

## Supplementary material

### A decision-making framework for wildfire suppression

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In this paper, we find the optimal suppression resource allocation for one or several simultaneous wildfires. We utilise a wildfire simulation, HFire, to assess how the suppression resources affect the final size of the fire. This supplement contains a more complete description of HFire (see also Morais 2001; Peterson *et al.* 2009) and our parameter choices.

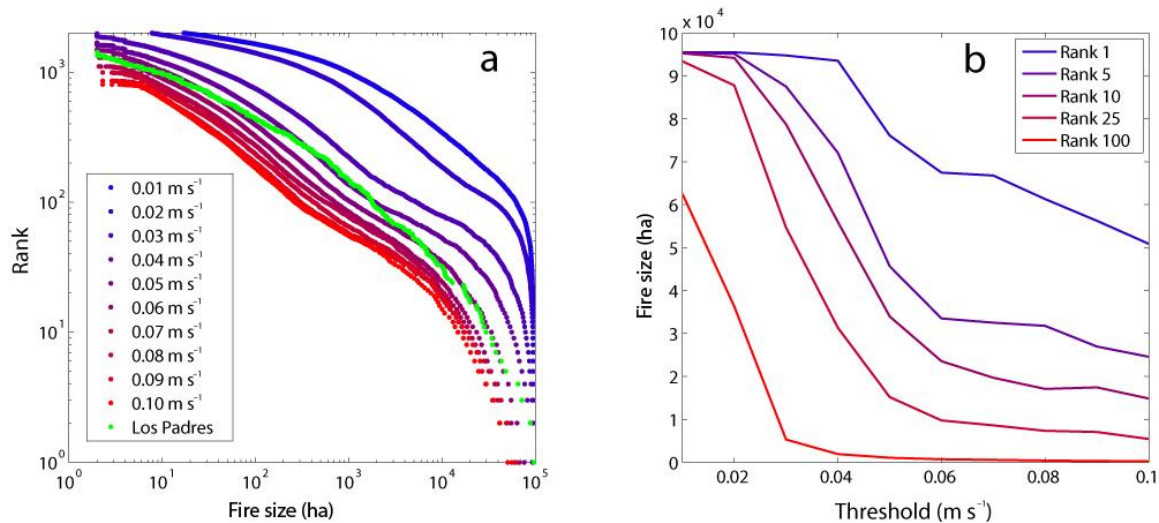
#### Details of HFire simulation

HFire is a spatially explicit, raster-based simulation that individually propagates each fire in a season. Raster schemes of two-dimensional fire growth partition the modelling domain into a regular grid, and fire spreads from cell to neighbouring cell, using cell contact or heat accumulation (HFire combines both methods). Each cell contains information about spatial variables that affect the progression of the fire, such as vegetation (fuel type, structure and accumulation) as well as topography (slope and aspect). At each time step, the wind speed, direction and humidity are stochastically sampled from an empirical distribution. These variables are inputs for Rothermel's one-dimensional equation of fire spread (Rothermel 1983). The empirical double-ellipse formulation of Anderson (Anderson 1982) is used to generalise the fire spread to a two-dimensional grid and dictate how fast the fire spreads from burning cells to adjacent unburned cells. As Rothermel's equation does not include an explicit parameter for stopping the fire (outside of burning all available cells), a stopping criterion is implemented to perform this function at the cell level. The fire terminates in any cell with a spread rate lower than a user-defined extinction threshold.

HFire is similar to FARSITE, the model used by the US Forest Service (Finney 1998), in its use of the Rothermel equation and the two models have been confirmed to yield comparable results for individual fires. However, FARSITE uses a vector-based approach and simulates fire as a continually expanding fire polygon. HFire's raster-based algorithm is more computationally efficient and allows for multicentury fire regime modelling (Peterson *et al.* 2009). HFire produces

fire catalogues that have good statistical agreement with available data (Moritz *et al.* 2005; Peterson *et al.* 2011). Therefore, the statistical data generated by HFire can be used to study the effect of long-term wildfire response strategies.

To generate Fig. S1a, we run a simplified version of HFire for 10 different values of stopping criterion or extinction threshold. The landscape is a flat square,  $315 \times 315$  ha in size, with a random sampling of 80% chaparral and 20% grass, and four ignitions and four Santa Anas per year. The fuel mix is chosen to model National Forests in southern California at the wildland–urban interface, which are primarily composed of chaparral. The fast-burning grass emulates the effect of younger chaparral. Each distribution in Fig. S1a is generated by fixing an extinction threshold and running a large ensemble of fires over many years, sampling different conditions for ignitions and weather. To obtain statistically averaged behaviour, we run 10 realisations of the simulation with different random number seeds and find the average size for each rank.



