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A fuzzy goal programming approach in stochastic multivariate stratified sample surveys

Neha Gupta, Irfan Ali, Shafiullah and Abdul Bari

Department of Statistics & Operations Research, Aligarh Muslim University, Aligarh, India.

Abstract

This paper deals with fuzzy goal programming (FGP) approach to stochastic multivariate stratified sampling with non linear objective function and probabilistic non linear cost constraint which is formulated as a multiobjective non linear programming problem (MONLPP). In the model formulation of the problem, we first determine the individual best solution of the objective functions subject to the system constraints and construct the non linear membership functions of each objective. The non linear membership functions are then transformed into equivalent linear membership functions by first order Taylor series at the individual best solution point. Fuzzy goal programming approach is then used to achieve maximum degree of each of the membership goals by minimizing negative deviational variables and finally obtain the compromise allocation. A numerical example is presented to illustrate the computational procedure of the proposed approach.

Keywords: *Multiobjective programming, multivariate stratified sampling, compromise allocation, fuzzy goal programming*

1. Introduction

Fuzzy programming is based on the basic idea to determine a feasible solution that minimizes the largest weighted deviation from any goal. This is an optimization programme. It can be thought of as an extension or generalization of linear programming to handle multiple, normally conflicting objective measures. The use of the fuzzy set theory for decision problems with several conflicting objectives was first introduced by Zimmermann (1978). Thereafter, various versions of fuzzy programming (FP) have been investigated and widely circulated in literature. The use of the concept of membership function of fuzzy set theory for satisfactory decisions was first introduced by Lai in 1996. To formulate the FGP Model of the problem, the fuzzy goals of the objectives are determined by determining individual optimal solution. The fuzzy goals are then characterized by the associated membership functions which are transformed into linear membership functions by first order Taylor series. Recently many authors discuss fuzzy goal programming approach in different fields, some of them are Parra *et al*. (2001) who use this approach to portfolio selection problem, Sharma *et al.* (2007) work in the field of agriculture land allocation problems, Pramanik *et al*. (2011) apply FGP approach to Quadratic Bi-Level Mutiobjective Programming Problem (QBLMPP), Paruang *et al.* (2012) presents FGP model for machine loading problem and minimize an average machine error and the total setup time, Pramanik & Banerjee (2012) in transportation, Haseen *et al*. (2012) and Gupta *et al.* (2012) in sample surveys etc.

The problem of allocation for a multivariate stratified survey becomes complicated because an allocation that is optimal for one characteristic is usually far from optimal for other characteristics unless the characteristics are highly correlated. In such situations, i.e. in multivariate stratified surveys, we need a compromise criterion that gives an allocation which is optimum for all characteristics in some sense and we have to consider the allocation problem as a Multiobjective Non Linear Programming Problem (MNLPP) in which individual variances are to be minimized simultaneously subject to the cost constraint. Such an allocation may be called a "Compromise Allocation". Many authors have discussed the multivariate sample allocation problem. Among them are Kozak (2006), Diaz-Garcia and Cortez (2008), Khan *et al.* (2010), Khowaja *et al.* (2011). Diaz-Garcia *et al.* (2007) dealt with the case when sampling variances are random in the constraints. Javaid and Bakhshi (2009) considered the case of random costs and used modified E- model for solving the problem. Bakhshi *et al*. (2010) find the optimal Sample Numbers in Multivariate Stratified Sampling with a Probabilistic Cost Constraint. Recently, some other authors who discuss stochastic programming in sample surveys are Ali *et al*. (2013), Khan *et al*. (2011, 2012), Ghufran *et al*. (2011), Raghav *et al*. (2014), etc.

In the present paper the problem of finding the optimum compromise allocation is formulated as Multiobjective Non Linear Programming Problem (MNLPP) and a Fuzzy Goal Programming (FGP) approach is used to work out the compromise allocation in multivariate stratified surveys in which we define the membership functions of each objective function and then transform membership functions into equivalent linear membership functions by first order Taylor series and finally by forming a fuzzy goal programming model obtain a desired compromise allocation. A numerical example is also worked out to illustrate the computational details of the proposed approach.

