy, the transition is probably driven by the electron interactions. This is consistent with the prediction of Abrahams *et al.* (1999) that without these interactions there can be no conducting state in 2D no matter how small the level of defects.

There are a number of novel properties associated with the localisation–delocalisation transition. At a critical value of the electron density n_e the system becomes metallic. However, the metallic phase only exists for a restricted range of r_s and when the disorder level is low. The magnitude of the drop in the resistivity diminishes with decreasing sample mobility. When the disorder is high, corresponding to a peak mobility $\mu \lesssim 10^6 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$, all electronic states are localised.

At low levels of disorder, when $r_s < 9$ the system undergoes a second transition into an insulating state. In contrast with the low density transition, this second

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transition is broad and gradual. There is a unique relation between correlations and disorder at both transitions.

In the limit of extremely low electron densities Wigner solidification occurs. The exchange-correlation hole is known to be very strong at such densities (Tanatar and Ceperley 1989). Wigner localisation is caused by the strong density exclusion around each electron which leads to a localisation of the density fluctuations. The mechanism for this localisation is purely the mutual repulsion of the electrons, so that Wigner localisation can be thought of as a complete opposite to Anderson localisation where the interaction between the electrons is usually neglected. Wigner localisation has close parallels with the close-packing solidification of charged hard spheres where the dominant mechanism arises from fluctuations in the density variable.

We have previously shown that when there are low levels of defects, electron localisation is still driven by electron correlations at low electron densities. The impurities act as pinning centres and facilitate the localisation. However, there is no long-range order when there is disorder and the electron solid is a frozen glass with liquid-like short-range order. We have found in the range of electron densities $10 \lesssim r_s \lesssim 25$ that weak substrate disorder causes a glass transition (Thakur and Neilson 1996).

In this paper we concentrate on intermediate electron densities, $r_s \lesssim 9$, where there is no discontinuous transition from a fully delocalised liquid state to a strongly localised disordered solid. Even though we have moved away from the limit of very strong correlations, to leading order the basis set remains the density fluctuations, $\rho(\mathbf{q}, t) = \sum_{\mathbf{k} < k_F} a_{\mathbf{k}+\mathbf{q}}^\dagger(t) a_{\mathbf{k}}(t)$. We incorporate scattering off defects in such a way that the Ward identities and particle conservation are exactly satisfied within the density basis. By restricting the basis to density fluctuations we average over phase information of processes where the particles and holes propagate independently (Neilson *et al.* 1991). We use a memory function formalism with mode-coupling theory (Bengtzelius *et al.* 1984; Thakur and Bosse 1987, 1991) generalised to the quantum case (Thakur and Neilson 1996; Neilson *et al.* 1991).

2. Theoretical Method

The density correlation function $\langle \rho(r, t) \rho(0, 0) \rangle$ gives the time dependence of the decay of the density fluctuations. In the liquid phase $\langle \rho(r, t) \rho(0, 0) \rangle$ goes to zero when $t \rightarrow \infty$ because of the propagation of the diffusive mode. In the glass phase the diffusion constant is zero, so if we approach the glass transition the decay of density fluctuations becomes very slow. The local structure and local density fluctuations no longer vanish when $t \rightarrow \infty$, and $\langle \rho(r, t) \rho(0, 0) \rangle$ does not go to zero. A narrow quasi-elastic peak builds up in the dynamic structure factor $S(q, \omega)$, so that $S(q, \omega)$ develops a singularity at zero frequency.

We define the Kubo relaxation function,

$$\Phi(q, t) \equiv \left(N_q(t) \middle| N_q(0) \right), \quad (1)$$

for the normalised density dynamical variables $N(q, t) \equiv \rho(q, t)/\sqrt{\chi(q)}$, where $\chi(q)$ is the static susceptibility. We introduce the order parameter $f(q)$ for the transition

$$f(q) = -\lim_{z \rightarrow 0} z\Phi(q, z), \quad (2)$$

with

$$\Phi(q, z) = \left(N_q \left| \frac{1}{\mathcal{L} - z} \right| N_q \right). \quad (3)$$

In Laplace space the limit in equation (2) corresponds to the long-time limit of $\Phi(q, t)$.