**Fig. S1.** HFire simulation data. (a) 1000-year synthetic catalogues generated for 10 different threshold values. Fire areas are ordered from largest to smallest and plotted as rank *v.* area. (b) Relationship between threshold value and the final fire size for five fires chosen from the HFire catalogue by rank. In general, this relationship is not linear.

In Fig. S1a, the lower the threshold, the larger the average fire size. For comparison, we also plot fires recorded in Los Padres National Forest in 1911–2007, one of the most complete available datasets. The 0.05- $\text{m s}^{-1}$  threshold distribution most closely matches the data for Los Padres National Forest. The slight dip in the middle of the HFire distributions is due to the simplicity of the landscape and is eliminated when more realistic elements (fuel map, slope variation) are added. It was found that the simplified version of HFire produces wildfire catalogues with similar statistical characteristics to real data and to catalogues produced by

versions with realistic topography and vegetation obtained from GIS measurements (E. D. Sherwin, J. M. Carlson, N. Petrovic, unpubl. data).

### **Selection of threshold range for suppression model**

HFire is used to compute optimal suppression allocations in several scenarios. In particular, the extinction threshold parameter is used as a proxy for suppression; the higher the threshold, the greater the suppression response, and the smaller the average size of the fire. The extinction threshold acts uniformly on the fire perimeter; we therefore define the level of suppression resources to be measured per unit length on the perimeter. We consider three response levels  $r$ : low ( $r = 1$ ), intermediate ( $r = 2$ ) and high ( $r = 3$ ), which correspond to extinction thresholds 0.04, 0.05 and  $0.06 \text{ m s}^{-1}$ . We select a value of fire rank that serves as a proxy for external conditions and, using the HFire catalogue in Fig.S1a, determine average fire area  $A(r)$  based on resource allocation  $r$ .

Discrete levels of suppression response are motivated by possible decisions made in a realistic situation. For example, when a fire is detected, a fleet of a standard size consisting of several engines or aircraft is sent to the site. It can be supplemented with reinforcements if the fire grows out of control; this would also occur in discrete steps. For this initial study, specific choices were made for the sake of clarity and computational feasibility. Three suppression levels allow clear visual representation and subsequent discussion of solutions for two fires. The level of realism could be increased in several ways (e.g. spatially localised resource assignments or different types of resources), as discussed in the Conclusion. For a particular situation and set of suppression resources, a finer gradation for  $r$  may be necessary and meaningful. However, for more than three threshold levels, finding the optimal solution for 10 fires via brute-force methods becomes computationally lengthy.

From a mathematical perspective, it is appealing to minimise cost based on a continuous  $r$  and to find the minimum by taking derivatives with respect to  $r$ . However, owing to the complexity of the problem and the number of inputs, an analytic form for  $A(r)$  cannot be easily derived. Additionally, although HFire is more efficient than its predecessors in performing statistical runs with multiple fires, dynamically rerunning this program during the optimisation procedure itself would be computationally overwhelming. The only feasible way to obtain a continuous relationship between fire area and suppression is to run HFire for some discrete set of threshold values and then interpolate a functional form for  $A(r)$ . This would create an added layer of assumptions. Therefore, discrete resource allocations alleviate computational issues.

The threshold range was chosen using figure IV-2 in Rothermel's 1983 paper 'How to predict the spread and intensity of forest and range fires' (Rothermel 1983). In this figure, empirical data determine which fires can be contained by various suppression mechanisms given their heat release per unit area and rate of spread. Heat per unit area varies widely depending on environmental factors such as fuel moisture. For chaparral fires, it may range from 1500 to 3000 Btu foot<sup>-2</sup> ( $1.7 \times 10^7$  to  $3.4 \times 10^7$  J m<sup>-2</sup>) (Pyne *et al.* 1996). Hence, according to figure IV-2 in Rothermel (1983), a broad range of fire spread rates from 0.01 to 0.1 m s<sup>-1</sup> approximately corresponds to fires that can potentially be controlled by human suppression.

Fig. S1a contains fire size distributions with thresholds ranging from 0.01 to 0.1 m s<sup>-1</sup>. High values of threshold (0.07 to 0.1 m s<sup>-1</sup>) are closely spaced and indicate diminishing returns. Low values of threshold (0.01 and 0.02) result in an excess of large fires that burn to the system size. Thus, the relationship between fire size and threshold is non-linear. This is further illustrated in Fig. S1b, where the relationship between threshold and fire area is shown for five representative fires. In general, fire area decreases steeply with threshold in the middle range, whereas the relationship between threshold and fire area levels out for low and high threshold values, indicating decreased sensitivity of area to resource allocation in this range.

