

Supplementary Material

BARA: cellular automata simulation of multidimensional smouldering in peat with horizontally varying moisture contents

Dwi M. J. Purnomo^A, Eirik G. Christensen^A, Nieves Fernandez-Anez^B and Guillermo Rein^{A,}*

^ALeverhulme Centre for Wildfires, Environment and Society and Department of Mechanical Engineering, Imperial College London, London, SW7 2AZ, UK

^BDepartment of Safety, Chemistry and Biomedical Laboratory Sciences, Western Norway University of Applied Sciences, Bjørsonsgate 45, 5528, Haugesund, Norway

*Correspondence to: Email: g.rein@imperial.ac.uk

2 **Appendix**

3 **BARA: Cellular Automata Simulation of Multidimensional Smouldering in**
4 **Peat with Horizontally Varying Moisture Contents**

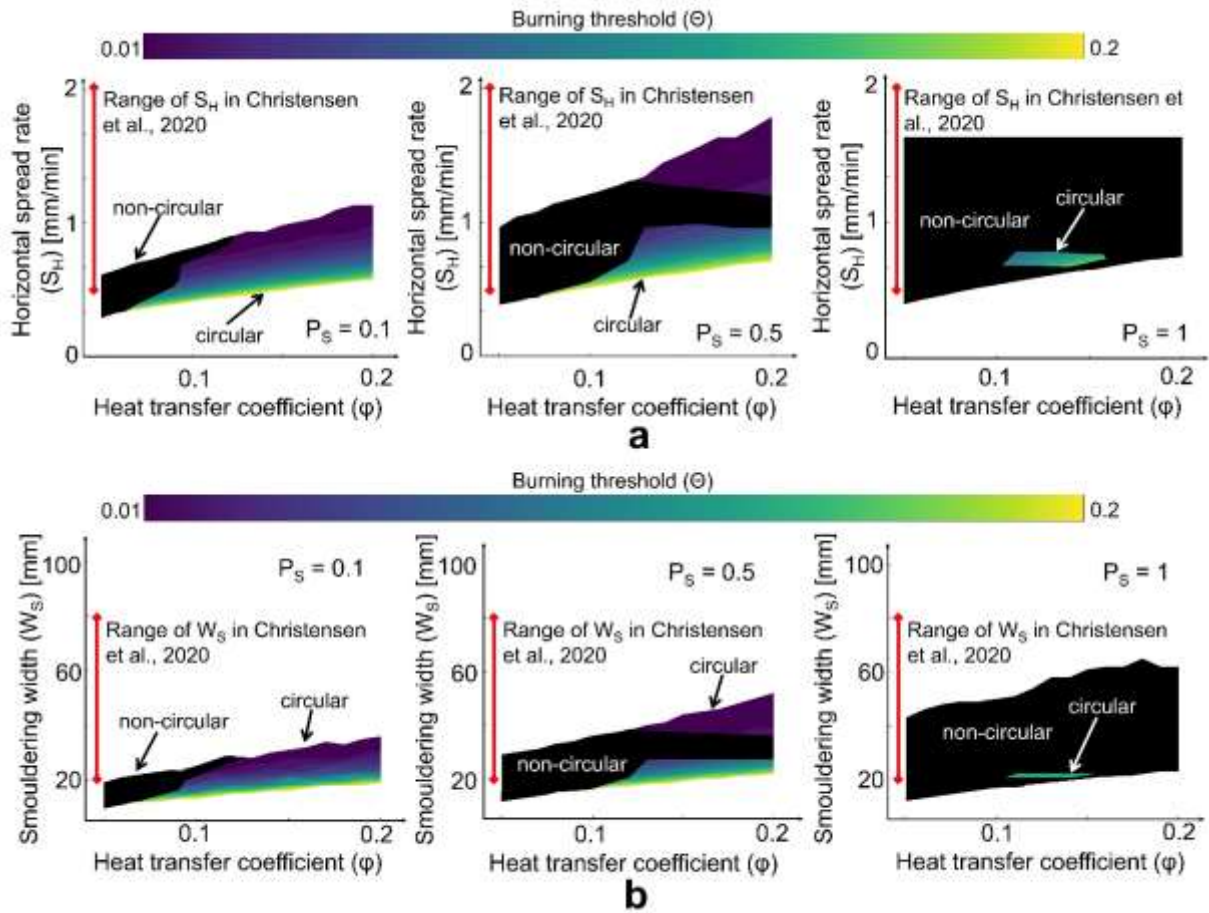
5 Dwi M J Purnomo¹, Eirik G Christensen¹, Nieves Fernandez-Anez², Guillermo Rein^{1,*}