The Liouvillian \mathcal{L} is for the system Hamiltonian

$$H = \sum_k \epsilon_k a_k^\dagger a_k + \frac{1}{2} \sum_q V(q) \rho_q \rho_{-q} + \sum_q U(q) \rho_{-q}, \quad (4)$$

where $\epsilon_k = \hbar^2 k^2 / 2m^*$ is the single-particle kinetic energy, $V(q)$ is the Coulomb interaction between electrons, and $U(q) = \{W(q) \sum_i \exp(-i\mathbf{q} \cdot \mathbf{r}_i) + U_{sr}(q)\}$ is the defect potential for the impurities plus surface roughness term $U_{sr}(q)$. The impurities are randomly distributed at points \mathbf{r}_i (which may lie outside the plane).

The Kubo relaxation function is expressed in terms of force–force relaxation functions. Using mode-coupling theory the force–force relaxation functions can be approximated by linear and bi-linear products of the relaxation functions. This results in a set of non-linear self-consistent equations which we use to calculate the $f(q)$.

We start with the exact identities (see e.g. Thakur and Pathak 1986)

$$\mathcal{P} = \left[\mathcal{P}\mathcal{L}\mathcal{P} - z - \mathcal{P}\mathcal{L}\bar{\mathcal{P}} \frac{1}{\bar{\mathcal{P}}\mathcal{L}\bar{\mathcal{P}} - z} \bar{\mathcal{P}}\mathcal{L}\mathcal{P} \right] \mathcal{P} \frac{1}{\mathcal{L} - z} \mathcal{P}, \quad (5)$$

$$\mathcal{J} = \left[\mathcal{J}\tilde{\mathcal{L}}\mathcal{J} - z - \mathcal{J}\tilde{\mathcal{L}}\bar{\mathcal{J}} \frac{1}{\bar{\mathcal{J}}\tilde{\mathcal{L}}\bar{\mathcal{J}} - z} \bar{\mathcal{J}}\tilde{\mathcal{L}}\mathcal{J} \right] \mathcal{J} \frac{1}{\tilde{\mathcal{L}} - z} \mathcal{J} \quad (6)$$

for the projection operators \mathcal{P} and \mathcal{J} ,

$$\begin{aligned} \mathcal{P} &= |N_q\rangle \langle N_q|, & \bar{\mathcal{P}} &= 1 - \mathcal{P}, \\ \mathcal{J} &= |\mathcal{L}N_q\rangle \frac{1}{(\mathcal{L}N_q|\mathcal{L}N_q)} (\mathcal{L}N_q|, & \bar{\mathcal{J}} &= 1 - \mathcal{J}, \end{aligned} \quad (7)$$

and where $\tilde{\mathcal{L}} \equiv \bar{\mathcal{P}}\mathcal{L}\bar{\mathcal{P}}$. The \mathcal{P} and \mathcal{J} project dynamical variables into the subspace spanned by density fluctuations $|N_q\rangle$, and current density fluctuations $|\mathcal{L}N_q\rangle$, respectively.

Taking a scalar product with density fluctuation $|N_q\rangle$ equation (5) becomes

$$\begin{aligned} (N_q|\mathcal{P}|N_q) &= (N_q|N_q) = 1 \\ &= \left[(N_q|\mathcal{P}\mathcal{L}\mathcal{P}|N_q) - (N_q|z|N_q) - (N_q|\mathcal{P}\mathcal{L}\bar{\mathcal{P}}\frac{1}{\bar{\mathcal{P}}\mathcal{L}\bar{\mathcal{P}}-z}|\bar{\mathcal{P}}\mathcal{L}\mathcal{P}|N_q) \right] \\ &\quad \times (N_q|\mathcal{P}|N_q) \left(N_q|\frac{1}{\mathcal{L}-z}N_q \right) (N_q|N_q), \end{aligned} \quad (8)$$

or, using the definition for $\Phi(q, z)$ given by equation (3),

$$1 = \left[(N_q|\mathcal{L}|N_q) - z - (N_q|\mathcal{L}\bar{\mathcal{P}}\frac{1}{\bar{\mathcal{P}}\mathcal{L}\bar{\mathcal{P}}-z}\bar{\mathcal{P}}\mathcal{L}|N_q) \right] \Phi(q, z). \quad (9)$$

