Generation capacity adequacy evaluation based on peak load consideration

R K Saket¹, R C Bansal², G Singh³

¹ Electrical Engineering Department, Banaras Hindu University, UP, India

²School of Engineering and Physics, The University of the South Pacific, Suva, Fiji

³College of Engineering & Technology, Allahabad Agricultural Institute- Deemed University, Allahabad (UP) India

ABSTRACT

This paper describes a new methodology based on peak load consideration for reliability evaluation of a generation system. The methodology has been developed using Simpson's $(1/3)^{rd}$ rule for loss of load probability (LOLP) evaluation. Load models are considered as straight line and stepped load duration curve for reliability evaluation. The LOLP evaluation is based on peak load consideration. The probability of load exceeding the generating capacity has also been considered in LOLP evaluation.

Keywords: Generation capacity, loss of load probability, mean capacity, mean load, peak load, stepped load duration curve, success probability

1 INTRODUCTION

Loss of load probability (LOLP) is one of the most commonly used index for planning generation capacity. This index is generally obtained by convolving generation model with a load model. The generation system model used is known as capacity outage probability table. The traditional and well accepted algorithms for generation system modeling based on recursive procedures have been discussed in (Billinton 1970; Endreyni 1978; Dhilon and Singh 1981; Bansal et al. 2002; Saket and Bansal 2006; Wang and Billinton 2003; Li and Billinton 2003; Wang et al. 2002). Such procedures are theoretically accurate for calculating the discrete probability distribution of the generation capacity outages. These discrete probability distributions are the outcome of (i) failure density functions, (ii) repair density functions, and (iii) derived availability and unavailability functions. Moreover, in such modeling the failure and repair distribution functions of each unit is assumed to be independent. Hence, for identical units binomial distribution is adopted. Recently many researchers have used continuous probability distribution as an approximation for capacity outage probability distribution (Alavi-Sereshki and Singh 1991). (Alavi-Sereshki and Singh 1991) have presented a generalized approach to use continuous distribution generating capacity approximation for reliability evaluation. Gram-Charlier's expression is shown to have a special of this formulation. Potential of this method lies in the fact that any continuous distribution function can be examined for its suitability for modeling generation systems or in fact any discrete distribution function. LOLP distribution is based on two and three parameters Gamma distribution functions. Loss of load expression in days per year has been calculated based on daily peaks. Approach of (Tian et al. 1989) is based on Laguerre polynomials, which uses Gamma distribution as the basic building blocks.

(Saket *et al.* 2001) have presented a methodology for evaluating the probability of failure based on continuous load and generation model at safety factor considerations. A frequency and duration approach for generation reliability evaluation using methods of stages has been developed in (Kim and Singh 1993). Their approach postulates multi parameter distribution for the probability and frequency of generating capacity outages and parameters of these distributions are obtained from the generating unit parameters by using the moment matching techniques. (Malik *et al.* 1997) modeled adequately pumped storage units and used proper estimates of their load leveling benefits in generation capacity planning. The technique used is frequency and duration approach coupled with the equivalent load duration curve method.

This paper presents a new methodology based on peak load considerations for reliability evaluation of a generation system. The methodology has been developed using Simpson's $(1/3)^{rd}$ rule for LOLP evaluation. The probability of load exceeding the generating capacity has also been considered in LOLP evaluation.

1.1 NOMENCLATURE

- α_i = Relative frequency of occurrence.
- P_{fi} = Failure probability for L_i^{th} load level

 σ_c = Standard deviation of generation capacity

- σ_d = Standard deviation of demand
- $\sigma_{\rm L}$ = Standard deviation of load
- C = Capacity of the generating station
- \overline{C} = Mean capacity of the plant
- Ci = Average generation capacity available for L_ith load level
- f(L) = Normal distribution
- LOLP = Loss of load probability
- L = Total load in MW
- L = Mean load in MW
- \overline{L}_i = Mean L_i^{th} load level
- n = Total duration of study
- $n_i = Duration for which L_i^{th} load level occurs$
- P_d = Demand at the generating stations
- $\overline{\mathbf{P}}_{d}$ = Average or mean load at the plant
- P_F = Probability of failure of generating power station
- P_S = Probability of success of generating power station
- S = Safety factor

2 LOAD MODELS OF POWER SYSTEM

The capacity and load models are essentially merged using a suitable analytical technique to obtain a reliability index. Various load models very often used in reliability study (Billinton 1970) are:

