## Supplementary material

## Propagation probability and spread rates of self-sustained smouldering fires under controlled moisture content and bulk density conditions

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## Nomenclature

$c_{w} \quad$ Specific heat of water $\left(\mathrm{kJ} \mathrm{kg}^{-1} \mathrm{~K}\right)$
$c_{p} \quad$ Specific heat of peat $\left(\mathrm{kJ} \mathrm{kg}^{-1} \mathrm{~K}\right)$
$E \quad$ Energy required to dry and heat a mass of peat sample $\left(\mathrm{kJ} \mathrm{kg}^{-1}\right)$
$E^{\prime} \quad$ Total energy required to dry and heat an entire sample (kJ)
$E^{\prime \prime} \quad$ Energy required to dry and heat a unit volume of peat $\left(\mathrm{kJ} \mathrm{cm}^{-3}\right)$
$E_{w} \quad$ Energy required to heat and dry a mass of water $\left(\mathrm{kJ} \mathrm{kg}^{-1}\right)$
$E_{p} \quad$ Energy required to heat a mass of peat $\left(\mathrm{kJ} \mathrm{kg}^{-1}\right)$
$L_{w} \quad$ Latent heat of water evaporation $\left(\mathrm{kJ} \mathrm{kg}^{-1}\right)$
$p \quad$ Mass of dry peat in the peat sample (kg)
$\rho \quad$ Bulk density $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$
$T_{0} \quad$ Ambient temperature (K)
$T_{1} \quad$ Temperature of water evaporation at ambient pressure (K)
$T_{2} \quad$ Temperature of the start of peat thermal decomposition to char (K)
$t^{L} \quad$ Time when the leading edge of the smouldering front reach a pixel (h)
$w \quad$ Mass of water in the peat sample (kg)
$\omega \quad$ Peat sample volume $\left(\mathrm{cm}^{3}\right)$

## Detection of changes in spread rate during long burns

To detect changes in the spread rate during long burns (defined as burns lasting more than 7 h ), we first calculated the median $t^{L}$ for groups of pixels between distance $x$ and $x+0.5 \mathrm{~cm}$ from the igniter that were more than 2 cm from the sides of the burnbox and more than 6 cm from the igniter $(T)$, and then analyse the variation of $T$ along the $x$-direction. To avoid issues of autocorrelation we analysed the spatial relationship between subsets $T$ in each experimental burn (Legendre and Legendre 2000). Gaussian and Spherical spatial semivariogram models were fitted. Both models indicated no spatial autocorrelation beyond 1 cm (semviariogram range) for all experimental burns. We then selected values of $T$ with 1 cm separation in distance, $x$. We fitted the following linear model:

$$
\begin{equation*}
T_{j}=\beta_{\delta 0}+\beta_{\delta 1} x_{j}+\beta_{\delta 2} x_{j}^{2}+\varepsilon_{j} \tag{S1}
\end{equation*}
$$

where $T_{j}$ is the median of $t^{L}$ for the group of pixels corresponding to the distance $x_{j}$ and $\varepsilon_{j}$ is a residual, assumed to be taken from a normal distribution. The coefficient of the quadratic term, $\beta_{\delta 2}$, is expected to be zero if spread is constant. The hypothesis $\beta_{\delta 1}=0$ was tested using F-tests.

## Estimation of energy required for smouldering combustion of peat

Table S1. Summary of the moisture evaporation tests for peat conditions that sustained smouldering fires for more than $\mathbf{7 h}$.