Solving equation (9) for $\Phi(q, z)$, and using $(N_q|\mathcal{L}|N_q) = 0$ and

$$\bar{\mathcal{P}}\mathcal{L}|N_q) = \{\mathcal{L}|N_q) - \mathcal{P}\mathcal{L}|N_q)\} = \{\mathcal{L}|N_q) - |N_q)(N_q|\mathcal{L}|N_q)\} = |\mathcal{L}N_q),$$

both of these relations coming from time reversal symmetry, we obtain

$$\begin{aligned} \Phi(q, z) &= - \frac{1}{z + (\mathcal{L}N_q|\frac{1}{\bar{\mathcal{P}}\mathcal{L}\bar{\mathcal{P}}-z}|\mathcal{L}N_q)} \\ &= - \frac{1}{z + K(q, z)}. \end{aligned} \quad (10)$$

The current relaxation function $K(q, z)$ is given by

$$K(q, z) = (\mathcal{L}N_q|\frac{1}{\bar{\mathcal{L}}-z}|\mathcal{L}N_q). \quad (11)$$

Now taking a scalar product of equation (6) with current fluctuation $|\mathcal{L}N_q\rangle$ we get

$$\begin{aligned} (\mathcal{L}N_q|\mathcal{J}|\mathcal{L}N_q) &= (\mathcal{L}N_q|\mathcal{L}N_q) \\ &= (N_q|\mathcal{L}|\left[\mathcal{J}\tilde{\mathcal{L}}\mathcal{J} - z - \mathcal{J}\tilde{\mathcal{L}}\bar{\mathcal{J}}\frac{1}{\bar{\mathcal{J}}\tilde{\mathcal{L}}\bar{\mathcal{J}}-z}\bar{\mathcal{J}}\tilde{\mathcal{L}}\mathcal{J} \right] \\ &\quad \times \left\{ |\mathcal{L}N_q) \frac{1}{(\mathcal{L}N_q|\mathcal{J}\mathcal{L}N_q)} (\mathcal{L}N_q| \right\} \frac{1}{\bar{\mathcal{L}}-z} |\mathcal{L}N_q). \end{aligned} \quad (12)$$

Using the definition of $K(q, z)$ given by equation (11) and solving for $K(q, z)$ we obtain

$$K(q, z) = - \frac{(\mathcal{L}N_q | \mathcal{L}N_q)}{z + \frac{1}{(\mathcal{L}N_q | \mathcal{L}N_q)} \left(\mathcal{L}N_q \tilde{\mathcal{L}} \tilde{\mathcal{J}} \left| \frac{1}{\tilde{\mathcal{J}} \tilde{\mathcal{L}} \tilde{\mathcal{J}} - z} \right| \tilde{\mathcal{J}} \tilde{\mathcal{L}} \mathcal{L}N_q \right)}, \quad (13)$$

where we have used $\mathcal{J}|\mathcal{L}N_q\rangle = |\mathcal{L}N_q\rangle$ and

$$\begin{aligned} (\mathcal{L}N_q | \tilde{\mathcal{L}} | \mathcal{L}N_q) &= (\mathcal{L}N_q | \bar{\mathcal{P}} \mathcal{L} \bar{\mathcal{P}} | \mathcal{L}N_q) \\ &= \left\{ (\bar{\mathcal{P}} \mathcal{L}N_q | \mathcal{L}^2 N_q) - (\bar{\mathcal{P}} \mathcal{L}N_q | N_q) (N_q | \mathcal{L}N_q) \right\} = 0, \end{aligned}$$

both terms vanishing because of time inversion symmetry. Now

$$(\mathcal{L}N_q | \mathcal{L}N_q) = \chi^{-1}(q) (\mathcal{L}\rho_q | \mathcal{L}\rho_q) \equiv \chi^{-1}(q) \langle [\rho_q, [\rho_q, H]] \rangle = \chi^{-1}(q) (q^2/m),$$

the last step following from the f -sum rule. We define $\Omega_q = q^2/m\chi(q)$. Using $\mathcal{P}|\mathcal{L}N_q\rangle = 0$ and

$$\begin{aligned} |\tilde{\mathcal{J}} \tilde{\mathcal{L}} \mathcal{L}N_q\rangle &= \{1 - \Omega_q^{-1} | \mathcal{L}N_q\rangle (\mathcal{L}N_q | \cdot) | \bar{\mathcal{P}} \mathcal{L} \bar{\mathcal{P}} \mathcal{L}N_q\rangle \\ &= \{1 - \Omega_q^{-1} | \mathcal{L}N_q\rangle (\mathcal{L}N_q | \cdot) | \bar{\mathcal{P}} \mathcal{L}^2 N_q\rangle = | \bar{\mathcal{P}} \mathcal{L}^2 N_q\rangle \end{aligned}$$