- 1. **CONSTANT LOAD**: This indicates that load remains constant throughout the period of study. This is a very simplified model of load and is not true especially for modern large power network, which is interconnected, and possesses very diversified loads.
- LOAD HAVING NORMAL DISTRIBUTION: It includes loads, which have variations around constant mean. Applicability of such loads can be where isolated system needing a specific or non-diversified load pattern. The load distribution function is written as

$$f(L) = \frac{1}{\sigma_L \sqrt{2\pi}} e^{-0.5 \left(\frac{L-\bar{L}}{\sigma_L}\right)^2} \qquad (1)$$

- LOAD DURATION CURVE: This is one of the most practical load model used in practice to evaluate LOLP index. It is constructed from chronological daily load curve.
- 4. **TWO LEVEL REPRESENTATION OF DAILY LOAD**: In two level daily load duration curve, which is a cumulative curve of daily peak loads, the variation of load within a day are not recognized in it. Hence, LOLP is a crude approximation of system failure probability and, hence, system failure frequency is not calculated using load duration curve. For system failure frequency calculations a two level load representation is used and Markov modeling is used to represent transition of load from one load level to another.

It is assumed that low load is always same every day and designated as L_o . The peak load ' L_i ' vary daily and may occur in random sequence with relative frequency of occurrence. The mean duration t_i of the peak is described by the exposure factor $e = t_i / d_o$. Where ' d_o ' is the length of the load cycle typically may be 24 hours. The factor 'e' is considered the same every day, its magnitude lies between 0 and 1. It has been observed that results are not much sensitive to the value of 'e'. If we assume that total number of days are 'n' and peak level L_i occurs for n_i days, then relative frequency of occurrence of this peak level can be written as (Billinton 1970),

$$\alpha_{i} = \frac{n_{i}}{n}$$
(2)
$$\sum_{i} \alpha_{i} = 1$$
(3)

and

Now it is observed that at any instant of time, the load will be in any of states represented by levels L_0 , L_1 , L_2 , L_3 L_i . Using Markov modeling the probabilities of each state rate of transitions can be put on the diagram.

Total duration of study = nd_o

Duration of load level $L_0 = (1-e)d_0n$

Hence, Probability of L_o,
$$P_{L_o} = \frac{(1-e)d_on}{n.d_o} = (1-e)$$
 (4)

Duration of load level $L_i = n_i e d_o$

Probability of L_i,
$$P_{Li} = \frac{n_i \cdot e \cdot d_o}{n \cdot d_o} = \alpha_i e$$
 (5)

Further it is pointed out that transition from L_o to any higher-level L_i state will take place at a rate

$$(\lambda + i) = [L_i / (1-e) d_o]$$
(6)

Transition from higher level $L_{\rm i}$ will take place to $L_{\rm o}$ only with transition rate –

$$(\lambda + i) = 1 / e d_o \tag{7}$$

This way (6) and (7) along with steady state probabilities (4) and (5) specify the Markov modeling of two level representation of the daily load. This type of load modeling is a part of frequency and duration calculation.

3 EVALUATION OF PROBABILITY OF FAILURE: LOAD AS WELL AS GENERATION CAPACITY ASSUMED AS NORMAL DISTRIBUTION

The load on the generating station is assumed to have a Gaussian distribution for a specified time interval.

$$f(P_d) = \frac{1}{\sigma_d \sqrt{2\pi}} e^{-0.5 \left(\frac{P_d - \overline{P_d}}{\sigma_d}\right)^2}$$
(8)

The aggregate generation capacity model is approximated as Gaussian (Khan 1998).

$$f(C) = \frac{1}{\sigma_c \sqrt{2\pi}} e^{-0.5 \left[\frac{C-\overline{C}}{\sigma_c}\right]^2}$$
(9)

The $P_{\rm F}$ for the above generation and load models can be written as

 $P_{F}=(1-P_{s})$ (10) P_s can be expressed as

$$P_{s} = \int_{-\infty}^{\infty} \int_{-\infty}^{C} \frac{1}{2\pi\sigma_{c}\sigma_{d}} e^{-0.5\left[\frac{C-\overline{C}}{\sigma_{c}}\right]^{2}} e^{-0.5\left(\frac{P_{d}-\overline{P_{d}}}{\sigma_{d}}\right)^{2}} dcdP_{d}$$
(11)

Putting. $x = \frac{C - \overline{C}}{\sigma_c}$ and $y = \frac{P_d - \overline{P_d}}{\sigma_d}$

Equation (11) can be written as follows:

$$P_{s} = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{z} e^{-0.5\left(x^{2}+y^{2}\right)} dy \right] dx \qquad (12)$$

where $Z = \frac{\sigma_c x + \overline{C} - \overline{P_d}}{\sigma_d}$

Further making substitution in (12) as follows.

$$x' = x \cos \theta + y \sin \theta$$
$$y' = -x \sin \theta + y \cos \theta$$

above transformation corresponds to rotation of axis by angle θ (Saket *et al.* 2001).