Test is the number of moisture evaporation test, $M C$ treatment is the initial moisture content of the peat sample at the start of the experiment, $\rho$ is the bulk density of the peat sample, time is the number of hours since the start of the test, propagation indicates if smouldering propagation was self-sustained $(\mathbf{Y})$ or not $(\mathrm{N})$ according to Fig. S2, MC is the \% moisture content estimated at each hour according to the mass loss rate of the peat sample, s.d. is the standard deviation of $M C$.
Test $M C$ treatment $\quad \rho \quad$ time propagation $M C \quad$ s.d.

| (num) | (\%) | $\left(\mathrm{kg} \mathrm{m}^{-3}\right)$ | (hours) | $(\mathbf{Y} / \mathrm{N})$ | (\%) | (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 100 | 129 | 2 | $Y$ | 100.2 | 0.1 |
| 1 | 100 | 129 | 4 | $Y$ | 99.7 | 0.1 |
| 1 | 100 | 129 | 6 | $\boldsymbol{Y}$ | 99.0 | 0.1 |
| 1 | 100 | 129 | 8 | $\boldsymbol{Y}$ | 98.3 | 0.1 |
| 1 | 100 | 129 | 10 | $\boldsymbol{Y}$ | 98.0 | 0.0 |
| 1 | 100 | 129 | 12 | N | 97.6 | 0.0 |
| 1 | 100 | 129 | 14 | N | 97.3 | 0.1 |
| 2 | 100 | 132 | 2 | Y | 101.0 | 0.0 |
| 2 | 100 | 132 | 4 | $\boldsymbol{Y}$ | 100.9 | 0.0 |
| 2 | 100 | 132 | 6 | $\boldsymbol{Y}$ | 100.7 | 0.0 |
| 2 | 100 | 132 | 8 | $\boldsymbol{Y}$ | 100.2 | 0.1 |
| 2 | 100 | 132 | 10 | $\boldsymbol{Y}$ | 99.7 | 0.1 |
| 2 | 100 | 132 | 12 | N | 99.1 | 0.1 |
| 2 | 100 | 132 | 14 | N | 98.6 | 0.1 |
| 3 | 100 | 138 | 2 | $\boldsymbol{Y}$ | 101.7 | 0.3 |
| 3 | 100 | 138 | 4 | $\boldsymbol{Y}$ | 102.0 | 0.0 |
| 3 | 100 | 138 | 6 | Y | 101.7 | 0.1 |
| 3 | 100 | 138 | 8 | $\boldsymbol{Y}$ | 101.3 | 0.0 |
| 3 | 100 | 138 | 10 | $\boldsymbol{Y}$ | 101.1 | 0.1 |
| 3 | 100 | 138 | 12 | N | 100.7 | 0.1 |
| 3 | 100 | 138 | 14 | N | 100.3 | 0.1 |
| 4 | 150 | 73 | 2 | $\boldsymbol{Y}$ | 148.3 | 0.3 |
| 4 | 150 | 73 | 4 | Y | 147.8 | 0.2 |
| 4 | 150 | 73 | 6 | $Y$ | 146.7 | 0.1 |
| 4 | 150 | 73 | 8 | $\boldsymbol{Y}$ | 144.8 | 0.1 |
| 4 | 150 | 73 | 10 | N | 144.2 | 0.1 |
| 4 | 150 | 73 | 12 | N | 143.4 | 0.1 |
| 4 | 150 | 73 | 14 | N | 143.0 | 0.1 |
| 5 | 150 | 81 | 2 | $\boldsymbol{Y}$ | 152.2 | 0.1 |
| 5 | 150 | 81 | 4 | $Y$ | 152.5 | 0.0 |
| 5 | 150 | 81 | 6 | $\boldsymbol{Y}$ | 152.4 | 0.0 |
| 5 | 150 | 81 | 8 | $\boldsymbol{Y}$ | 152.1 | 0.0 |
| 5 | 150 | 81 | 10 | N | 151.8 | 0.1 |
| 5 | 150 | 81 | 12 | N | 151.5 | 0.1 |
| 5 | 150 | 81 | 14 | N | 151.0 | 0.1 |
| 6 | 150 | 82 | 2 | Y | 150.9 | 0.0 |
| 6 | 150 | 82 | 4 | $\boldsymbol{Y}$ | 150.7 | 0.1 |
| 6 | 150 | 82 | 6 | $\boldsymbol{Y}$ | 150.2 | 0.1 |
| 6 | 150 | 82 | 8 | $\boldsymbol{Y}$ | 149.6 | 0.1 |
| 6 | 150 | 82 | 10 | N | 149.1 | 0.1 |
| 6 | 150 | 82 | 12 | N | 148.8 | 0.1 |
| 6 | 150 | 82 | 14 | N | 148.3 | 0.1 |



Fig. S1. Expected fraction of peat burnt $\left(P_{y D}\right)$ for peats with $150 \%$ moisture content (Eqn S 1 , Table S 1 ). Panels are predictions for distance $D$ away from the ignition region of (a) 6 cm , (b) 8 cm , (c) 10 cm and (d) 12 cm . Line shows mean prediction and shaded areas are quantile $=25 \%$ and quantile $=75 \%$. Points are fractions of peat burnt $(y)$ along a transect at distance $D$.