2. Problem Formulations

Consider a multivariate population consisting of N units which is divided into L disjoint strata of sizes $N_1, N_2, ..., N_L$ such that $N = \sum_{h=1}^{L} N_h$. Suppose that p characteristics $(j = 1,..., p)$ are measured on each unit of the population. We assume that the strata boundaries are fixed in advance. Let n_h units be drawn without replacement from the h^{th} stratum $h = 1,..., L$. For j^{th} character, an unbiased estimate of the population mean $\bar{Y}_j$ $(j = 1,..., p)$, denoted by $\bar{y}_{jst}$, has its sampling variance

$$V(\bar{y}_{jst}) = \sum_{h=1}^{L}\left(\frac{1}{n_h} - \frac{1}{N_h}\right) W_h^2 S_{hj}^2 \quad , \quad j = 1,..., p \quad (1)$$

where $W_h = \frac{N_h}{N}$ is the stratum weight and $S_{hj}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_i}\left(y_{hji} - \bar{Y}_{ij}\right)^2$ is the variance for the j^{th} character in the h^{th} stratum. Let C be the upper limit on the total cost of the survey. The problem of optimal sample allocation involves determining the sample sizes $n_1, n_2, \ldots, n_L$ that minimize the variances of various characters under the given sampling budget C. Within any stratum the linear cost function is appropriate when the major item of cost is that of taking the measurements on each unit. If travel costs between units in a given stratum are substantial, empirical and mathematical studies indicate that the costs are better represented by the expression $\sum_{h=1}^{L} t_h \sqrt{n_h}$, where t_h is the travel cost incurred in enumerating a sample unit in the h^{th} stratum (Beardwood *et al.*, 1959; Cochran, 1977).

Assuming this non linear cost function one should have

$$C = c_0 + \sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} \quad (2)$$

where c_h; $h = 1, 2, ..., L$ denote the per unit cost of measurement in the h^{th} stratum, t_h is the travel cost for enumerating on a unit the j^{th} character in the h^{th} stratum and c_0 is the overhead cost.

The restrictions $2 \le n_h \le N_h$; $h = 1, 2, \ldots, L$ are introduced to obtain the estimates of the stratum variances and to avoid the problem of oversampling.

Thus the problem with non-linear cost function and ignoring the term independent of n_h, the allocation problem can be written as the following problem:

$$\left.\begin{array}{ll} \textit{Minimize} & \sum_{h=1}^{L} \frac{W_h^2 S_{jh}^2}{n_h} \\ \textit{Subject to} & \sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0 \le C \\ \textit{and} & 2 \le n_h \le N_h; \quad h = 1, 2, \ldots, L \end{array}\right\} j = 1, 2, \ldots, p \quad (3)$$

In many practical situations the measurement cost c_h and the travel cost t_h in the various strata are not fixed and may be considered as random. Let us assume that c_h and t_h, $h = 1,..., L$ are independently normally distributed random variables.

The formulated MNLPP (3) can be written in the following chance constrained programming form as:

$$\left.\begin{array}{ll} \textit{Minimize} & \sum_{h=1}^{L} \frac{W_h^2 S_{jh}^2}{n_h} \\ \textit{Subject to} & P\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0 \le C\right) \ge p_0 \\ \textit{and} & 2 \le n_h \le N_h; \quad h = 1, 2, \ldots, L \end{array}\right\} j = 1, 2, \ldots, p \quad (4)$$

where p_0, $0 < p_0 \le 1$ is a specified probability.

The costs c_h and t_h, $h = 1,..., L$ have been assumed to be independently normally distributed random variables. Then the function defined in (2), will also be normally distributed with mean

$E\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0\right)$ and variance $V\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0\right)$.