Define,

$$\sin\theta = \frac{\sigma_C}{\sqrt{P_d^2 + \sigma_C^2}} \quad \text{and} \ \cos\theta = \frac{\sigma_C}{\sqrt{P_d^2 + \sigma_C^2}}$$

In view of the above substitution (12) can be written as

$$P_{s} = \int_{-\infty-\infty}^{\infty} \int_{2\pi}^{p} \frac{1}{2\pi} e^{-0.5(x'^{2}+y'^{2})} dy' dx' \qquad (13)$$

where $\beta = \frac{C - P_d}{\sqrt{\sigma_d^2 + \sigma_c^2}}$

The limit β comes out to be independent of x'. Further (13) can be simplified as

$$P_{s} = \int_{-\infty}^{\beta} \left[\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-0.5(x'^{2})} dx' \right] e^{-0.5(y'^{2})} dy'$$
$$= \int_{-\infty}^{+\beta} \frac{1}{\sqrt{2\pi}} e^{-0.5(y'^{2})} dy' = \phi(\beta) \qquad (14)$$

in fact $\varphi(\beta)$ is the area under the normal distribution curve having mean = 0 and standard deviation $\int_{-\infty}^{\beta} [N(0,1)] = 1$ and this value can be conveniently obtained from standard tabulated data. Daily variation in the load can be accounted by predicting the various load levels P_{di} . Relative frequency of occurrence of these levels are assumed to be L_0 , L_1 , L_2 ,..., L_i and frequency of occurrence as $\alpha_0 \alpha_1 \alpha_2$,, α_i . For each load level probability of failure can be calculated and overall probability of failure is given as

$$P_F = \sum_i \alpha_i P_{fi} \tag{15}$$

From (15) it follows that

$$P_{fi} = 1 - \phi(\beta_i)$$

Hence, overall failure probability is given as follows

$$\mathbf{P}_{\mathrm{F}} = \sum_{i} \alpha_{i} \left[1 - \phi(\boldsymbol{\beta}_{i}) \right] \tag{16}$$

Above expression can be used to calculate failure probabilities. It is obvious that failure probability dependents on α_i (can be found using eqn. 2), and β_i given as

$$\beta_i = \frac{\overline{C_i - L_i}}{\sqrt{(\sigma_{d_i}^2 + \sigma_{C_i}^2)}}$$
(17)

4 GENERATION CAPACITY ADEQUACY EVALUATION BASED ON SAFETY FACTOR CONCEPTS AND PEAK LOAD CONSIDERATION

The probability distribution function of generation capacity is assumed to be Gaussian as earlier. Further, it is assumed that peak loading dominates over low-level loading, whereas, the probability of failure under low load level condition is negligible. Let us assume that P_{dmax} is the peak loading on the generating system. The safety factor 'S' is defined as

$$S = C/P_{dmax}$$

It is obvious that the generating capacity 'C' has normal distribution. Obviously 'S' is also a random variable. Since P_{dmax} is assumed to be constant. The distribution 'S' will also be normal and is given as (Alavi-Sereshki and Singh 1991).

$$f_s = \frac{P_{d\max}}{\sqrt{2\pi\sigma_c}} e^{-0.5\left(\frac{P_{d\max}S-\overline{C}}{\sigma_c}\right)}$$
(18)

This means that safety factor 'S' again is a Guassion distribution with mean.

$$\overline{S} = \frac{C}{P_{d \max}} \text{ and } \sigma_s = \frac{\sigma_c}{P_{d \max}}$$
 (19)

Probability of failure is given as

$$P_{F} = \int_{-\infty}^{1} \frac{P_{d \max}}{\sqrt{2\pi}\sigma_{s}} e^{-0.5\left(\frac{s-\bar{c}/Pd\max}{\sigma_{s}}\right)^{2}} ds = \phi \left[\frac{1-\frac{\bar{C}}{P_{d \max}}}{\frac{\sigma_{c}}{P_{d \max}}}\right]$$
(20)

A curve can be plotted $P_F v/s \frac{\overline{C}}{P_{d max}}$, which can be used as

standard curve for evaluating the generating capacity, using (20).