6 ¹Leverhulme Centre for Wildfires, Environment and Society and Department of Mechanical
7 Engineering, Imperial College London, London, SW7 2AZ, United Kingdom

8 ²Department of Safety, Chemistry and Biomedical Laboratory Sciences, Western Norway
9 University of Applied Sciences, Bjørsonsgate 45, 5528, Haugesund, Norway

10

11 **A. Supplementary figures**



12

13 Figure A1. Sensitivity analysis of P_S on different Θ and φ based on (a) horizontal spread rate
 14 (S_H) and (b) smouldering width (W_S). Regions shaded in black represent simulations with non-
 15 circular shape (unexpected results).

16

17 **B. Derivation from first principle**

18 *Heat release rate*

19 Fig. 3 illustrates consumed peat at one time-step and its components. Consumed peat
 20 with the volume of ($\delta V = \Delta x^2 \delta$) contains water with a volume of δV_w , inorganic content with a
 21 volume of δV_i , air with a volume of δV_a , and organic content with a volume of δV_o . The heat
 22 release rate (Q_R) depends on the heat generation per m^3 (ΔH_c) and δV_o as shown in Eq. A1. By
 23 using mathematical operations, δV_o can be formulated as a function of organic density (ρ_o),
 24 particle density of the peat (ρ'_o), and δV . The thickness of the consumed peat is a function of
 25 vertical spread rate (S_d), therefore, Q_R can be formulated as shown in Eq. A6 where b_1 is a
 26 constant. By assuming constant ΔH_c , ρ'_o and Δx , Eq. A6 can be simplified to become Eq. 3.

$$27 \quad Q_R = \Delta H_c \delta V_o \quad (\text{A1})$$

$$28 \quad \frac{\delta V_o}{\delta V} = \frac{\rho_b}{\rho'_o} \cdot \frac{m_o}{m_b} = \frac{1}{\rho'_o} \cdot \frac{m_o}{\delta V} \quad (\text{A2})$$

$$29 \quad \frac{\delta V_o}{\delta V} = \frac{\rho_o}{\rho'_o} \quad (\text{A3})$$

$$30 \quad Q_R = \Delta H_c \frac{\rho_o}{\rho'_o} \delta V \quad (\text{A4})$$

$$31 \quad Q_R = \frac{\Delta H_c}{\rho'_o} \rho_o \Delta x^2 \delta \quad (\text{A5})$$

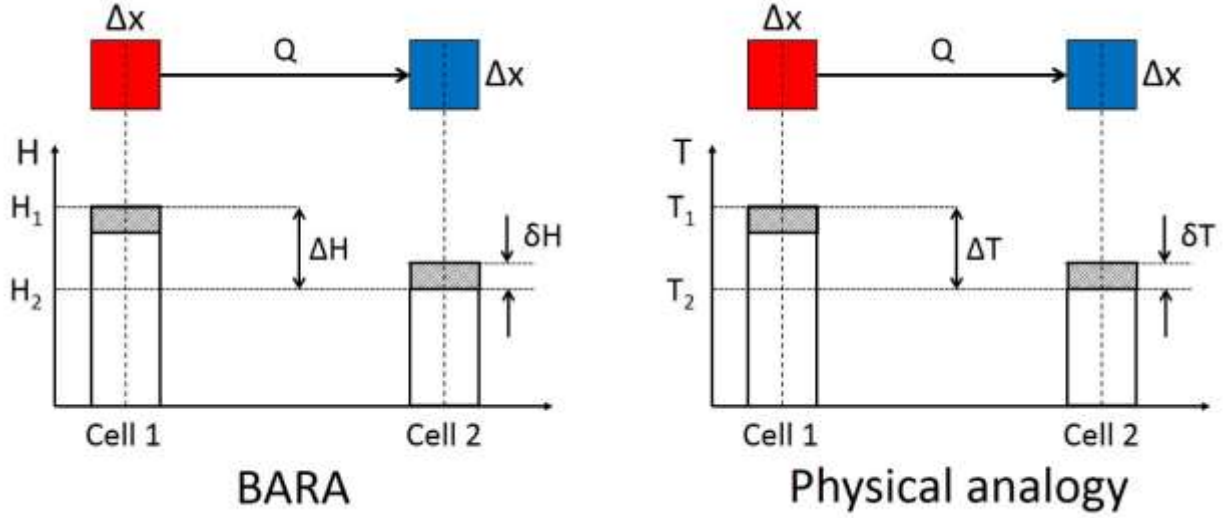
$$32 \quad Q_R = \left(b_1 \Delta x^2 \frac{\Delta H_c}{\rho'_o} \right) \rho_o S_d \quad (\text{A6})$$