If $c_h \sim N(\mu_{ch}, \sigma_{ch}^2)$ *and* $t_h \sim N(\mu_{th}, \sigma_{th}^2)$, then the mean of the function $\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0\right)$ is obtained as:

$$\begin{aligned} E\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0\right) &= E\left(\sum_{h=1}^{L} c_h n_h\right) + E\left(\sum_{h=1}^{L} t_h \sqrt{n_h}\right) + c_0 \\ &= \sum_{h=1}^{L} n_h E(c_h) + \sum_{h=1}^{L} \sqrt{n_h} E(t_h) + c_0 \\ &= \sum_{h=1}^{L} n_h \mu_{ch} + \sum_{h=1}^{L} \sqrt{n_h} \mu_{th} + c_0 \end{aligned} \tag{5}$$

and the variance as:

$$\begin{aligned} V\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0\right) &= V\left(\sum_{h=1}^{L} c_h n_h\right) + V\left(\sum_{h=1}^{L} t_h \sqrt{n_h}\right) + c_0 \\ &= \sum_{h=1}^{L} n_h^2 E(c_h) + \sum_{h=1}^{L} n_h E(t_h) + c_0 \\ &= \sum_{h=1}^{L} n_h^2 \sigma_{ch}^2 + \sum_{h=1}^{L} n_h \sigma_{th}^2 + c_0 \end{aligned} \tag{6}$$

Now let $f(t) = \sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0$, then the chance constraint in (4) is given by

$P(f(t) \le C) \ge p_0$,

$or\ P\left\{\frac{f(t) - E(f(t))}{\sqrt{V(f(t))}} \le \frac{C - E(f(t))}{\sqrt{V(f(t))}}\right\} \ge p_0$,

where $\frac{f(t) - E(f(t))}{\sqrt{V(f(t))}}$ is a standard normal variate with mean zero and variance one. Thus the probability of realizing $f(t)$ less than or equal to C can be written as:

$$P(f(t) \le C) = \phi\left\{\frac{C - E(f(t))}{\sqrt{V(f(t))}}\right\}, \tag{7}$$

where $\phi(z)$ represents the cumulative density function of the standard normal variable evaluated at z. If K_α represents the value of the standard normal variate at which $\phi(K_\alpha) = p_0$, then the constraint (7) can be written as

$$\phi\left\{\frac{C - E(f(t))}{\sqrt{V(f(t))}}\right\} \ge \phi(K_\alpha) \tag{8}$$

The inequality will be satisfied only if

$$\left\{\frac{C - E(f(t))}{\sqrt{V(f(t))}}\right\} \ge (K_\alpha)$$

Or equivalently, $E(f(t)) + K_\alpha \sqrt{V(f(t))} \le C.$ (9)

Substituting from (5) and (6) in (9), we get

$$\left(\sum_{h=1}^{L} n_h \mu_{ch} + \sum_{h=1}^{L} \sqrt{n_h} \mu_{th} + c_0\right) + K_\alpha \sqrt{\sum_{h=1}^{L} n_h^2 \sigma_{ch}^2 + \sum_{h=1}^{L} n_h \sigma_{th}^2} \le C \tag{10}$$

Since the constants $\mu_{ch}, \mu_{th}, \sigma_{ch}$ *and* σ_{th} in (10) are unknown (by hypothesis). So we will use the estimators of mean $E\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0\right)$ and variance $V\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0\right)$ given by

$$\hat{E}\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0\right) = \sum_{h=1}^{L} \bar{c}_h n_h + \sum_{h=1}^{L} \bar{t}_h \sqrt{n_h} + c_0$$

and

$$\hat{V}\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0\right) = \left(\sum_{h=1}^{L} \sigma_{ch}^2 n_h^2 + \sum_{h=1}^{L} \sigma_{th}^2 n_h\right), \; say$$

where $\bar{c}_h, \bar{t}_h, \sigma_{ch}^2$ and σ_{th}^2 are the estimated means and variances from the sample.

Thus an equivalent deterministic constraint to the stochastic constraint is given by

$$\left(\sum_{h=1}^{L} \bar{c}_h n_h + \sum_{h=1}^{L} \bar{t}_h \sqrt{n_h} + c_0\right) + K_\alpha \sqrt{\left(\sum_{h=1}^{L} \sigma_{ch}^2 n_h^2 + \sum_{h=1}^{L} \sigma_{th}^2 n_h\right)} \le C \tag{11}$$