5 EVALUATION OF PROBABILITY OF FAILURE: LOAD DURATION CURVE AS LOAD MODEL

Failure probability has been evaluated with more realistic model as load duration curve as shown in Figure 1 (a). Generating model adopted is Normal distribution function as in Section 1. The evaluation is based on maximum average generation capacity available. The probability of load exceeding the capacity using load duration curve of Figure 1 (a). can be written as

$$LOLP = \int_{0}^{100} \frac{t}{100} \left[\int_{-\infty}^{P_{d}(t)} \frac{1}{\sqrt{2\pi\sigma_{c}}} e^{-0.5 \left(\frac{C-\bar{C}}{\sigma_{c}}\right)^{2}} dc \right] dt \qquad (21)$$

Putting, $\frac{C-\bar{C}}{\sigma_{c}} = Z$

The LOLP expression of (21) can be expressed as

$$LOLP = \int_{0}^{100} \frac{t}{100} \left[\int_{-\infty}^{P_{d}(t)} \frac{1}{\sqrt{2\pi\sigma_{c}}} e^{-0.5(Z)^{2}} dZ \right] dt$$
$$= \int_{0}^{100} \frac{t}{100} \varphi \left(\frac{P_{d}(t) - \overline{C}}{\sigma_{c}} \right) dt$$
(22)

From the Figure 1 (a). inner integral represents the area under the [N(0,1)] curve for the various values of 't' and, hence, has been replaced by the $\oint \left[\frac{\mathbf{P}_{d}(t)-\bar{C}}{\sigma_{c}}\right]^{\sigma_{c}}$

function.

Thus (22) can be evaluated by any one of the methods of area evaluation. Simpson's rule has been used to evaluate the LOLP. Evaluation of (22) is simplified if load duration curve is approximated using stepped load duration curve. This may be possible if daily load curve can be approximated as multilevel representation as is the case with Markov modeling of load in frequency and duration calculations.



Figure 1(a) Load duration curve

6 RESULTS AND DISCUSSION

The probability of failure has been evaluated for a load model as load duration curve as a straight line and a stepped load duration curves shown in Figure 1 (b) and (c). respectively. The generation capacity data for evaluation of LOLP are given in Table 1.



Figure 1(b) Load duration curve as a straight line



Figure 1(c) Stepped load duration curve

capacity	$\sigma_{\rm C} =$				
in MW	5% of	10%	15 %	20%	25%
	\bar{C}	of \bar{C}	of \bar{C}	of \bar{C}	of \bar{C}
750	107.14	75	112.5	150	187.5
800	114.28	80	120.0	160	200.0
850	121.43	85	127.5	170	212.5
900	128.57	90	135.0	180	225.0
950	135.71	95	142.5	190	237.5
1000	142.86	100	150.0	200	250.0
1050	150.00	105	157.5	210	262.5
1100	157.44	110	165.0	220	275.0
1150	164.28	115	172.5	230	287.5
1200	171.43	120	180.0	240	300.0
1250	178.00	125	187.5	250	312.5
1300	185.71	130	195.0	260	325.0
1350	192.86	135	202.5	270	337.5

Table 1. Generation capacity data for evaluating lolp with load duration curve

In first case, load duration curve is assumed to be a straight line assuming $P_{dmax} = 1000$ MW and $P_{dmin} = 500$ MW as shown in Figure 1 (b). Simpson's $(1/3)^{rd}$ rule has been utilized to evaluate the LOLP for various values of generating capacity using (22). The Figure 2 (a) and (b). show the plots of LOLP v/s generating capacity for load duration curve for 20 steps and 10 steps respectively. The curves A, B, C, D, and E show the relation between LOLP and \bar{c} for the $\sigma_C = 5\%$, 10%, 15%, 20% and 25% respectively.



Figure 2 (a) LOLP v/s generating capacity for load duration curve (20 steps)

It is observed from curves B and C for 20 steps of load duration curve in Figure 2 (a). that for the same generating capacity of the power system, the LOLP with $\sigma_C = 15\%$ of \bar{C} is higher than $\sigma_C = 10\%$ of \bar{C} . Similarly from curves D and E it is observed that the LOLP of curve E is greater than curve D at same generating capacity. This is due to

the fact that large uncertainty involved in generating capacity function. From Figure 2 (b). it is seen for 10 steps of load duration curve that LOLP of curve E is higher than A, B, C or D at same capacity. In summary, if the standard deviation of generating capacity increases, the failure probability of system increases at same generating capacity.



Figure 2(b) LOLP v/s generating capacity for load duration curve (10 steps)

The LOLP has been evaluated for stepped load duration curve for same generating capacity of Data given in Table I. The failure probability for 5 steps has been evaluated as shown in Figure 3 (a).



The curves A, B, C and D show the relations of LOLP and \bar{C} at $\sigma_{\rm C} = 10\%$, 15% 20%, and 25% of capacity respectively. The failure probability of the system increases with increasing values of σ_c .

