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Supplementary Material

Quantile Regression: an alternative approach to modeling forest area burned by individual fires

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Preliminary experiment: method

Variable consideration

In my preliminary experiment of the model fitting process, I considered as many model variables that are commonly used in empirical modelling following the larger literature framework. The size of an individual forest fire is a function of probability of occurrence (ignition and establishment) and escape. Wotton and Martell (2005) modelled the probability of occurrence with selected fire weather indices (i.e. fine fuel moisture code, FFMC; duff moisture code, DMC; drought code, DC; initial spread index, ISI) and some other variables (e.g. wind speed, relative humidity). Podur and Martell (2007) used fire weather index (FWI) and Fire Behaviour Prediction (FBP) Systems (Forestry Canada Fire Danger Group 1992) to model the escaped fires. Martell and Sun (2008) modelled the annual fire size with composite of the FWI and the FBP considering the probability of occurrence and escape. Flannigan *et al*. (2005) modelled monthly area burned using several fire weather indices (FWI) and weather attributes. Although FWI is derived from the other indices and weather attributes (Van Wagner 1987; Natural Resources Canada 2015), the empirical model examinations show that the different set of model attributes are significant regionally (e.g. Flannigan *et al*. 2005) and model forms (e.g. Martell and Sun 2008). Therefore, it would be worthwhile to examine the model attributes that would best explain to the selected study area. In addition, I introduced geographic variables including elevation (Garcia *et al*. 1995) and slope. Although elevation and slope are the components of Canadian FBP system, another subsystem of Canadian fire danger rating system, their applications in empirical modelling are less in the current literature. Hence the complete set of possible model variables in my full model included seven variables of FWI (namely, BUI, build-up index; DC; DMC; FFMC; FWI; ISI; and MSR, monthly severity rating), four variables of weather reading (temperature, precipitation, relative humidity and wind speed) and two geographic variables (elevation and slope).

Variable selection: multicollinearity and principal component analysis

It is likely that the fitted model with many unsorted variables and small sample sizes may lead to overparameterised results. Quality of model performance may also decrease due to multicollinearity among the variables. Because the derivation of the FWI component indices are related to each other and with the weather attributes (Van Wagner 1987), it is worthwhile to examine the multicolinearity so as we take only the variables that have variance inflation factor less than a rule of thumb (O'Brien 2007). Therfore, I first performed principal component analysis to reduce the number of variables using the *PCA* and *dimdesc* functions in the *FactoMineR* package (Sebastien *et al*. 2008) of R (R Foundation for Statistical Computing, Vienna, Austria, see http://www.R-project.org/). I selected the number of dimensional axes that would explain the cumulative variance up to 95% and variance inflation factor (VIF) less than 5. Hence selected variables were used to construct a full model using multiple linear regression.

The variables from the full mdoel were selected using sequential analysis of variance tests using the *step* function in R to fit the best reduced model, following a simplest procedure. Fitted parameters of empirical models often have the opposite sign than what these should be to reflect the physical meaning of fire growth dynamics processes. I ensured that the reduced model would have significant parameters ($\alpha \leq 0.05$) and carry term-wise physical significances (but limited to sign of parameters) of the reduced model. To reduce the complexity and likely misleading inferences while fitting the model with a small number of data, I did not consider the interaction effects among or between the variables.

I designed the hierarchical model structure at the spatial scale of HFR and temporal scale of month to address possible spatial and temporal autocorrelations (Fig. S1) in my main experiment. Pinheiro and Bates (2009) explain that the impact of such autocorrelation can be reduced either by specifying autoregressive parameters (e.g. Martell and Sun 2008) or modelling by group using mixed-effects technique. The latter grouping approach often performs better (Rijal *et al*. 2012). The preliminary examination also supported it (result not shown here) while comparing with autoregressive model (e.g. Martell and Sun 2008). In addition, specifying month as a model (fixed) covariate can serve as the fire season (Wotton *et al*. 2010). For model simplicity during fitting with small numbers of data, I considered only the prominent hierarchies, namely HFR and month (Fig. S1). Fire propagations and extinguishments do not only depend on the weather at the time when it starts (ignition), but also rely on the fuel sources and climatic situations through time and space dynamically that impacts on the duration of burnings. Because of lack of such information, each individual fire was linked with geographic attributes and weather indices as a fire point where and when it started (ignition) or first reported.