Now in multivariate stratified sample surveys the problem of allocation with p independent characteristics is formulated as a Multiobjective Nonlinear programming problem (MNLPP). The objective is to minimize the individual variances of the estimates of the population means of p characteristics simultaneously, subject to the non linear probabilistic cost constraint. The formulated problem is given as:

$$\left.\begin{aligned}
&\text{Minimize} && \begin{pmatrix} V(\bar{y}_{1st}) \\ \vdots \\ V(\bar{y}_{pst}) \end{pmatrix} \\
&\text{Subject to} && \hat{E}\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h} + c_0\right) + K_\alpha \sqrt{\hat{V}\left(\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} t_h \sqrt{n_h}\right)} \le C \\
& && 2 \le n_h \le N_h \\
&\text{and} && n_h \text{ are integers}; \quad h = 1, 2, \ldots, L.
\end{aligned}\right\} \tag{12}$$

To solve the problem (12) using stochastic programming, we first solve the following p Non Linear Programming Problems (NLPPs) for all the 'p' characteristics separately. The equivalent deterministic non linear programming problem to the stochastic programming problem is given by

$$\left.\begin{aligned}
&\text{Minimize} && \sum_{h=1}^{L} \frac{W_h^2 S_{jh}^2}{n_h} \\
&\text{Subject to} && \left(\sum_{h=1}^{L} \bar{c}_h n_h + \sum_{h=1}^{L} \bar{t}_h \sqrt{n_h} + c_0\right) + K_\alpha \sqrt{\left(\sum_{h=1}^{L} \sigma_{ch}^2 n_h^2 + \sum_{h=1}^{L} \sigma_{th}^2 n_h\right)} \le C \\
& && 2 \le n_h \le N_h \\
&\text{and} && n_h \text{ are integers}; \quad h = 1, 2, \ldots, L.
\end{aligned}\right\} \tag{13}$$

Let $\underline{n}_{jh}^* = \left(n_{j1}^*, n_{j2}^*, \ldots, n_{jL}^*\right)$ denote the solution to the j^{th} NLPP in (6) with V_j^* as the value of the objective function given by

$$V_j^* = \sum_{h=1}^{L} \frac{W_h^2 S_{hj}^2}{n_{hj}}; \quad j = 1, 2, \ldots, p \tag{14}$$

A reasonable criterion to work out a compromise allocation may be to 'Minimize the sum of the variances V_j; $j = 1,2,..., p$'. But in this paper a new approach called "Fuzzy Goal Programming" is used to obtain a compromise allocation and discussed in next section.

3. Compromise Solution Using Fuzzy Goal Programming

Present approach is discussed by Pramanik *et al*. (2011) and Pramanik and Banerjee (2012) and here the approach is used in accordance with the above formulated problem.

We now formulate the fuzzy programming model of multiobjective programming problem by transforming the objective functions $V_1, V_2, \dots, V_j$; $j = 1,2,\dots,p$ into fuzzy goals by means of assigning an imprecise aspiration level to each of them. Let $V_1^*, V_2^*, \dots, V_j^*$ be the optimal solutions of the each objective functions when calculated in isolation subject to the system constraints.

Then the fuzzy goals appear in the form: $V_j \gtrapprox V_j^*$; $j = 1,2,\dots,p$

Using the individual best solutions, we formulate a payoff matrix as follows:

$$\begin{bmatrix} & V_1(\bar{n}) & \dots & V_j(\bar{n}) \\ n_{1h}^* & V_1(n_{1h}^*) & \dots & V_j(n_{1h}^*) \\ \vdots & \vdots & \ddots & \vdots \\ n_{jh}^* & V_1(n_{jh}^*) & \dots & V_j(n_{jh}^*) \end{bmatrix}, \text{where } h = 1,2,\dots,L \text{ and } j = 1,2,\dots,p$$

where n_{jh}^* , $j = 1,2,\dots,p$ are the individual optimal points of each objective functions.

The maximum value of each column gives the upper tolerance limit for the objective functions and the minimum value of each column gives lower tolerance limit for the objective functions respectively.

The objective value, which is equal to or larger than V_j^* should be absolutely satisfactory to the objective functions. If the individual best solutions are identical, then a satisfactory optimal solution of the system is reached. However, this situation arises rarely because the objectives are conflicting in general.