[since by symmetry $(\mathcal{L}N_q | \bar{\mathcal{P}} \mathcal{L}^2 N_q) = 0$], equation (13) becomes

$$\begin{aligned} K(q, z) &= - \frac{\Omega_q}{z + \frac{m}{q^2} \left(\bar{\mathcal{P}} \mathcal{L}^2 \rho_q \left| \frac{1}{\tilde{\mathcal{J}} \tilde{\mathcal{L}} \tilde{\mathcal{J}} - z} \right| \bar{\mathcal{P}} \mathcal{L}^2 \rho_q \right)} \\ &= - \frac{\Omega_q}{z + M(q, z)}. \end{aligned} \quad (14)$$

The force–force correlation function $M(q, z)$ is given by

$$M(q, z) = \frac{m}{q^2} \left(\bar{\mathcal{P}} \mathcal{L}^2 \rho_q \left| \frac{1}{\tilde{\mathcal{J}} \tilde{\mathcal{L}} \tilde{\mathcal{J}} - z} \right| \bar{\mathcal{P}} \mathcal{L}^2 \rho_q \right). \quad (15)$$

From equation (10) we thus obtain for $\Phi(q, z)$,

$$\Phi(q, z) = - \frac{1}{z - \frac{\Omega_q}{z + M(q, z)}}. \quad (16)$$

From the equation of motion for the Hamiltonian H (equation 4) we have

$$\begin{aligned}\mathcal{L}^2 \rho_q &= [H, [H, \rho_q]] \\ &= \left\{ \frac{1}{2m} \sum_k [q^2 + 2\mathbf{q} \cdot \mathbf{k}] a_{q+k}^\dagger a_k + \frac{1}{m} \sum_k V(k)(\mathbf{q} \cdot \mathbf{k}) \rho_k \rho_{\mathbf{q}-\mathbf{k}} \right. \\ &\quad \left. + \frac{1}{m} \sum_k W(k)(\mathbf{q} \cdot \mathbf{k}) \rho_{\mathbf{q}-\mathbf{k}} \right\}, \quad (17)\end{aligned}$$

so equation (15) becomes

$$\begin{aligned}M(q, z) &= \frac{1}{mq^2} \left(\left[\sum_k V(k)(\mathbf{q} \cdot \mathbf{k}) \rho_k \rho_{\mathbf{q}-\mathbf{k}} + \sum_k W(k)(\mathbf{q} \cdot \mathbf{k}) \rho_{\mathbf{q}-\mathbf{k}} \right] \bar{\mathcal{P}} \left| \frac{1}{\bar{\mathcal{J}} \bar{\mathcal{L}} \bar{\mathcal{J}} - z} \right| \bar{\mathcal{P}} \right. \\ &\quad \left. \times \left[\sum_k V(k)(\mathbf{q} \cdot \mathbf{k}) \rho_k \rho_{\mathbf{q}-\mathbf{k}} + \sum_k W(k)(\mathbf{q} \cdot \mathbf{k}) \rho_{\mathbf{q}-\mathbf{k}} \right] \right). \quad (18)\end{aligned}$$

For weak scattering from defects we neglect the cross terms in equation (18) and obtain the decoupled expression:

$$M(q, z) = M_{ee}(q, z) + M_{de}(q, z), \quad (19)$$

where $M_{de}(q, z)$ is the contribution to $M(q, z)$ from electrons scattering from defects. It is a two-point density relaxation function:

$$M_{de}(q, z) = \frac{1}{mq^2} \sum_{kk'} W(k)(\mathbf{q} \cdot \mathbf{k}) \left(\rho_{\mathbf{q}-\mathbf{k}} \bar{\mathcal{P}} \left| \frac{1}{\bar{\mathcal{J}} \bar{\mathcal{L}} \bar{\mathcal{J}} - z} \right| \bar{\mathcal{P}} \rho_{\mathbf{q}-\mathbf{k}'} \right) W(k')(\mathbf{q} \cdot \mathbf{k}'). \quad (20)$$