Preliminary experiment: result

Principal component analysis showed that all of the examined covariates were significantly correlated (at 5%) with various principal axes with the correlation coefficient ranging between –0.63 and 0.98 (Table S2). However, among the 13 evaluated covariates, 95% of the variability was explained by the first eight dimensional components and the first three components explained 43, 19 and 9% of the total variability (Fig. S3). Based on the dimensional correlations and variance inflation factor less than 5, eight model covariates, namely *DC*, *Elev*, *FWI*, *RH*, *Precip*, *Slope*, *Temp* and *WS* were used in the full model (Table S2). The local models fitted for each HFR showed that different sets of model covariates were significant for different HFRs, but elevation, FWI and relative humididty were the most common among the 20 fitted local models including the regional model using multiple linear regression (Table S4). Hence, the regional reduced model yielded only three significant parameters associated with the covariates *FWI*, *RH* and *Elev*. Refitting the model by dropping the positive *RH* parameter did not reduce model perfromance as there were no differences between the deviances (17 595 *v.* 17,845 compared to the null deviance 31 601). Hence, only the two covariates, elevation and FWI were taken for my main experiment.

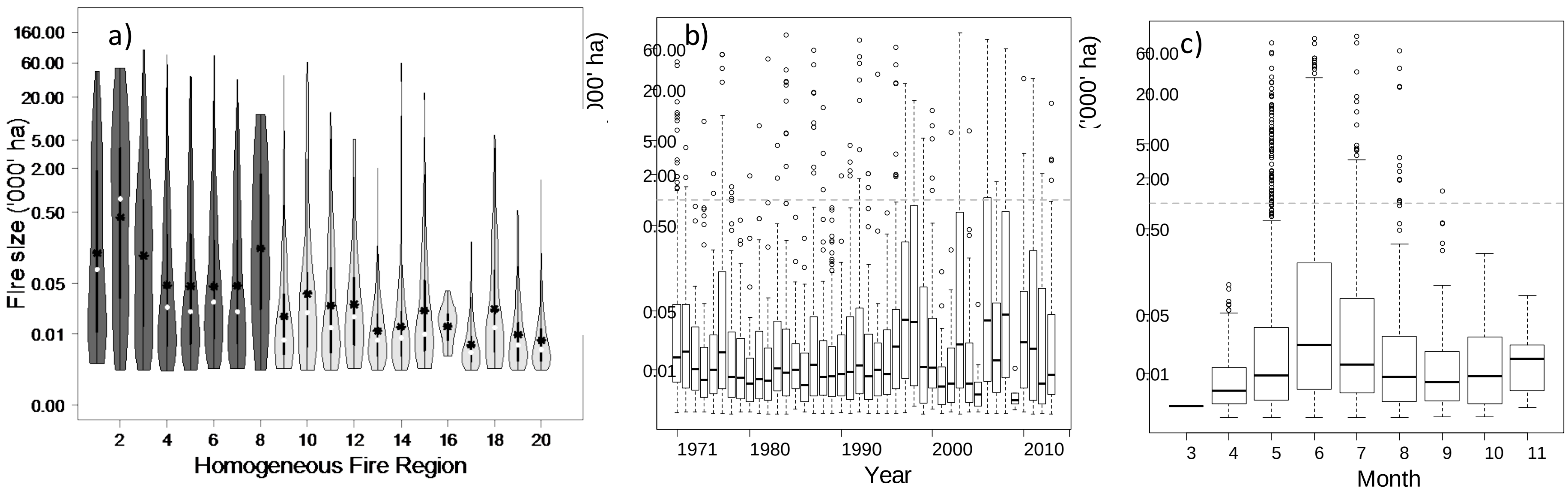

Fig. S1. Spatial and temporal variations of area burned by individual forest fires: (a) Violin plots showing spatial variations of fire in 20 homogeneous fire regions, (b) boxplots showing the annual distributions between 1971 and 2014, and (c) boxplots showing monthly distribution between March and November (fire season).