The non linear membership function $\mu_j(\bar{n}), j = 1,\dots,p$ corresponding to the objective function $V_j(\bar{n}), j = 1,\dots,p$ can be formulated as:

$$\mu_j(\bar{n}) = \begin{cases} 0, & \text{if } V_j(\bar{n}) \geq V_j^U(\bar{n}) \\ 1 - \dfrac{V_j(\bar{n}) - V_j^L(\bar{n})}{V_j^U(\bar{n}) - V_j^L(\bar{n})}, & \text{if } V_j^L(\bar{n}) \leq V_j(\bar{n}) \leq V_j^U(\bar{n}), \qquad j = 1,\dots,p \\ 1, & \text{if } V_j(\bar{n}) \leq V_j^L(\bar{n}) \end{cases}$$

Here $V_j^U(\bar{n})$ and $V_j^L(\bar{n})$ are the upper and lower tolerance limits of the fuzzy objective goals.

Now the problem can be given as:

$$\left.\begin{aligned} & \max \mu_j(\bar{n}) \\ s.t.\ & \left(\sum_{h=1}^{L} \bar{c}_h n_h + \sum_{h=1}^{L} \bar{t}_h \sqrt{n_h} + c_0\right) + K_\alpha \sqrt{(}\sum_{h=1}^{L} \sigma_{ch}^2 n_h^2 + \sum_{h=1}^{L} \sigma_{th}^2 n_h) \leq C \\ & 2 \leq n_h \leq N_h \\ & and\ n_h\ integers; h = 1,2,\dots,L; j = 1,\dots,p \end{aligned}\right\} \tag{15}$$

3.1 Linearization of the Non Linear Membership Functions by First Order Taylor Series

Let $\bar{n}_h^{(j)*}$, j=1,…, p ; h=1,2,…,L be the individual best solutions of the non linear membership functions subject to the constraints. Now, we transform the non-linear membership functions $\mu_j(\bar{n})$, j=1,2,…,p into equivalent linear membership functions at individual best solution point by first order Taylor series as follows:

$$\mu_j(\bar{n}) \cong \mu_j(\bar{n}_h^{(j)*}) + (n_1 - n_1^{(j)*}) \frac{\partial}{\partial n_1} \mu_j(\bar{n}_h^{(j)*}) + \dots + (n_L - n_L^{(j)*}) \frac{\partial}{\partial n_L} \mu_j(\bar{n}_h^{(j)*}) = \xi_j(\bar{n})$$

3.2 Fuzzy Goal Programming Model of Multiobjective NLPP

The NLPP represented by (15) reduces to the following problem:

$$\left.\begin{array}{c} \max \xi_j(\bar{n}) \\ s.t.\ \left(\sum_{h=1}^{L} \bar{c}_h n_h + \sum_{h=1}^{L} \bar{t}_h \sqrt{n_h} + c_0\right) + K_\alpha \sqrt{(\sum_{h=1}^{L} \sigma_{ch}^2 n_h^2 + \sum_{h=1}^{L} \sigma_{th}^2 n_h)} \le C \\ 2 \le n_h \le N_h \\ and\ \ n_h\ integers; h = 1,2,\dots,L; j = 1,\dots,p \end{array}\right\} \quad (16)$$

The maximum value of a membership function is unity (one), so for the defined membership functions in (16), the flexible membership goals having the aspiration level unity can be presented as:

$\xi_j(\bar{n}) + \delta_j = 1$

Here $\delta_j \ge 0, j = 1,\dots,p$ represent the deviational variables.

Then our Fuzzy Goal Programming (FGP) model is given as:

$$\left.\begin{array}{ll} Minimize & \sum_{j=1}^{p} \delta_j \\ Subject\ \ to & \xi_j + \delta_j = 1;\ j = 1,2,\dots,p \\ & \left(\sum_{h=1}^{L} \bar{c}_h n_h + \sum_{h=1}^{L} \bar{t}_h \sqrt{n_h} + c_0\right) + K_\alpha \sqrt{\left(\sum_{h=1}^{L} \sigma_{ch}^2 n_h^2 + \sum_{h=1}^{L} \sigma_{th}^2 n_h\right)} \le C \\ & 2 \le n_h \le N_h \\ & \delta_j \ge 0 \\ \text{and} & n_h \text{ are integers}; \quad h = 1,2,\dots,L. \end{array}\right\}$$

4. Numerical Illustration

In the table below the stratum sizes, stratum weights, stratum standard deviations, measurement costs, and the travel costs within stratum are given for four different characteristics under study in a population stratified in five strata. The data are mainly from Chatterjee (1968). The values of strata sizes are added assuming the population size as 6000. The total budget of the survey is assumed to be 1500 units with an overhead cost = 300 units.