We write

$$M_{de}(q, t) = \frac{1}{mq^2} \sum_{kk'} \langle W(k)(\mathbf{q} \cdot \mathbf{k}) W(k')(\mathbf{q} \cdot \mathbf{k}') \rangle \left(\rho_{\mathbf{q}-\mathbf{k}}(t) \rho_{\mathbf{q}-\mathbf{k}'}(0) \right) \quad (21)$$

$$= \sum_k \left[W^2(k) S_i(k)(\mathbf{q} \cdot \mathbf{k})^2 \right] \times \left[\chi(|\mathbf{q} - \mathbf{k}|) \Phi(|\mathbf{q} - \mathbf{k}|, t) \right], \quad (22)$$

where $S_i(k)$ is the impurity structure factor. This is unity for a random distribution of uncorrelated impurities.

In equation (19) $M_{ee}(q, z)$ is the contribution from electron-electron scattering. It is a four-point relaxation function:

$$M_{ee}(q, z) = \frac{1}{mq^2} \sum_{kk'} V(k)(\mathbf{q} \cdot \mathbf{k}) \left(\rho_k \rho_{\mathbf{q}-\mathbf{k}} \bar{\mathcal{P}} \left| \frac{1}{\bar{\mathcal{J}} \bar{\mathcal{L}} \bar{\mathcal{J}} - z} \right| \bar{\mathcal{P}} \rho_{k'} \rho_{\mathbf{q}-\mathbf{k}'} \right) V(k')(\mathbf{q} \cdot \mathbf{k}'). \quad (23)$$

In order to numerically evaluate an expression like equation (23), one would usually start by factorising four-point density correlation functions as products of two-point correlation functions. For a classical system this means the four-point relaxation function $M_{ee}(q, z)$ in equation (23) could also be factorised since the functions are equivalent. The same approximation cannot be used for a quantum system since the two functions are not identical except in the high temperature limit:

$$\langle A|B\rangle_\omega = \frac{1 - e^{-\beta\hbar\omega}}{\hbar\omega} (A|B)_\omega. \quad (24)$$

However, in order to identify the transition we only need $\omega = 0$, and in this limit the two quantum functions are identical. This permits us to factorise the quantum four-point relaxation function. With this factorisation equation (23) becomes

$$\begin{aligned} M_{ee}(q, t) = \frac{1}{mq^2} \sum_{kk'} V(k)(\mathbf{q}, \mathbf{k}) & \left[\langle \rho_k(t) \rho_{k'}(0) \rangle \langle \rho_{\mathbf{q}-\mathbf{k}}(t) \rho_{\mathbf{q}-\mathbf{k}'}(0) \rangle \right. \\ & \left. + \langle \rho_k(t) \rho_{\mathbf{q}-\mathbf{k}'}(0) \rangle \langle \rho_{\mathbf{q}-\mathbf{k}}(t) \rho_{k'}(0) \rangle \right] V(k')(\mathbf{q}, \mathbf{k}'). \end{aligned} \quad (25)$$

The glass state is identified with infinite relaxation time. We define

$$M(q) \equiv \lim_{t \rightarrow \infty} M(q, t) = M_{ee}(q) + M_{de}(q), \quad (26)$$

obtaining for our final expressions,

$$\begin{aligned} M_{ee}(q) = \frac{1}{2mq^2} \sum_{q'} & \left[V(q')(\mathbf{q} \cdot \mathbf{q}') + V(q - \mathbf{q}')(\mathbf{q} \cdot (\mathbf{q} - \mathbf{q}')) \right]^2 \\ & \times \chi(q') f(q') \chi(|\mathbf{q} - \mathbf{q}'|) f(|\mathbf{q} - \mathbf{q}'|), \end{aligned} \quad (27)$$

$$M_{de}(q) = \frac{n_i}{mq^2} \sum_{q'} \left[W^2(q')(\mathbf{q} \cdot \mathbf{q}')^2 \right] \chi(|\mathbf{q} - \mathbf{q}'|) f(|\mathbf{q} - \mathbf{q}'|). \quad (28)$$

Using equations (2), (10) and (14), we write the order parameter $f(q)$ in terms of $M(q)$,

$$f(q) = \frac{1}{1 + \Omega_q/M(q)}. \quad (29)$$

Since $M(q)$ itself depends on $f(q)$, equation (29) when combined with equations (26)–(28) form a closed set of non-linear equations for $f(q)$. We numerically solve these iteratively until the solutions are self-consistent.