From the broader range of thresholds in Fig. S1, we select thresholds that: (1) lie within a plausible range for chaparral fires, (2) are not affected by system size effects and (3) are not affected by diminishing returns. The range 0.04 to 0.06 m s<sup>-1</sup> fits all of these criteria. Additionally, the middle threshold value of 0.05 m s<sup>-1</sup> corresponds closely to Los Padres National Forest data.

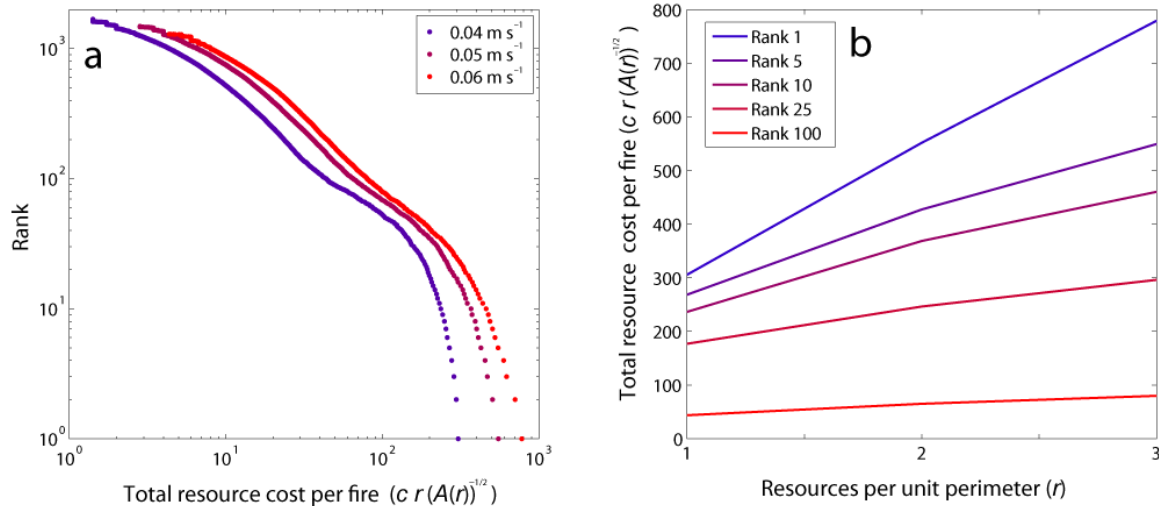
### Cost function behaviour in threshold range

Using the range of thresholds selected, the cost function:

$$\text{Total cost} = \sigma \times A(r) + c \times r \times \sqrt{A(r)} \quad (1)$$

is minimised for a progressive set of scenarios. The first term, or loss due to the fire, scales with area and will clearly decrease as  $r$  increases. However, it is less clear whether the second term, or total resource cost per fire, will increase or decrease with  $r$ . In principle, resources may be so effective that a small increase in resource allocation significantly decreases the final fire perimeter, and allocating the maximum  $r$  minimises both terms. However, Fig. S2a demonstrates that this is not the case in the chosen threshold range. Fig. S2a contains data for thresholds 0.04, 0.05 and 0.06 m s<sup>-1</sup> (corresponding to  $r = 1, 2$  and 3). In contrast to Fig. S1a, the horizontal axis is the total resource cost for each fire ( $c \times r \times \sqrt{A(r)}$ ), which is computed using the stated

relationship between threshold and  $r$ . For each horizontal slice (fire selected by rank), the total resource cost per fire increases monotonically with threshold and therefore with  $r$ . Fig. S2b illustrates this relationship for five representative fires.



**Fig. S2.** Examination of total resource costs per fire using HFire simulation data. (a) For each fire rank selected from a distribution with a given threshold value, we compute the total resource cost ( $c \times r \times \sqrt{A(r)}$ ) where  $c$  is cost per unit resource,  $r$  is resource allocation per unit perimeter and  $A(r)$  is fire size. Thresholds 0.04, 0.05 and 0.06  $\text{m s}^{-1}$  correspond to  $r = 1, 2$  and 3. We assume  $c = \text{US}\$1$  per unit resource. Resource costs are ordered from largest to smallest and plotted as rank  $v$ . resource cost per fire. For each fire rank (horizontal cut), the total resource cost per fire increases monotonically with threshold and therefore with  $r$ . (b) Total resource cost per fire  $v$ . resource allocation per unit perimeter for five representative fires chosen from the HFire catalogue by rank. For all fires, the resource cost increases with  $r$ .

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