Table 1. Values of N_h , W_h , c_h , t_h and S_{jh} for five strata and four characteristics.

h	N_h	W_h	S_{jh}			
			S_{1h}	S_{2h}	S_{3h}	S_{4h}
1	1500	0.25	28	206	38	120
2	1920	0.32	24	133	26	184
3	1260	0.21	32	48	44	173
4	480	0.08	54	37	78	92
5	840	0.14	67	9	76	117

In this problem $c_1, c_2, c_3, c_4, c_5, t_1, t_2, t_3, t_4$ and t_5 are independently normally distributed random variables with known means and standard deviations

$E(c_1) = 1$, $E(c_2) = 1$, $E(c_3) = 1.5$, $E(c_4) = 1.5$ and $E(c_5) = 2$

$E(t_1)= 0.5$, $E(t_2)= 0.5$, $E(t_3)= 1$, $E(t_4) =1$, $E(t_5) =1.5$

$V(c_1) = 0.25$, $V(c_2) = 0.25$, $V(c_3) = 0.35$, $V(c_4) = 0.35$ and $V(c_5) = 0.45$.

$V(t_1)= 0.125$, $V(t_2)= 0.125$, $V(t_3)= 0.175$, $V(t_4)= 0.175$ $V(t_5)= 0.225$.

Using the values given in Table 1 the MONLPP and their optimal solutions $\underline{n}_h^*; j = 1,2,\dots,5$ with the corresponding values of V_j^* are listed below. These values are obtained by software LINGO.

$$minimize\ V_1 = \frac{49}{n_1} + \frac{58.9824}{n_2} + \frac{45.1584}{n_3} + \frac{18.6624}{n_4} + \frac{87.9844}{n_5}$$

$$minimize\ V_2 = \frac{2652.25}{n_1} + \frac{1811.3536}{n_2} + \frac{101.6064}{n_3} + \frac{8.7616}{n_4} + \frac{87.9844}{n_5}$$

$$minimize\ V_3 = \frac{90.25}{n_1} + \frac{69.2224}{n_2} + \frac{85.3776}{n_3} + \frac{38.9376}{n_4} + \frac{113.2096}{n_5}$$
$$minimize\ V_4 = \frac{900}{n_1} + \frac{3466.8544}{n_2} + \frac{1319.8689}{n_3} + \frac{54.1696}{n_4} + \frac{268.3044}{n_5}$$
$$subject\ to$$
$$\left(1n_1 + 1n_2 + 1.5n_3 + 1.5n_4 + 2n_5 + 0.5\sqrt{n_1} + 0.5\sqrt{n_2} + 1\sqrt{n_3} + 1\sqrt{n_4} + 1.5\right) + 2.33\sqrt{\frac{(0.25n_1^2 + 0.25n_2^2 + 0.35n_3^2 + 0.35n_4^2 + 0.45n_5^2) +}{(0.125n_1 + 0.125n_2 + 0.175n_3 + 0.175n_4 + 0.225n_5)}} \leq 1200$$
$$and \quad 2 \leq n_1 \leq 1500$$
$$2 \leq n_2 \leq 1920$$
$$2 \leq n_3 \leq 1260$$
$$2 \leq n_4 \leq 480$$
$$2 \leq n_5 \leq 840$$

The optimum allocation $\underline{n}_1^* = (n_{11}^*, n_{12}^*, n_{13}^*, n_{14}^*, n_{15}^*)$ is $n_{11}^* = 132.999, n_{12}^* = 143.2324, n_{13}^* = 107.7228, n_{14}^* = 72.3840, n_{15}^* = 127.6964$.