Table S2. Correlation coefficients of model variables with various dimensional axes of the principal component

Only the significant correlations at 5% are displayed. The bold numbers are the highest (one positive and one negative) correlations with the respective dimension, and the variables associated with the numbers with bold and italic were taken to construct the full model. The eight model variables that gave variance inflation factor less than 5 are marked with bold. Variables are: BUI, Build-up Index; DC, Drought Code; DMC, Duff Moisture Code; Elev, elevation; FFMC, Fine fuel moisture code; FWI, fire weather index; ISI, initial spread index; MSR, monthly severity rating; Prcp, precipitation (mm); RH, relative humidity (%); Slope, slope (%); WS, wind speed (km h^{-1}). For the details please refer to http://cwfis.cfs.nrcan.gc.ca/maps/fw

SN	Variables	Dimensional axis												
		1	2	3	4	5	6	7	8	9	10	11	12	13
1	BUI	0.81	0.49	–0.05	0.18	–0.17	–0.06	–0.13	0.06	–0.12			–0.04	**0.04**
2	**DC**	0.45	***0.73***	–0.04	0.19	–0.18	0.12	**0.29**	**–0.31**					
3	DMC	0.83	0.35	–0.05	0.18	**–0.18**	–0.11	–0.26	0.17	–0.13		0.05		
3	**Elev**		0.47	0.54	–0.23	0.18	***–0.63***	0.09						
5	FFMC	0.86	–0.17	0.04	**–0.31**	0.12	0.09	0.21		**–0.19**	**0.16**	–0.05		
6	**FWI**	***0.98***			0.08	0.08				0.14		–0.04	**–0.07**	
7	ISI	0.81	**–0.51**	0.05	0.04	0.19			–0.04	0.13	0.07	**0.13**		
8	MSR	0.94			0.18	0.05	–0.04	–0.12		**0.22**		**–0.08**	**0.06**	
9	**Prcp**	–0.59	0.31	**–0.14**	0.34	0.51	–0.04	***–0.35***	–0.17	–0.05	0.09			
10	**RH**	***–0.62***	0.60		0.32	–0.13	0.10	0.23	**0.19**	0.16	0.14			
11	**Slope**		0.06	***0.88***	0.06		**0.43**	–0.17						
12	**Temp**	0.31	0.58	–0.13	–0.24	***0.61***	0.27	0.13	0.16		**–0.09**			
13	**WS**	0.10	***–0.50***[A]	0.17	***0.73***	0.23	–0.11	0.29	0.07	–0.13	–0.05			

[A]WS in the second dimension was selected instead of ISI due to highly correlated with dimension 1, from where FWI was selected; likewise FFMC and DMC were not selected.