The energy required to dry and heat a mass of peat sample $(E)$, the entire peat sample ( $E^{\prime}$ ) and a unit volume ( $E^{\prime \prime}$ ) were estimated for each combination of peat moisture content ( $M C$ ) and bulk density $(\rho)$. The energy required to heat and dry the water from the peat is

$$
\begin{equation*}
E_{w}=\left(c_{w}\left(T_{1}-T_{0}\right)\right) L_{w}\left(\frac{M C}{100+M C}\right) \tag{S2}
\end{equation*}
$$

where $c_{w}$ is a constant $4.186 \mathrm{~kJ} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}, T_{0}$ is assumed to be constant at $288.15 \mathrm{~K}, T_{l}$ is assumed to be 373.15 K and $L_{w}$ is $2260 \mathrm{~kJ} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$ (Huang et al., 2014). The energy required to heat the peat from $T_{0}$ to $T_{2}$ is

$$
\begin{equation*}
E_{p}=\left(c_{p}\left(T_{2}-T_{1}\right)\right)\left(1-\frac{M C}{100+M C}\right) \tag{S3}
\end{equation*}
$$

where $c_{p}$ is a constant $1.84 \mathrm{~kJ} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}, T_{0}$ is assumed to be constant at 288 K and $T_{2}$ is 423 K , which is considered the temperature at which peat starts the thermal decomposition to char (Rein 2013). $E_{p}$ was calculated for a higher $T_{2}$ ( 473 K ), however the temperature increase of 50 K did not have a qualitative effect on $E$ (Eqn S4). The total energy required to evaporate all water from a unit mass of peat sample and heat the peat to a temperature $\mathrm{T}_{2}$ is

$$
\begin{equation*}
E=E_{w}+E_{p} \tag{S4}
\end{equation*}
$$

The values were estimated for each moisture content treatment (Fig. S2).


Fig. S2. Energy required ( $E$, Eqn S3) to heat 1 kg of peat to 423 K as a function of peat moisture content.

The energy required to start thermal decomposition of the entire sample is

$$
\begin{equation*}
E^{\prime}=E_{w} \times w+E_{p} \times p \tag{S5}
\end{equation*}
$$

where $p$ is the mass of peat and $w$ is the mass of water of the entire sample. Assuming homogeneous peat conditions within the burnbox, the energy required to start thermal decomposition per unit volume is

$$
\begin{equation*}
E^{\prime \prime}=\frac{E^{\prime}}{\omega} \tag{S6}
\end{equation*}
$$

where $\omega$ is a constant volume of $1800 \mathrm{~cm}^{3}$. The values were estimated for each peat moisture content and bulk density treatment (Fig. S3).


Fig. S3. Energy required per unit volume of peat sample ( $E^{\prime \prime}$, Eqn S5) as a function of peat bulk density. Circle, triangle, square, diamond and star correspond to $25 \%$, $100 \%, 150 \%, 200 \%$ and $250 \%$ moisture content, respectively.

## References

Huang X, Rein G (2014) Smouldering combustion of peat: inverse modelling of the drying and the thermal and decomposition kinetics. Combustion and Flame 161, 1633-1644.

Legendre P, Legendre L (2000) Numerical ecology (Elsevier: Amsterdam)

Rein G (2013) Smouldering Fires and Natural Fuels. In 'Fire phenomena and the earth system: an interdisciplinary approach to fire sicence'. (Eds CM Belcher) pp. 1534. (Wiley-Blackwell: West Sussex)