Figure 3 (b) and (c). show the relation between LOLP and generating capacity for 10 steps and 4 steps respectively. LOLP of power system decreases with increases in generating capacity. For same generating capacity available, if σ_C increases, the LOLP also increases.

Figure 4 (a), (b) and (c) represent the relation between LOLP and various generating capacity available in Table 1 with effects of various σ_c . The LOLP has been evaluated for evaluation for various steps of load duration curve. The curves have been plotted for 10, 20, and 40 steps using Simpson's $(1/3)^{rd}$ rule. The LOLP of system increases with increments in σ_c due to large uncertainty involved in the generating capacity distribution factor.



Figure 3(b) LOLP v/s generating capacity for load duration curve (10 steps)



Figure 4(a) LOLP v/s generating capacity using Simpson's 1/3 rule (10 steps)



Figure 3(c) LOLP v/s generating capacity for load duration curve (4 steps)



Figure 4(b) LOLP v/s generating capacity using Simpson's 1/3 rule (20 steps)



Figure 4(c) LOLP v/s generating capacity using Simpson's 1/3 rule (40 steps)

From above discussions, it is obvious that failures probability of system decreases with increasing the values of generating capacity so the reliability of the system increases. The failure probability of the system increases with increase in σ_C due to large uncertainty involved in the generating capacity distribution factor.

7 CONCLUSIONS

Methodologies have been introduced to evaluate generating system reliability using different load and generation models. The evaluation of LOLP have been based on load generation models as

- 1. Peak load considerations assuming that low load level, failure probability is negligible and normal distribution is assumed as generation capacity model.
- 2. Safety factor considerations having constant peak load and Normal distribution as generating capacity.
- 3. Load duration curve as load models and Normal distribution model as generating capacity.
- 4. Stepped duration curve as load model of the system and continuous distribution function as generating capacity model.

Further in all cases it is confirmed that LOLP increases with an increase in variance of load and reliability of system decreases.

REFERENCES

- Alavi-Sereshki, M.M. and Singh, C. 1991. A generalized continuous distribution approach for generating capacity reliability evaluation and its applications. *IEEE Trans. Power Systems* **6**(1), 16-21.
- Bansal, R.C. Bhatti, T.S. and Kothari, D.P. 2002. Discussion of bibliography on the application of probability method in power system reliability evaluation. *IEEE Trans. Power Systems* **17(3)**, 924.
- Billinton, R. 1970. *Power system reliability evaluation*. Gordon and Breach Publishers, New York.
- Dhilon, B.S. and Singh, C. 1981. *Engineering reliability*. John Wiley, New York.
- Endreyni, J. 1978. *Reliability modeling in electric power* systems. John Wiley and Sons, New York.
- Khan, M.E. 1998. Bulk load points reliability evaluation using a security based model. *IEEE Trans. Power Systems* **13(2)**, 456-463.
- Kim, J.O. and Singh, C. 1993. A frequency and duration approach for generation reliability evaluation using the method of stages. *IEEE Trans. Power Systems* **8(1)**, 207-215.
- Malik, A.S. and Cory, B.J. 1997. An application of frequency and duration approach in generation planning. *IEEE Trans. Power Systems* **12(3)**, 1076-1081.
- Li, W. and Billinton, R. 2003. Common cause outage models in power system reliability evaluation, *IEEE Transactions on Power Systems* 18(2), 2003, 966-968.
- Saket, R.K. Arya, L.D. and Choube, S.C. 2001. Generation system adequacy evaluation using probability theory. *Journal of the Institution of Engineers (India)* **81**, 170-174.
- Saket, R.K. and Bansal, R.C. March 2006. Reliability evaluation of interconnected composite power system considering voltage stability at continuation power flow. *International Conference on Power System Operation in Deregulated Regime*, Department of Electrical Engineering, Institute of Technology, Banaras Hindu University, Varanasi, India, 339-348.
- Tian, W.D., Sutanto, D., Lee, Y.B. and Outhred, H.R. 1989. Cumulant based probabilistic power system simulation using laguerre polynomials, *IEEE Trans. Energy Conversion* **4(4)**, 567-574.
- Wang, P. and Billinton, R. 2003. Reliability assessment of restructured power system using reliability network equivalent techniques. *IEE Proc.- Generation Transmission and Distribution* **150(5)**, 555-560.
- Wang, P., Billinton, R. and Goel, L. 2002. Unreliability cost assessment of an electric power system using reliability network equivalent approaches. *IEEE Transactions on Power Systems* **13(3)**, 549-556.