The correlations between electrons are taken into account through the static susceptibility $\chi(q)$. We have found that the key property in $\chi(q)$ determining the transition is the area occupied by the density exclusion region in the exchange-correlation hole. We assume in the strongly correlated region that the overall shape of the exchange-correlation hole is not greatly affected by low levels of disorder.

The static susceptibility terms in equations (27) and (28) insert into the vertex part of the memory function the information about the strong exchange-correlation hole. The static structure factor $S(q)$, taken from diffusion quantum Monte Carlo numerical simulations (Tanatar and Ceperley 1989), is used to determine a static local field factor $G(q)$ in the expression,

$$S(q) = \int_0^\infty d\omega \frac{\chi_0(q, i\omega)}{1 + V(q)[1 - G(q)]\chi_0(q, i\omega)}. \quad (30)$$

The $G(q)$ then determines $\chi(q)$ using

$$\chi(q) = \frac{\chi_0(q)}{1 + V(q)[1 - G(q)]\chi_0(q)}. \quad (31)$$

3. Results

For a given electron density and level of defects we can solve equations (26)–(29) for $f(q)$. If the disorder level is sufficiently small we find there are only the trivial solutions to the equations, that is $f(q) = 0$, implying a delocalised state. By increasing the disorder, at a certain critical level of disorder the $f(q)$ jumps discontinuously from zero. This behaviour implies that there is a discontinuous metal–insulator transition to a localised state. For $r_s > 9$ the transition from liquid to solid is discontinuous. For a lower level of disorder the transition occurs at a smaller carrier density (that is, for stronger electron correlations). If we continue to increase the electron density for a fixed level of disorder, then by $r_s \simeq 8$ the $f(q)$ grows continuously from zero. This leads to a continuous localisation transition from liquid to solid.

The Fourier transform of $f(q)$,

$$F(r) = \int dq e^{i\mathbf{q}\cdot\mathbf{r}} f(q), \quad (32)$$

gives the probability in the infinite time limit of finding an electron at distance r from the origin if there was an electron at the origin at $t = 0$. In Fig. 1 we show $rF(r)$ for fixed electron densities corresponding to $r_s = 10$ and $r_s = 5$ for a range of impurity densities n_i . We define an impurity concentration parameter $c_i = n_i/n_e$. For the lower density, $r_s = 10$, the $F(r)$ is everywhere zero when $c_i < 0.085$. At $c_i = 0.085$ there is a discontinuous jump in $F(r)$ corresponding to a localisation length of the order of the inter-particle spacing. Further increasing n_i increases the degree of localisation.

In contrast at $r_s = 5$, the $F(r)$ increases continuously from zero with impurity concentration. Note that the vertical scales of both parts of Fig. 1 are different. The overall localisation at $r_s = 5$ is much weaker than at $r_s = 10$ and corresponds to a significantly longer localisation length.

We associate this evolution of $F(r)$ at $r_s = 5$ with a continuous localisation–delocalisation transition as a function of c_i . We use the Ioffe–Regel criterion that localisation occurs when the electron localisation length drops below $rk_F \simeq 1$ (see Mott 1974). We determine a localisation length from the integrated probability