The corresponding value of the variance ignoring finite population correction (fpc) is $V_1^* = 2.148212$.

The optimum allocation $\underline{n}_2^* = (n_{21}^*, n_{22}^*, n_{23}^*, n_{24}^*, n_{25}^*)$ is $n_{21}^* = 303.1810, n_{22}^* = 259.2840, n_{23}^* = 60.5848, n_{24}^* = 18.3975, n_{25}^* = 6.6782$.

The corresponding value of the variance ignoring finite population correction (fpc) is $V_2^* = 18.12507$.

The optimum allocation $\underline{n}_3^* = (n_{31}^*, n_{32}^*, n_{33}^*, n_{34}^*, n_{35}^*)$ is $n_{31}^* = 142.0023, n_{32}^* = 126.7286, n_{33}^* = 117.2123, n_{34}^* = 82.6231, n_{35}^* = 117.3308$.

The corresponding value of the variance ignoring finite population correction (fpc) is $V_3^* = 3.346324$.
The optimum allocation $\underline{n}_4^* = (n_{41}^*, n_{42}^*, n_{43}^*, n_{44}^*, n_{45}^*)$ is $n_{41}^* = 139.7336, n_{42}^* = 246.2649, n_{43}^* = 139.3793, n_{44}^* = 31.8239, n_{45}^* = 59.5315$.

The corresponding value of the variance ignoring finite population correction (fpc) is $V_4^* = 36.19729$.

Now the payoff matrix is

$$\text{Payoff matrix} = \begin{bmatrix} 2.146266 & 33.66483 & 3.378915 & 46.07330 \\ 15.32374 & 18.12512 & 21.04245 & 81.24543 \\ 2.171515 & 33.95712 & 3.346323 & 47.89731 \\ 2.978546 & 27.36704 & 4.664726 & 36.19729 \end{bmatrix}$$

Here the upper and lower tolerance limits can be given as:

$$V_1^U = 15.32374, V_1^L = 2.146266$$
$$V_2^U = 33.95712, V_2^L = 18.12512$$
$$V_3^U = 21.04245, V_3^L = 3.346323$$
$$V_4^U = 81.24543, V_4^L = 36.19729$$

The non-linear membership functions can be formulated as:

$$\mu_1(\bar{n}) = 1 - \frac{V_1(\bar{n}) - 2.146266}{15.32374 - 2.146266}$$
$$\mu_2(\bar{n}) = 1 - \frac{V_2(\bar{n}) - 18.12512}{33.95712 - 18.12512}$$
$$\mu_3(\bar{n}) = 1 - \frac{V_3(\bar{n}) - 3.346323}{21.04245 - 3.346323}$$
$$\mu_4(\bar{n}) = 1 - \frac{V_4(\bar{n}) - 36.19729}{81.24543 - 36.19729}$$

The membership function $\mu_1(\bar{n})$ is maximal at the point (132.999, 143.2324, 107.7228, 72.3840, 127.6964), $\mu_2(\bar{n})$ is maximal at the point (303.1810, 259.2840, 60.5848, 18.3975, 6.6782), $\mu_3(\bar{n})$ is maximal at the point (142.0023, 126.7286, 117.2123, 82.6231, 117.3308) and $\mu_4(\bar{n})$ is maximal at the point (139.7336, 246.2649, 139.3793, 31.8239, 59.5315) respectively.

Then, the non-linear membership functions are transformed into linear at the individual best solution point by first order Taylor polynomial series as follows:

$$\mu_1(\bar{n}) \cong 1 + (n_1 - 132.999) \times 0.0002 + (n_2 - 143.2324) \times 0.0002 + (n_3 - 107.7228) \times 0.0003 + (n_4 - 72.3840) \times 0.0003 + (n_5 - 127.6964) \times 0.000 = \xi_1(\bar{n})$$
$$\mu_2(\bar{n}) \cong 1 + (n_1 - 303.1810) \times 0.0018 + (n_2 - 259.2840) \times 0.0017 + (n_3 - 60.5848) \times 0.0017 + (n_4 - 18.3975) \times 0.0016 + (n_5 - 6.6782) \times 0.0023 = \xi_2(\bar{n})$$
$$\mu_3(\bar{n}) \cong 1 + (n_1 - 142.0023) \times 0.0003 + (n_2 - 126.7286) \times 0.0002 + (n_3 - 117.2123) \times 0.0004 + (n_4 - 82.6231) \times 0.0003 + (n_5 - 117.3308) \times 0.000 = \xi_3(\bar{n})$$
$$\mu_4(\bar{n}) \cong 1 + (n_1 - 139.7336) \times 0.0010 + (n_2 - 246.2649) \times 0.0013 + (n_3 - 139.3793) \times 0.0015 + (n_4 - 31.8239) \times 0.0012 + (n_5 - 59.5315) \times 0.0017 = \xi_4(\bar{n})$$

Then, the FGP model for solving MNLPP is formulated as follows:

$$
\left.\begin{array}{l}
Minimize \quad \sum_{j=1}^{4} \delta_j \\
Subject\ to \\
1 + (n_1 - 132.999) \times 0.0002 + (n_2 - 143.2324) \times 0.0002 + (n_3 - 107.7228) \times 0.0003 + \\
(n_4 - 72.3840) \times 0.0003 + (n_5 - 127.6964) \times 0.0004 + \delta_1 = 1 \\
1 + (n_1 - 303.1810) \times 0.0018 + (n_2 - 259.2840) \times 0.0017 + (n_3 - 60.5848) \times 0.0017 + \\
(n_4 - 18.3975) \times 0.0016 + (n_5 - 6.6782) \times 0.0023 + \delta_2 = 1 \\
1 + (n_1 - 142.0023) \times 0.0003 + (n_2 - 126.7286) \times 0.0002 + (n_3 - 117.2123) \times 0.0004 + \\
(n_4 - 82.6231) \times 0.0003 + (n_5 - 117.3308) \times 0.0005 + \delta_3 = 1 \\
1 + (n_1 - 139.7336) \times 0.0010 + (n_2 - 246.2649) \times 0.0013 + (n_3 - 139.3793) \times 0.0015 + \\
(n_4 - 31.8239) \times 0.0012 + (n_5 - 59.5315) \times 0.0017 + \delta_4 = 1 \\
(1n_1 + 1n_2 + 1.5n_3 + 1.5n_4 + 2n_5 + 0.5\sqrt{n_1} + 0.5\sqrt{n_2} + 1\sqrt{n_3} + 1\sqrt{n_4} + 1.5\sqrt{n_5}) + 2.33 \\
\sqrt{\begin{array}{l}(0.25n_1^2 + 0.25n_2^2 + 0.35n_3^2 + 0.35n_4^2 + 0.45n_5^2) + \\ (0.125n_1 + 0.125n_2 + 0.175n_3 + 0.175n_4 + 0.225n_5)\end{array}} \le 1200 \\
2 \le n_1 \le 1500,\ 2 \le n_2 \le 1920,\ 2 \le n_3 \le 1260,\ 2 \le n_4 \le 480,\ 2 \le n_5 \le 840 \\
\delta_j \ge 0 \quad and \quad n_h\ are\ integers; \quad h = 1, 2, \ldots, L; \quad j = 1, \ldots, 4
\end{array}\right\}
$$

By solving the FGP model by software LINGO, we get the optimal solution as: $n_1 = 198, n_2 = 214, n_3 = 95, n_4 = 37$ and $n_5 = 79$ with a total of 623. Corresponding to this allocation the values of the variances for the four characters are obtained as $V_1 = 2.616561, V_2 = 23.18591, V_3 = 4.163389, V_4 = 39.49937$ with the total cost consumption for conducting the survey i.e. $C = 1200$ units.

5. Conclusion

In this paper Multiobjective non linear programming problem with probabilistic cost constraint is formulated. To obtain the compromise allocation a new approach is proposed called Fuzzy Goal Programming. In the proposed approach non linear membership functions are defined which are linearized by first order Taylor series. And the FGP model is solved by an optimizing software Lingo.

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Correspondence to: Irfan Ali
Email: irfii.ali@gmail.com