33

34 *Heat transfer coefficient*

35 Fig. A2 shows the physical analogy of heat transfer in BARA and heat transfer in the
36 physical term. If two adjacent cells have different heat values (H), this condition is analogous to
37 having different temperatures (T) in physical term. Due to the temperature difference (ΔT), there
38 is heat transfer (Q) from a cell with higher temperature (red) to a cell with lower temperature
39 (blue). By assuming a 1D transient conduction heat transfer (Eq. A7) with effective thermal
40 conductivity, k_e (see Eq. A8) for porous media following Huang *et al.* (2015), a finite difference
41 method is applied to the Eq. A7 which results in Eq. A9. In these equations, T is temperature, t is
42 time, ρ_b is bulk density, c is specific heat, x is distance, k is material thermal conductivity, Φ is
43 porosity, γ is the radiative conductivity coefficient, and σ is the Stefan–Boltzmann constant.
44 Meanwhile, ΔT_{1-2} is the temperature difference between cell 1 (west neighbour) and cell 2
45 (centre cell) and ΔT_{3-2} is the temperature difference between cell 2 and cell 3 (east neighbour).
46 These equations are implemented in BARA where the heat transfer is discretized to one time step
47 and to one neighbour (the other three sides of the cell are assumed to be insulated). Therefore, in
48 Eq. A9, $\Delta t = 1$ and $\Delta T_{3-2} = 0$, resulting in Eq. A10 when the temperature is translated to heat
49 value. These processes are repeated independently for the other three neighbours (in this example
50 North, East, South), therefore, although the heat transfers between a centre cell and each
51 neighbour are not directly connected, eventually they affect one another. This neighbourhood
52 concept is one of the reasons CA becomes computationally efficient. To further simplify the
53 model, k_e is set to be constant resulting in Eq. A11 where b_2 is a constant. In BARA, the heat
54 value transferred to the neighbour (Q) is $\delta H / \Delta l$, where Δl is the number of cells that separate the

55 two cells which are interacting. Therefore, Eq. A11 can be further modified to be Eq. A12 where
 56 φ is heat transfer coefficient in BARA which depends on the ρ_b and c of the sample.



57
 58 Figure A2. Schematic of analogy of heat transfer in BARA and physical analogy.

59
$$\frac{\partial T}{\partial t} = \frac{k_e}{\rho_b c} \cdot \frac{\partial^2 T}{\partial x^2} \quad (A7)$$

60
$$k_e = k(1 - \Phi) + \gamma \sigma T^3 \quad (A8)$$

61
$$\frac{\Delta T}{\Delta t} = \frac{k_e}{\rho_b c} \cdot \frac{\Delta T_{1-2} + \Delta T_{3-2}}{\Delta x^2} \quad (A9)$$

62
$$\delta H = \frac{k_e}{\Delta x^2} \frac{\Delta H}{\rho_b c} \quad (A10)$$

63
$$\delta H = \frac{b_2}{\rho_b c} \Delta H \quad (A11)$$

64
$$Q = \varphi \frac{\Delta H}{\Delta l} \quad (A12)$$