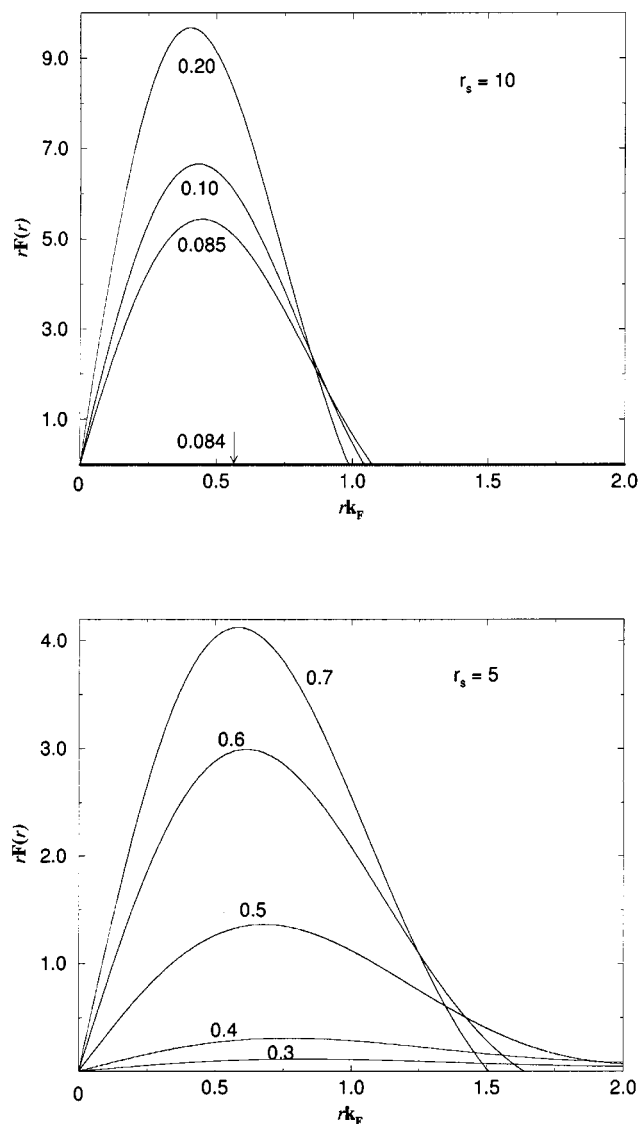


Fig. 1. Infinite time limit probability $F(r)$ of finding an electron at distance r from the origin for electron densities $r_s = 10$ and $r_s = 5$. Labels are the impurity concentration $c_i = n_i/n_e$.

$\int d^2r F(r)$ out to the nearest neighbour distance. Fig. 2 shows the boundary between the localised and delocalised states as a function of impurity and electron density. This phase boundary merges around $r_s = 9$ with the boundary for the discontinuous transition we have previously reported (Thakur and Neilson 1996).

We conclude that at low electron densities corresponding to $r_s > 10$ the electron correlations are strong enough to drive the system through a discontinuous localisation transition to the strongly localised disordered electron glass we have

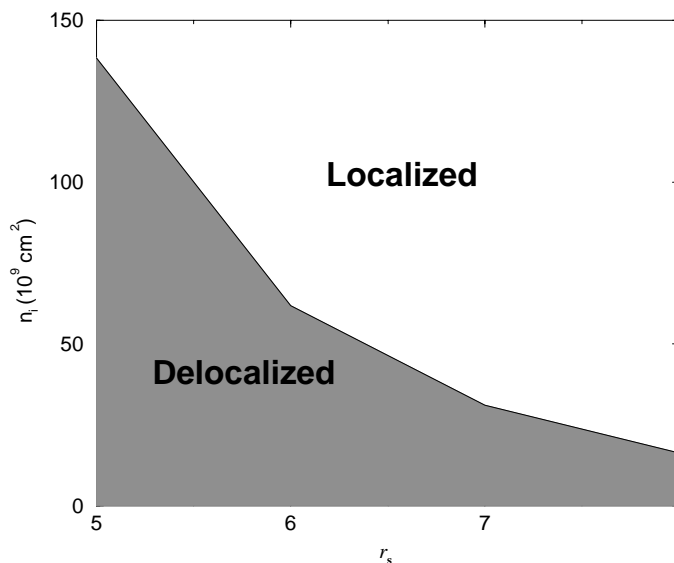


Fig. 2. Phase boundary for continuous localisation to delocalisation transition as a function of impurity density n_i and electron density parameter r_s .

previously discussed (Thakur and Neilson 1994, 1999). We now report that there is a range of intermediate electron densities above $r_s \lesssim 9$, down to at least $r_s \simeq 5$, where the electron correlations remain strong enough for a continuous delocalisation–localisation transition to occur when the density of impurities is increased.

Acknowledgments

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