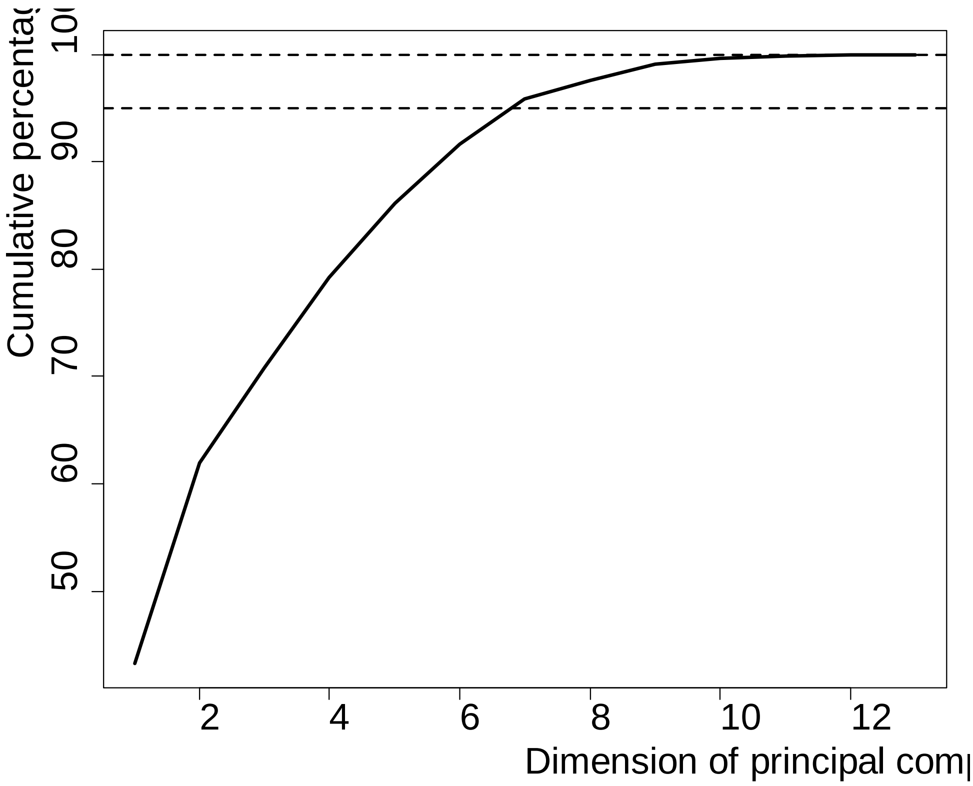

Fig. S2. Principal component analysis: The cumulative percent of the explained variations by 13 dimensional components.

Table S4. Parameter estimates of HFR wise multiple linear regression

The full model consisted of all of the eight covariates and the intercept in each of the local models. The displayed parameters are significant at 5% error level of the reduced models. Sample numbers corresponding to the local and regional models are presented in Table 1. Elev, elevation (m), FWI, fire weather index, DC, drought code, Temp, temperature (°C), WS, wind speed (km h^{-1}), relative humidity (%), Precip, precipitation (mm)

HFR	Intercept	Elev	Slope	FWI	DC	Temp	WS	RH	Precip	R^2
1	–	–	–	0.5546	–0.0136	–	–	0.0611	–	0.78
2	–	–	–	–	–	–	0.8822	–0.1165		0.84
3	–	–	–	0.1586	–	–	0.2170	–	0.0084	0.8
4	–	–	0.1222	0.3531	–	–0.1384	–	0.0646	–	0.76
5	–	–0.0127	–	0.2788	–0.0059	–	–	0.1120	–	0.74
6	–	–	–	0.2700	–	–	–	–	0.0254	0.79
7	–	–	–	0.2706	–	–0.1869	–	0.0613	0.0172	0.75
8	–	–	–	0.5859	–0.0343	–	0.4251	–	–	0.87
9	–	–	–	0.1988	–	–	–	–	0.0175	0.73
10	–	0.0046	–	0.2107	–	–	–	–	–	0.72
11	–	–	–	0.1701	–0.0045	–	–	0.0457	–	0.74
12	–	0.0048	–	–	–	–	–	–	–	0.75
13	–	0.0023	–	0.1498	–0.0044	–	–	0.0216	–	0.84
14	–	0.0070	–	0.0486	–	–	–	–	–	0.78
15	–	0.0026	0.0622	–	–	–	0.1051	–	–	0.78
16	–	0.0058	–	–	–	–	–	–	0.0167	0.88
17	–	–	–	0.0641	–	–	–	0.0269	–	0.87
18	–	0.0055	–	0.1137	–	–	–	–	–	0.74
19	–	0.0025	–	0.0894	–	–0.0647	–	0.0338	–	0.80
20	–	0.0014	–	0.0806	–	–0.0341	–	0.0308	–	0.84
Regional	–	0.0044	–	0.0951	–	–	–	0.0215	–	0.73

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